сообщения
объединенного
института
ядерных
исследований
дубна

E2-84-777

A.A.Akhundov,* D.Yu.Bardin, O.M.Fedorenko,**
T.Riemann

# SOME INTEGRALS FOR EXACT CALCULATION OF QED BREMSSTRAHLUNG

* Institute of Physics, Acad.Sci.Azerbaijan SSR, Baku, USSR
** Dept. of Physics and Mathematics, State Univ. of Petrozawodsk, USSR.

1984

## 1. INTRODUCTION

We have calculated the exact to order $\alpha^3$ in QED cross section of

$$e^-(k_1, m) + e^+(k_2, m) \to f^-(p_2, \mu) + f^+(p_1, \mu) + \gamma(p, 0)$$

without neglecting any of the masses [1,2]. This is a novel result compared to the literature [3-5]. The present note contains some technical results we have to work out studying the above process, namely, the Bremsstrahlung integrals. These integrals are useful for the study of related processes, both in QED and QCD. They have been used as input for a SCHOONSCHIP - program of the analytic calculation, which by itself helps to organise the fourfold exact analytic integration of the squared matrix element to get the spectrum $d\sigma/dX$ ($X$ is the energy of $f^+$ in the cms - system) and the total cross section $\sigma_{tot}$. The integrals have been calculated by hand and tested in several ways, e.g., numerically and by comparison of $\sigma_{tot}$ with a result previously obtained in the limit $m \to 0$ [5]. Furthermore, some of the integrals have been used earlier in the crossed channel [6].

## 2. KINEMATICS

We use the following invariants:

$$k_1^2 = k_2^2 = -m^2, \quad p_1^2 = p_2^2 = -\mu^2, \quad p^2 = 0;$$

$$S = -(k_1 + k_2)^2, \quad t' = (k_1 - p_2)^2, \quad t = -2k_1 \cdot p_1, \quad S_t = S - t;$$

$$X = -2p_1 \cdot (k_1 + k_2), \quad S_X = S - X, \quad X_t = X - t, \quad S_{Xt} = S - X + t;$$

$$\tau = -(p_2 + p)^2 = S - X + \mu^2.$$

The following denominators of propagators will be utilized in what follows:

$$M^2 = -(p_1 + p_2)^2 \quad \text{(photon propagator for initial state radiation)}$$

$$X_M = -2p_1 p = X - M^2 \quad (f^+ \text{ is radiating})$$

$S_X = -2p_2 p = S - X$ ($f^-$ is radiating)

$Z = -2k_1 \cdot p$ ($e^-$ is radiating)

$\bar{Z} = -2k_2 \cdot p = S - M^2 - Z$ ($e^+$ is radiating)

The cms energies and 3-momenta may be conveniently expressed using the $\lambda$-function:

$$\lambda(x, y, z) = (x - y - z)^2 - 4yz$$

$$4S|\vec{k}_{1,2}|^2 = \lambda(S, m^2, m^2) = S^2 - 4m^2 S \equiv \lambda_S,$$

$$4S|\vec{p}_1|^2 = \lambda(S, \tau, \mu^2) = X^2 - 4\mu^2 S \equiv \lambda_X,$$

$$4S|\vec{p}_2|^2 = \lambda(S, X_M + \mu^2, \mu^2),$$

$$4S|\vec{p}|^2 = \lambda(S, M^2, 0) = (S - M^2)^2,$$

$$4Sk_{10}^2 = 4Sk_{20}^2 = S^2, \quad 4Sp_{10}^2 = X^2,$$

$$t = \frac{X}{2} - \frac{\sqrt{\lambda_S}\sqrt{\lambda_X}}{2S}\cos\theta, \quad X_t = \frac{X}{2} + \frac{\sqrt{\lambda_S}\sqrt{\lambda_X}}{2S}\cos\theta,$$

where

$$\theta = (\widehat{\vec{k}_1 \vec{p}_1})_{cms} \in [0, \pi].$$

Furthermore, we will make extensive use of the so-called R-system, defined by $\vec{p}_2 + \vec{p} = 0$, where:

$$4\tau|\vec{k}_1|^2 = \lambda(-X_t + m^2 + \mu^2, \tau, m^2) = S_t^2 - 4m^2\tau \equiv \lambda_{St},$$

$$4\tau|\vec{k}_2|^2 = \lambda(-t + m^2 + \mu^2, \tau, m^2) = S_{Xt}^2 - 4m^2\tau \equiv \lambda_{SXt},$$

