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SIX- AND EIGHT-DIMENSIONAL GLUON CONDENSATES FROM LATTICE QCD

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1. INTRODUCTION

Gluon and quark condensates - i.e., non-vanishing vacuum expectation values of local, composite gluon and quark operators - are important parameters of non-perturbative QCD. Within the framework of QCD sum rules [1] they play the role of phenomenological numbers. Ultimately, one should be able to calculate them from a theory of the vacuum structure. So far models based on instanton solutions yield only rough estimates [2,3].

Recently, Nikolaev and Radyushkin [4] have computed the coefficients of higher dimensional operator contributions to the charmonium sum rules usually written down for moments, i.e., derivatives of the vacuum polarization function. Having estimated the $\langle 0|G^4|0\rangle$ matrix elements employing instanton gas models as well as the factorization hypothesis $\langle 0|G^4|0\rangle \sim \langle 0|G^2|0\rangle\langle 0|G^2|0\rangle$ they found these contributions to be very large and possibly to spoil the convergence of the operator expansion. In Ref. [5] it has been argued that the convergence can be considerably improved if the moments are defined at a momentum $Q^2 \neq 0$ instead of $Q^2=0$. However, the estimates were based on the factorization hypothesis as well. The latter has been criticized [6] with $1/N_c$ arguments. On the other hand naive factorization is violated due to operator mixing with respect to renormalization [7,8]. Thus, independent $\langle G^4\rangle$ estimates are highly desirable.

Monte-Carlo simulations in lattice QCD proved to be a powerful instrumentarium for calculating non-perturbative numbers such as the string tension, hadron and glueball masses, deconfinement temperature, etc. They provided a strong indication for chiral symmetry breaking [9] as well as for non-vanishing vacuum expectation values of $g^2 G^a_{\mu\nu} G^a_{\mu\nu}(x)$ [10] and $g^3 f^{abc} G^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu}(x)$ [11]. The numerical value for $\langle g^2 G^a_{\mu\nu} G^a_{\mu\nu}(x)\rangle$ in the SU(3) case [12] turned out to be very close to the phenomenological one.

In this paper we present first numerical estimates of the $\langle G^3\rangle$ and $\langle G^4\rangle$ matrix elements from Monte-Carlo simulations in pure SU(3) Yang-Mills theory. Our method is analogous to the one applied in Refs. [10,11], which has been shown [13] to yield values for the ratio $\langle g^2 G^2\rangle/\sigma^2$ (σ denoting the string tension) independent of the subtraction scheme and of the lattice size.

2. OUTLINE OF THE METHOD

The condensation parameter corresponding to an operator $O(x)$ is usually understood as the finite non-perturbative remainder after having subtracted all perturbative (divergent) diagrams. Therefore let us formally denote it by $\langle 0|:O(x):|0\rangle$ according to a "normal product" prescription with respect to the perturbative vacuum state. Then, one imagines distinct large-scale fluctuations in the vacuum (instantons, etc.) due to which $\langle 0|:O(x):|0\rangle \neq 0$. In the following we consider the local composite operators of the gluon field $G_{\mu\nu}(x) = G^a_{\mu\nu}(x)\, T^a$ (T^a the generators of SU(3) in the fundamental representation)

$$S(x) = g^3 f^{abc} G^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu} ,$$

$$O_1(x) = (g^2 G^a_{\lambda\mu} G^a_{\mu\nu}(x))^2, \quad O_3(x) = (g^2 f^{abc} G^b_{\lambda\mu} G^c_{\mu\nu}(x))^2,$$

$$O_5(x) = (g^2 d^{abc} G^b_{\lambda\mu} G^c_{\mu\nu}(x))^2, \tag{1a}$$

$$O_2(x) = (g^2 G^a_{\lambda\mu} G^a_{\nu\rho}(x))^2, \quad O_4(x) = (g^2 f^{abc} G^b_{\lambda\mu} G^c_{\nu\rho}(x))^2,$$

$$O_6(x) = (g^2 d^{abc} G^b_{\lambda\mu} G^c_{\nu\rho}(x))^2,$$

f^{abc}, d^{abc} represent the antisymmetric and symmetric structure constants of SU(3). Summation over colour and Lorentz indices is understood. The operators $O_1, \ldots, O_6$ represent a complete set of $D=8$ scalar operators.