$$4\tau|\vec{p}_1|^2 = \lambda(S, \tau, \mu^2) \equiv \lambda_X,$$

$$4\tau|\vec{p}_2|^2 = 4\tau|\vec{p}|^2 = \lambda(0, \tau, \mu^2) = S_X^2,$$

$$4\tau k_{10}^2 = S_t^2, \quad 4\tau k_{20}^2 = S_{Xt}^2,$$

$$4\tau p_{10}^2 = (X - 2\mu^2)^2, \quad 4\tau p_{20}^2 = (\tau + \mu^2)^2, \quad 4\tau p_0^2 = (\tau - \mu^2)^2 = S_X^2,$$

$$\cos\theta_1 = \frac{1}{\sqrt{\lambda_X}\sqrt{\lambda_{St}}} \cdot [S_t(X - 2\mu^2) - 2t\tau],$$

$$\theta_1 = (\widehat{\vec{k}_1 \vec{p}_1})_R \in [0, \pi].$$



The following quantity had to be calculated:

$$\Gamma(A) = \frac{\pi^2}{4S}\mathcal{J}^X\{\mathcal{J}^\theta[\mathcal{J}^R(A)]\},$$

where A is the corresponding squared Bremsstrahlung matrix element and

$\mathcal{J}^X(B) = \int_{2\mu\sqrt{S}}^{S} dX \cdot B$ – integration over the $f^+$-energy in the cms,

$\mathcal{J}^\theta(B) = \frac{1}{2}\int_{-1}^{+1} d\cos\theta \cdot B$ – integration over the cms-angle between $f^+$ and $e^-$,

$\mathcal{J}^R(B) = \frac{S_X}{4\pi}\int_{-1}^{+1} d\cos\theta_R \int_0^{2\pi} d\phi_R \cdot B$ – integration over the photon angles in the R-system.

## 3. THE INTEGRALS $\mathcal{J}^R(A) \equiv [A]$

To carry out the first (twofold) integration we use, e.g., the following quantities being defined in the R-system:

$$X_M = -2p_1 \cdot p = 2p_0[p_{10} - |\vec{p}_1|\cos\theta_R],$$

$$Z = -2k_1 \cdot p = 2p_0[k_{10} - |\vec{k}_1|(\cos\theta_1\cos\theta_R + \sin\theta_1\sin\theta_R\cos\phi_R)],$$

which may be expressed through invariants by using the formulae of section 2.

**Table of R-integrals**

$$[1] = \frac{S_X}{\tau}$$

$$[Z] = \frac{S_X^2 S_t}{2\tau^2}$$

$$[X_M] = \frac{S_X^2 a}{2\tau^2}$$

$$\left[\frac{1}{Z}\right] = \frac{1}{\sqrt{\lambda_{St}}} \cdot L_{St}$$

$$\left[\frac{1}{\bar{Z}}\right] = \left[\frac{1}{Z}\right]$$

$$\left[\frac{1}{M^2}\right] = \frac{1}{\sqrt{\lambda_X}}(L_X - L)$$

$$\left[\frac{1}{X_M}\right] = \frac{1}{\sqrt{\lambda_X}}(L_X + L)$$

$$\left[\frac{1}{Z^2}\right] = \left[\frac{1}{\bar{Z}^2}\right] = \frac{1}{m^2 S_X}$$

$$\left[\frac{1}{X_M^2}\right] = \frac{1}{\mu^2 S_X}$$

$$\left[\frac{1}{M^4}\right] = \frac{S_X}{\mu^2 S^2}$$

$$\left[\frac{1}{Z\bar{Z}}\right] = \frac{2}{S_X\sqrt{\lambda_S}} L_S$$

$$\left[\frac{1}{ZX_M}\right] = \frac{2}{S_X\sqrt{\lambda_t}} L_t$$

$$\left[\frac{1}{\bar{Z}X_M}\right] = \left[\overline{\frac{1}{ZM_M}}\right]$$

$$\left[\frac{1}{ZM^2}\right] = \frac{1}{S\sqrt{\lambda_{Xt}}} L_{Xt}$$

$$\left[\frac{1}{\bar{Z}M^2}\right] = \left[\overline{\frac{1}{ZM^2}}\right]$$

$$\left[\frac{X_M}{Z}\right] = \frac{S_X}{\lambda_{St}}(S_t \cdot t - 2m^2 a)\left[\frac{1}{Z}\right] + \frac{S_X}{\lambda_{St}}\left[\frac{S_t(2S - X)}{r} - 2S\right]$$

$$\left[\frac{X_M}{\bar{Z}}\right] = \left[\overline{\frac{X_M}{Z}}\right]$$

$$\left[\frac{Z}{X_M}\right] = \frac{S_X b}{\lambda_X}\left[\frac{1}{X_M}\right] + \frac{S_X}{\lambda_X}\left[\frac{S_t(S - \mu^2)}{r} - S - t\right]$$

$$\left[\frac{Z}{X_M^2}\right] = \frac{1}{\lambda_X}\left[X(S + t) - 2S(t + \mu^2)\right]\cdot\left[\frac{1}{X_M}\right] + \frac{b}{\mu^2\lambda_X}$$

$$\left[\frac{Z^2}{X_M}\right] = \frac{S_X^2}{\lambda_X}(t^2 + 2\mu^2 c)\left[\frac{1}{X_M}\right] - \frac{S_X^2}{\lambda_X r}\left[2tS_t + a\left(c - \frac{S_t^2}{2r}\right)\right]$$



$$\left[\frac{Z^2}{X_M^2}\right] = -\frac{2S_X}{\lambda_X}(tS_t + a\cdot c)\left[\frac{1}{X_M}\right] + \frac{S_X}{\lambda_X}\left(\frac{S_t^2}{r} + \frac{t^2}{\mu^2} + 4c\right)$$

$$\left[\frac{Z}{M^2}\right] = \frac{S}{\lambda_X}\bar{b}\left[\frac{1}{M^2}\right] - \frac{S_X}{\lambda_X}\left[\frac{S_t(S - \mu^2)}{r} - S - t\right]$$

$$\left[\frac{1}{Z^2M^2}\right] = \frac{1}{S\lambda_{Xt}}\bar{b}\left[\frac{1}{ZM^2}\right] + \frac{1}{S\lambda_{Xt}}\left(\frac{\lambda_{Xt} + X_t S_X}{m^2 S_X} - \frac{2X}{S}\right)$$

$$\left[\frac{1}{\bar{Z}^2M^2}\right] = \left[\overline{\frac{1}{Z^2M^2}}\right]$$

$$\left[\frac{1}{ZM^4}\right] = \frac{1}{S\lambda_{Xt}}\left[S_t X_t - 2m^2\left(\frac{S_X X}{S} + 2\mu^2\right)\right]\left[\frac{1}{ZM^2}\right] + \frac{S_X\bar{b}}{\mu^2 S^3\lambda_{Xt}}$$

$$\left[\frac{1}{\bar{Z}M^4}\right] = \left[\overline{\frac{1}{ZM^4}}\right]$$

$$\left[\frac{1}{Z^2M^4}\right] = \frac{1}{S\lambda_{Xt}}\left[\frac{S_X t}{S} - X_t + 2\mu^2 + \frac{3\bar{b}}{S\lambda_{Xt}}\left(S_t X_t - 4m^2\mu^2 - \frac{2m^2 S_X\cdot X}{S}\right)\right]\left[\frac{1}{ZM^2}\right] +$$

$$+ \frac{1}{m^2 S^2\lambda_{Xt}}\left(S_t + X_t - 4m^2 + \frac{\lambda_{Xt}}{S_X}\right) + \frac{S_X\lambda_X}{\mu^2 S^4\lambda_{Xt}} + \frac{12 S_X}{S^4\lambda_{Xt}^2}(m^2\lambda_X + \mu^2 S^2 - StX_t)$$

$$\left[\frac{1}{\bar{Z}^2M^4}\right] = \left[\overline{\frac{1}{Z^2M^4}}\right]$$

We employ the following abbreviations:

$a = X - 2\mu^2$,

$b = X\cdot t - 2\mu^2 S$,

$c = m^2 + \frac{3S}{\lambda_X}(\mu^2 S - tX_t)$,

$\lambda_t = t^2 - 4m^2\mu^2$,

$\lambda_{Xt} = S_t^2 - 4m^2\mu^2$,

$$L = \ln \frac{2S - X + \sqrt{\lambda_X}}{2S - X - \sqrt{\lambda_X}},$$

$$L_v = \ln \frac{v + \sqrt{\lambda_v}}{v - \sqrt{\lambda_v}}, \quad v = S,\ X,\ t,\ X_t,\ S_t,\ S_{Xt}.$$

The quantity $\bar{A}$ is defined as follows:

$$\overline{A(t, X_t, \ldots)} = A(X_t, t, \ldots).$$

At the end of this section we would like to note that some of the integrals contained in the above table have been calculated for the crossed channel in the earlier publication /6/.