The renormalization properties of S, $O_1, \ldots, O_6$ have been studied in detail in Refs. /7,8/. In pure SU(3) gauge theory the six operators $O_1, \ldots, O_6$ have been shown to mix with each other under renormalization.

Renormalization-invariant combinations are at the lowest order

$$\Sigma(x) \equiv g^{\frac{2\gamma_\Sigma}{b}} S(x), \quad \gamma = -6N_C = -18, \tag{2a}$$

$$\Omega_i(x) \equiv g^{\frac{2\gamma_i}{b}} Q_i(x) \equiv g^{\frac{2\gamma_i}{b}} \sum_{j=1} C_{ij} O_j(x), \quad i = 1,2,\ldots,6. \tag{2b}$$

The anomalous dimensions γ_i and the matrix C are quoted in the appendix ($b = \frac{11N}{3}$). In fact, we shall determine first the invariant condensate matrix elements $\langle 0|:\Sigma:|0\rangle$, $\langle 0|:\Omega_i:|0\rangle$ from lattice data. Then, by inverting (2b) the quantities $\langle 0|:O_i:|0\rangle$ will be estimated at different subtraction scales.

On a hyper-cubic 4D lattice with lattice spacing a let us define operators tending to expressions (1) in the naive continuum limit.

$$S^L(n) = 2\,\mathrm{tr}((U_{\mu\nu} - 1)[U_{\nu\rho} - 1, U_{\rho\mu} - 1]) = 2\,\mathrm{tr}(U_{\mu\nu}[U_{\nu\rho}, U_{\rho\mu}]), \tag{3a}$$

$$\left.\begin{aligned}
&O^L_1(n) = 4(\mathrm{tr}((U_{\lambda\mu} - 1)(U_{\mu\nu} - 1)))^2, \quad O^L_2(n) = 4(\mathrm{tr}((U_{\lambda\mu} - 1)(U_{\nu\rho} - 1)))^2, \\
&O^L_3(n) = -2\,\mathrm{tr}([U_{\lambda\mu} - 1, U_{\mu\nu} - 1]^2), \quad O^L_4(n) = -2\,\mathrm{tr}([U_{\lambda\mu} - 1, U_{\nu\rho} - 1]^2), \\
&O^L_5(n) = 2\,\mathrm{tr}(\{U_{\lambda\mu} - 1, U_{\mu\nu} - 1\}^2) - \tfrac{2}{3} O^L_1(n), \\
&O^L_6(n) = 2\,\mathrm{tr}(\{U_{\lambda\mu} - 1, U_{\nu\rho} - 1\}^2) - \tfrac{2}{3} O^L_2(n).
\end{aligned}\right\} \tag{3b}$$

$U_{\mu\nu} = e^{ia^2 g_0 G_{\mu\nu}(x_n) + O(a^3)} \in$ SU(3) represents the Wilson loop attached to a given lattice site $x_n \equiv n$ around an elementary plaquette in the $\mu-\nu$ plane. It can be easily realized that for $a \to 0$, $S^L \to a^6 S + O(a^7)$, $O^L_i \to a^8 O_i + O(a^9)$, $i = 1,2,\ldots,6$. The choice of lattice operators (3) is not unique. Different lattice operators deviate from each other in the continuum limit by the higher order contributions $O(a^7)$ and $O(a^9)$, respectively. In order to check the consistency of our calculations we employ also the modified operators

$$\left.\begin{aligned}
&\hat{O}^L_1(n) = 4(\mathrm{tr}((U_{\lambda\mu} - 1)(U^+_{\mu\nu} - 1)))^2, \\
&\hat{O}^L_3(n) = -2\,\mathrm{tr}([U_{\lambda\mu} - 1, U^+_{\mu\nu} - 1]^2), \\
&\hat{O}^L_5(n) = 2\,\mathrm{tr}(\{U_{\lambda\mu} - 1, U^+_{\mu\nu} - 1\}) - \tfrac{2}{3} \hat{O}^L_1 ,
\end{aligned}\right\} \tag{3c}$$

having the same continuum limit as O^L_i $(i=1,3,5)$. Furthermore, we introduce the lattice analogue of expression (2b)

$$Q^L_i(n) \equiv \sum_{j=1}^{6} C_{ij} O^L_j(n), \tag{4}$$

which can be modified replacing O^L_i by $\hat{O}^L_i$ $(i = 1,3,5)$, respectively.