## 4. THE INTEGRALS $\mathcal{J}^{\theta}(A) \equiv [A]$

$$[1] = 1$$

$$[t] = \frac{X}{2}$$

$$[t^2] = \frac{1}{4}\left(X^2 + \frac{\lambda_S \lambda_X}{3S^2}\right)$$

$$\left[\frac{1}{\lambda_t}\right] = \frac{S}{2m\mu} \cdot \frac{1}{\sqrt{\lambda_S}\sqrt{\lambda_X}} \cdot \ln \frac{\mu\sqrt{\lambda_S} + m\sqrt{\lambda_X}}{\mu\sqrt{\lambda_S} - m\sqrt{\lambda_X}}$$

$$\left[\frac{1}{\lambda_t^2}\right] = \frac{1}{8m^2\mu^2}\left[\frac{S}{d^2}(\mu^2 S^2 + m^2 X^2) - \left[\frac{1}{\lambda_t}\right]\right]$$

$$\left[\frac{t}{\lambda_t}\right] = \frac{S}{\sqrt{\lambda_S}\sqrt{\lambda_X}} \ln \frac{S\sqrt{\lambda_X} + X\sqrt{\lambda_S}}{S\sqrt{\lambda_X} - X\sqrt{\lambda_S}}$$

$$\left[\frac{t}{\lambda_t^2}\right] = \frac{S^2 X}{2d^2}$$

$$\left[\frac{1}{\sqrt{\lambda_t}} L_t\right] = \frac{S}{\sqrt{\lambda_S}\sqrt{\lambda_X}} L_S L_X$$

$$\left[\frac{1}{\sqrt{\lambda_{St}}} L_{St}\right] = \frac{S}{\sqrt{\lambda_S}\sqrt{\lambda_X}} L_S L$$



$$[\sqrt{\lambda_t} L_t] = -2m^2\mu^2\left[\frac{1}{\sqrt{\lambda_t}} L_t\right] + \frac{X(S - 2m^2)}{2\sqrt{\lambda_S}} L_S + \frac{X^2 - 2\mu^2 S}{2\sqrt{\lambda_X}} L_X - \frac{X}{2}$$

$$[\sqrt{\lambda_{St}} L_{St}] = -2m^2 r\left[\frac{1}{\sqrt{\lambda_{St}}} L_{St}\right] + \frac{(2S - X)(S - 2m^2)}{2\sqrt{\lambda_S}} L_S + \frac{S(S - 2\mu^2) + S_X^2}{2\sqrt{\lambda_X}} L - \frac{2S - X}{2}$$

$$\left[\frac{1}{\lambda_t^{3/2}} L_t\right] = \frac{1}{2m^2\mu^2}\left[\frac{t}{\lambda_t}\right] + \frac{S}{2d}\left(\frac{X}{\mu^2\sqrt{\lambda_S}} L_S - \frac{S}{m^2\sqrt{\lambda_X}} L_X\right)$$

$$\left[\frac{t}{\lambda_t^{3/2}} L_t\right] = 2\left[\frac{1}{\lambda_t}\right] + \frac{S}{d}\left(\frac{S}{\sqrt{\lambda_S}} L_S - \frac{X}{\sqrt{\lambda_X}} L_X\right)$$

$$\left[\frac{1}{\lambda^{5/2}} L_t\right] = \frac{1}{2m^2\mu^2}\Big\{-\frac{1}{2}\left[\frac{1}{\lambda_t^{3/2}} L_t\right] + \frac{1}{12m^2\mu^2}\left[\frac{t}{\lambda_t}\right] + \frac{1}{3}\left[\frac{t}{\lambda_t^2}\right] +$$

$$+ \frac{S}{12d^3}\Big[\frac{1}{\mu^2}(m^4X^2\lambda_X + 3\mu^4S^2\lambda_S)\frac{X}{\sqrt{\lambda_S}} L_S -$$

$$- \frac{1}{m^2}(\mu^4S^2\lambda_S + 3m^4X^2\lambda_X)\frac{S}{\sqrt{\lambda_X}} L_X\Big]\Big\}$$

$$\left[\frac{t}{\lambda_t^{5/2}} L_t\right] = \frac{2}{3}\left[\frac{1}{\lambda_t^2}\right] + \frac{S}{12d^3}\left[\frac{S}{\sqrt{\lambda_S}}(S^2\lambda_X + 3X^2\lambda_S) L_S - \frac{X}{\sqrt{\lambda_X}}(X^2\lambda_S + 3S^2\lambda_X) L_X\right]$$

Here

$$d = \mu^2 S - m^2 X.$$

We tabulated only a subset of integrals. The others may be obtained using the property

$$\int_{-1}^{+1} d\cos\theta\, f(\cos\theta) = \int_{-1}^{+1} d\cos\theta\, f(-\cos\theta)$$

that allows us to replace $t$ and $X_t$ under the integral wherever it occurs:

$$[A(X_t)] = [A(t)].$$

## 5. THE INTEGRALS $\mathcal{J}^X(A) \equiv [A]$

$$[\sqrt{\lambda_X}] = S\left(\frac{1}{2}\sqrt{\lambda^F_S} - \mu^2 L^F_S\right)$$

$$[X\sqrt{\lambda_X}] = \frac{1}{3}(\lambda^F_S)^{3/2}$$

$$\left[\frac{\sqrt{\lambda_X}}{r}\right] = (S - 2\mu^2) L^F_S - \sqrt{\lambda^F_S}$$

$$\left[\frac{\sqrt{\lambda_X}}{r^2}\right] = \frac{\sqrt{\lambda^F_S}}{\mu^2} - \frac{S + 2\mu^2}{S - \mu} L^F_S$$

$$[L] = \sqrt{\lambda^F_S} - 2\mu^2 L^F_S$$

$$[XL] = \frac{1}{4}(3S - 2\mu^2)\sqrt{\lambda^F_S} - \mu^2(S + \mu^2) L^F_S$$

$$[L_X] = S L^F_S - 2\sqrt{\lambda^F_S}$$

$$[XL_X] = \frac{S}{2}\left[(S - 2\mu^2) L^F_S - \sqrt{\lambda^F_S}\right]$$

$$[X^2 L_X] = \frac{S^3}{3} L^F_S - \frac{2}{9}(\lambda^F_S)^{3/2} - \frac{8}{3}\mu^2 S \sqrt{\lambda^F_S}$$

Here

$$L^F_S \equiv L_X|_{X=S} = \ln\frac{S + \sqrt{\lambda^F_S}}{S - \sqrt{\lambda^F_S}},$$

$$\lambda^F_S \equiv \lambda(S, m^2, \mu^2) = S^2 - 4\mu^2 S.$$

The above integrands are finite in the limit $X \to S$ and, thus, yield finite integrals. Two integrals, however, are IR-divergent and must be regularized:

$$\left[\frac{\sqrt{\lambda_X}}{S - X}\right]^{reg} = \sqrt{\lambda^F_S}\ln\frac{\lambda^F_S}{2\mu^2\sqrt{S}\,\bar\omega} - \sqrt{\lambda^F_S} - \frac{S}{2} L^F_S$$

$$\left[\frac{L_X}{S - X}\right]^{reg} = L^F_S \ln\frac{S}{2\mu\bar\omega} + 2L^F_S \ln\frac{S + \sqrt{\lambda^F_S}}{2S} - \frac{1}{4}(L^F_S)^2 -$$



$$- \frac{3}{2}\Phi(1) + 2\Phi\left(\frac{S - \sqrt{\lambda^F_S}}{S + \sqrt{\lambda^F_S}}\right) + \Phi\left(-\frac{S - \sqrt{\lambda^F_S}}{S + \sqrt{\lambda^F_S}}\right),$$

where

$$L^F_X = \frac{1}{2}(L + L_X) = \frac{1}{2}\ln\frac{X - 2\mu^2 + \sqrt{\lambda_X}}{X - 2\mu^2 - \sqrt{\lambda_X}},$$

$\Phi(X) = -\int_0^X y^{-1}\cdot dy \cdot \ln|1-y|$ is the Spence function, and the range of integration has been limited to $[2\mu\sqrt{S},\ S - 2\mu\bar\omega]$. The limit $\bar\omega \to 0$ has been taken wherever this is possible.