In the following we want to compute the vacuum expectation values $\langle X^L \rangle = \int\{du\} e^{-S} X^L / \int\{du\} e^{-S}$, $X^L = S^L$, O^L_i, numerically by Monte-Carlo simulations on a finite lattice. Wilson's action $S = \frac{1}{g^2} \sum_{n;\mu\neq\nu} \mathrm{tr}(1 - U_{\mu\nu}(n))$ is assumed. The behaviour of $\langle X^L \rangle$ as functions of the bare coupling g_0 can be analytically determined at $g_0 \ll 1$, applying usual perturbation theory as well as at $g_0 \gg 1$ using the high temperature expansion. In the weak coupling regime we have

$$\langle S^L\rangle_{pert.} = \sum_{n=2,3,\ldots} b_n g_0^{2n}\,, \quad \langle O_i^L\rangle_{pert.} = \sum_{n=2,3,\ldots} a_{in} g_0^{2n}\,, \quad i=1,\ldots,6. \quad (5)$$

Applying the Feynman rules according to Ref./14/ we obtained for the lowest order coefficients in the $SU(N_C)$ case

$$b_2 = 72N_C(N_C^2-1)\left(\frac{I}{4}-\frac{I^2}{2}-\frac{1}{64}+2I'\right), \;*$$

$$a_{12} = (N_C^2-1)((48I^2+9)N_C^2+48I^2+3),$$

$$a_{22} = (N_C^2-1)((96I^2+6)N_C^2+36), \quad a_{32} = (N_C^2-1)N_C\cdot 6,$$

$$a_{42} = (N_C^2-1)N_C(-96I^2+30), \quad a_{52} = \frac{1}{N_C}(N_C^2-1)(N_C^2-4)(96I^2+12),$$

$$a_{62} = \frac{1}{N_C}(N_C^2-1)(N_C^2-4)(96I^2+42),$$

where

$$I = \int_{-\pi}^{\pi} \frac{d^4p}{(2\pi)^4}\,\frac{\hat{p}_\mu \hat{p}_\nu^*}{|\hat{p}|^2},$$

$$I' = \int_{-\pi}^{\pi} \frac{d^4p}{(2\pi)^4}\int_{-\pi}^{\pi} \frac{d^4q}{(2\pi)^4}\,\frac{\hat{p}_\mu \hat{k}_\mu (1+\frac{1}{2}\hat{p}_\rho^*)\hat{q}_\nu(\hat{q}_\nu^*-\hat{k}_\nu^*)}{(\hat{p}\hat{p}^*)(\hat{q}\hat{q}^*)(\hat{k}\hat{k}^*)},$$

($\mu \neq \nu \neq \rho \neq \mu$, without summation), $\hat{p}_\mu = e^{ip_\mu}-1$, $k_\mu = -p_\mu - q_\mu$, $(\hat{p}\hat{p}^*) = \sum_{\alpha=1}^{4} \hat{p}_\alpha \hat{p}_\alpha^*$. A numerical computation yields for a 4^4 lattice $I \simeq 0.107$, $I' \simeq 0.00148$. Thus we have for SU(3) $b_2 = 14.6$, $a_{12} = 716.$, $a_{22} = 799.$, $a_{32} = 144$, $a_{42} = 694.$, $a_{52} = 175.$, $a_{62} = 575.$

The leading terms of the high temperature expansions have been determined as well in order to check our Monte-Carlo computer programme. There is no need to quote them here.