## 6. INTEGRALS CONNECTED WITH THE IR-DIVERGENCE

If the IR-divergence is handled with the help of the general method developed in /7/ the following integrals arise:

$$\mathcal{J}^\theta\left[\frac{1}{\sqrt{\lambda_{t\alpha}}} L_{t\alpha}\right] = \frac{S}{\sqrt{\lambda_S}\sqrt{\lambda^F_S}} \cdot \frac{1}{u} \cdot L_u L^F_S$$

$$\mathcal{J}^\theta\left[\frac{t_\alpha}{\sqrt{\lambda_{t\alpha}}} L_{t\alpha}\right] = \frac{S}{\sqrt{\lambda_S}} \cdot \frac{1}{u} \cdot L_u + \frac{S}{\sqrt{\lambda^F_S}} L^F_S - 2$$

$$\mathcal{J}^\theta[\sqrt{\lambda_{t\alpha}} L_{t\alpha}] = -2\mu^2(-k^2_\alpha)\mathcal{J}^\theta\left[\frac{1}{\sqrt{\lambda_{t\alpha}}} L_{t\alpha}\right] + \frac{S^2 + u^2\lambda_S}{4\sqrt{\lambda_S}\,u} L_u +$$

$$+ \frac{S(S - 2\mu^2)}{2\sqrt{\lambda^F_S}} L^F_S - \frac{S}{2}$$

$$\mathcal{J}^\theta[t_\alpha\sqrt{\lambda_{t\alpha}} L_{t\alpha}] = \frac{8}{3}\mu^2(-k^2_\alpha) - \frac{S^2}{6} - \frac{(S - 4m^2)(S - 4\mu^2)}{18} u^2 +$$

$$+ \frac{S}{12}\left(\frac{\lambda^F_S + 3u^2\lambda_S}{u\sqrt{\lambda_S}} L_u + \frac{3\lambda^F_S + u^2\lambda_S}{\sqrt{\lambda^F_S}} L^F_S\right),$$

where

$$u = 2\alpha - 1,$$

$$-k^2_\alpha = m^2 + (S - 4m^2)\alpha(1 - \alpha),$$

$$t_\alpha = (S - t_0)\alpha + t_0 \cdot (1 - \alpha), \quad t_0 = \frac{S}{2} - \frac{\sqrt{\lambda_S}\sqrt{\lambda_S^F}}{2S}\cos\theta$$

$$\lambda_{t\alpha} = t_\alpha^2 - 4\mu^2(-k_\alpha^2),$$

$$L_{t\alpha} = \ln\frac{t_\alpha + \sqrt{\lambda_{t\alpha}}}{t_\alpha - \sqrt{\lambda_{t\alpha}}}, \quad L_u = \ln\frac{S + u\sqrt{\lambda_S}}{S - u\sqrt{\lambda_S}}.$$

The remaining integrals over the Feynman parameter $\alpha$ may be calculated using the following expressions:

$$\mathfrak{I}^\alpha\left[\frac{1}{-k_\alpha^2}\right] = \frac{2}{\sqrt{\lambda_S}} L_S$$

$$\mathfrak{I}^\alpha\left[\frac{1}{u} L_u\right] = \Phi\left(\frac{\sqrt{\lambda_S}}{S}\right) - \Phi\left(-\frac{\sqrt{\lambda_S}}{S}\right)$$

$$\mathfrak{I}^\alpha\left[\frac{u}{-k_\alpha^2} L_u\right] = \frac{S}{\lambda_S}\left[L_S \ln\frac{S}{m^2} + 2\Phi\left(\frac{S - \sqrt{\lambda_S}}{2S}\right) - 2\Phi\left(\frac{S + \sqrt{\lambda_S}}{2S}\right)\right]$$

$$\mathfrak{I}^\alpha\left[\frac{1}{u^2}\right] = \frac{1}{\epsilon} - 1$$

$$\mathfrak{I}^\alpha\left[\frac{1}{u^3} L_u\right] = \frac{\sqrt{\lambda_S}}{S}\left(\frac{2}{\epsilon} - 1\right) - \frac{2m^2}{S} L_S,$$

where

$$\mathfrak{I}^\alpha[A] \equiv \int_0^1 d\alpha \cdot A,$$

and $\epsilon$ is a regularization parameter.