The renormalization-invariant gluon condensation parameters should be determined in the intermediate coupling range ($g_0 \lesssim 1$) from

**Obviously, there are some misprints in the corresponding formula for b_2 in Ref./11/.*

$$a^6 g_0^{-\frac{2\gamma_\Sigma}{b}} \langle 0|:\Sigma:|0\rangle = \langle S^L\rangle - \langle S^L\rangle_{pert.}\,, \quad (6a)$$

$$a^8 g_0^{-\frac{2\gamma_i}{b}} \langle 0|:\Omega_i:|0\rangle = \langle Q_i^L\rangle - \langle Q_i^L\rangle_{pert.}\,, \quad (6b)$$

where we assumed the contributions $O(a^7)$ and $O(a^9)$, respectively, to be negligibly small. In the continuum limit the lattice spacing has to satisfy the renormalization group behaviour

$$a^2 = \frac{1}{\Lambda_L^2}\left(\frac{1}{\beta_0 g_0^2}\right)^{\frac{\beta_1}{2\beta_0^2}} \exp\left(-\frac{1}{\beta_0 g_0^2}\right)\cdot(1+O(g_0^2)), \quad (7)$$

where β_0 and β_1 are the one- and two-loop coefficients of the β-function. The scale constant Λ_L is usually related to the string tension σ. For definiteness we use here the estimate $\Lambda = (0.008 \pm 0.001)\sqrt{\sigma}$ /15/, $\sqrt{\sigma} = 420$ MeV. Recent calculations for large lattices /16/ point out to larger values of $\Lambda_L/\sqrt{\sigma}$ leaving the given number intact at least as a lower bound.

Eqs. (6,7) indicate an exponential, non-perturbative signal to be "seen" in the Monte-Carlo data for $\langle S^L\rangle$ and $\langle Q_i^L\rangle = \sum C_{ij}\langle O_j^L\rangle$ if the condensate values are sufficiently large. In practice we fit the condensate values in units of Λ_L together with a few analytically unknown higher perturbative coefficients of $\langle S^L\rangle_{pert.}$ and $\langle Q_i^L\rangle_{pert.}$, respectively.

3. RESULTS

The SU(3) vacuum expectation values of the operators S^L, O_i^L and $\hat{O}_j^L$ have been calculated with the Monte-Carlo heatbath procedure /17/ on a 4^4 lattice with periodic boundary conditions. Random upgrading of the lattice links has been applied in order to minimize correlations between subsequent iterations. (Per definition one iteration updates each link in the average twice).

The expectation values w.r. to averages over all lattice sites showed up only small statistical fluctuations. Thus, we limited the number of iterations typically to 70 per g_0 value. Our numerical results are presented in Figs. 1-5 and in Table 1. The error estimates in the Monte-Carlo data are usual statistical ones taking correlations between subsequent iterations into account.

In Fig. 1 the $\langle S^L\rangle$ data have been plotted together with the fitted curve. For comparison the perturbative tail $\langle S^L\rangle_{pert.}$ is indicated with a dashed line. Given the fixed coefficient b_2 two further coefficients had to be fitted in order to achieve an

Table 1

Results of the fits for the renormalization invariant condensation parameters $\langle 0|:\Sigma:|0\rangle$, $\langle 0|:\Omega_i:|0\rangle$.

renormalization-invariant condensate parameter	fitted value	number of fitted perturbative coefficients	$\chi^2_{d.f.}$	calculated from combination (4) and (6 b) with
$\langle 0|:\Sigma:|0\rangle$	$(-2.1 \pm .2)\ 10^{14}\ L^6$	2	0.55	–
$\langle 0|:\Omega_1:|0\rangle$	$(-0.9 \pm .2)\ 10^{22}\ L^8$	2	0.26	$O_1^L \rightarrow \hat{O}_1^L$
$\langle 0|:\Omega_2:|0\rangle$	$(-0.17 \pm .02)\ 10^{22}\ L^8$	2	0.69	$O_1^L \rightarrow \hat{O}_1^L$
$\langle 0|:\Omega_3:|0\rangle$	$(0.07 \pm .02)\ 10^{22}\ L^8$	2	0.13	$O_1^L \rightarrow \hat{O}_1^L$
$\langle 0|:\Omega_4:|0\rangle$	$(-0.90 \pm .05)\ 10^{22}\ L^8$	2	2.2	
$\langle 0|:\Omega_5:|0\rangle$	$(0.58 \pm .05)\ 10^{22}\ L^8$	4	4.9	$O_1^L, O_5^L \rightarrow \hat{O}_1^L, \hat{O}_5^L$
$\langle 0|:\Omega_6:|0\rangle$	$(-0.51 \pm .04)\ 10^{22}\ L^8$	4	4.4	$O_1^L, O_5^L \rightarrow \hat{O}_1^L, \hat{O}_5^L$