## REFERENCES


1. Akhundov A.A., Bardin D.Yu., Fedorenko O.M., Riemann T. Proc. XVIII Int.Symp. on Quantum Field Theory. GDR, Ahrenshoop, 1984; Berlin-Zeuthen, 1984, PHE 84-11, p. 38.
2. Akhundov A.A. et al. JINR, E2-84-787, Dubna, 1984.
3. Kuraev E.A., Meledin G.V. Nucl.Phys., 1977, B122, p. 485.
4. Berends F.A., Kleiss R. Nucl.Phys., 1981, B177, p. 237.
5. Berends F.A., et al. Acta Phys.Pol., 1983, B14, p. 413.
6. Bardin D.Yu., Shumeiko N.M., Fedorenko O.M. JINR, P2-10114, Dubna, 1976.
7. Bardin D.Yu., Shumeiko N.M. Nucl.Phys., 1977, B127, p. 242.



Received by Publishing Department
on December 6, 1984.




В Объединенном институте ядерных исследований начал выходить сборник *"Краткие сообщения ОИЯИ"*. В нем будут помещаться статьи, содержащие оригинальные научные, научно-технические, методические и прикладные результаты, требующие срочной публикации. Будучи частью "Сообщений ОИЯИ", статьи, вошедшие в сборник, имеют, как и другие издания ОИЯИ, статус официальных публикаций.

Сборник "Краткие сообщения ОИЯИ" будет выходить регулярно.

The Joint Institute for Nuclear Research begins publishing a collection of papers entitled *JINR Rapid Communications* which is a section of the JINR Communications and is intended for the accelerated publication of important results on the following subjects:

Physics of elementary particles and atomic nuclei.
Theoretical physics.
Experimental techniques and methods.
Accelerators.
Cryogenics.
Computing mathematics and methods.
Solid state physics. Liquids.
Theory of condenced matter.
Applied researches.

Being a part of the JINR Communications, the articles of new collection like all other publications of the Joint Institute for Nuclear Research have the status of official publications.

*JINR Rapid Communications* will be issued regularly.

COMMUNICATIONS, JINR RAPID COMMUNICATIONS, PREPRINTS, AND PROCEEDINGS OF THE CONFERENCES PUBLISHED BY THE JOINT INSTITUTE FOR NUCLEAR RESEARCH HAVE THE STATUS OF OFFICIAL PUBLICATIONS.

JINR Communication and Preprint references should contain:

- names and initials of authors,
- abbreviated name of the Institute (JINR) and publication index,
- location of publisher (Dubna),
- year of publication
- page number (if necessary).

For example:

1. *Pervushin V.N. et al. JINR, P2-84-649, Dubna, 1984.*

References to concrete articles, included into the Proceedings, should contain

- names and initials of authors,
- title of Proceedings, introduced by word "In:"
- abbreviated name of the Institute (JINR) and publication index,
- location of publisher (Dubna),
- year of publication,
- page number.

For example:

*Kolpakov I.F. In: XI Intern. Symposium on Nuclear Electronics, JINR, D13-84-53, Dubna, 1984, p.26.*

*Savin I.A., Smirnov G.I. In: JINR Rapid Communications, N2-84, Dubna, 1984, p.3.*

Ахундов А.А. и др. E2-84-777

Интегралы, встречающиеся при точном вычислении тормозного излучения в КЭД

Приведен исчерпывающий набор интегралов, который был использован при получении с помощью системы аналитических вычислений SCHOONSCHIP точного результата для энергетического спектра фермионов и полного сечения процесса $e^+e^- \to f^+f^-(\gamma)$ $(f = \mu, \tau, \dots)$ в порядке $\alpha^3$ в КЭД. Эти интегралы вычислены без пренебрежения массами частиц и представляют общий интерес для точных вычислений процессов тормозного излучения в КЭД, а также аналогичных процессов в квантовой хромодинамике.

Работа выполнена в Лаборатории теоретической физике ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1984

Akhundov A.A. et al. E2-84-777

Some Integrals for Exact Calculation of QED Bremsstrahlung

An exhaustive list of integrals is presented which has been used together with the system of analytical calculations SCHOONSCHIP to obtain exact results for the energetic fermion spectrum and the total cross section of the process $e^+e^- \to f^+f^-(\gamma)$ $(f = \mu, \tau, \dots)$ to order $\alpha^3$ in QED. These integrals are calculated without neglections in the particle masses. They are of general interest for exact calculations of bremsstrahlung processes in QED and in analogue processes of quantum chromodynamics.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1984