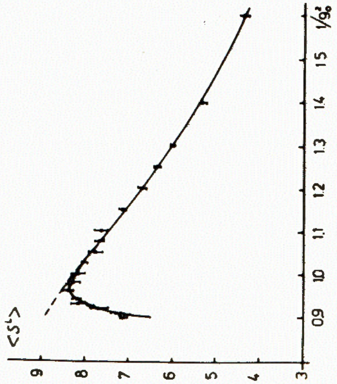

Fig.1. Monte-Carlo data for $\langle S^L\rangle$ and fitted curves (dashed line = fitted perturbative tail).

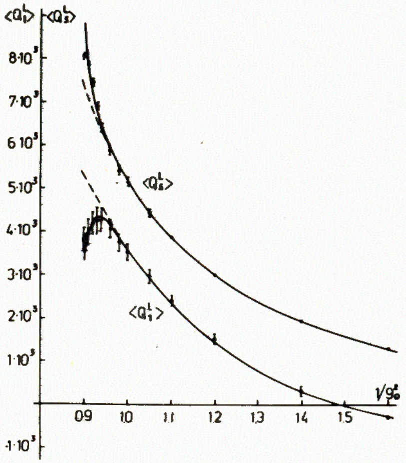

Fig. 2. The same as in Fig. 1 for $\langle Q_i^L\rangle$, i = 1, 5.

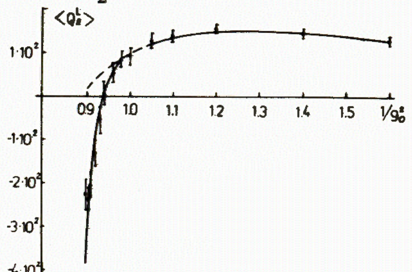

Fig. 3. The same as in Fig. 1 for $\langle Q_2^L\rangle$.

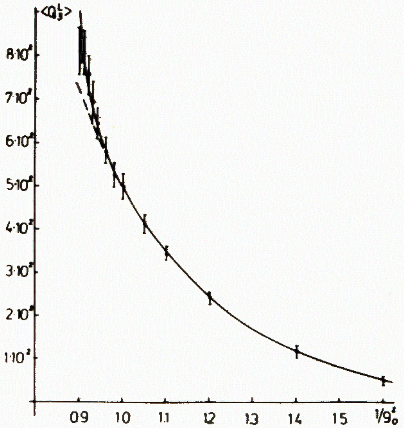

Fig. 4. The same as in Fig. 1 for $\langle Q_3^L\rangle$.

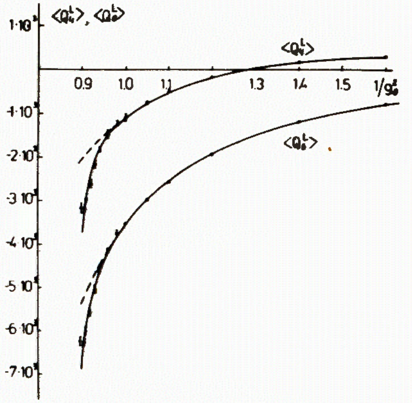

Fig. 5. The same as in Fig. 1 for $\langle Q_i^L\rangle$, i = 4, 6.

effective description of the tail and an excellent χ^2 value per degree of freedom (= number of data points – number of fitted parameters). Analogously to the SU(2) case [11] we observe a clear non-perturbative signal. As expected within Euclidean Yang-Mills theory $\langle 0|:\Sigma:|0\rangle$ turns out to be negative.

The $\langle O_i^L\rangle$ data have been linearly combined according to eq. (4) with all possible replacements $\langle O_i^L\rangle \rightarrow \langle \hat{O}_i^L\rangle$, i = 1, 3, 5. For each possible $\langle Q_i^L\rangle$ combination the condensate values $\langle 0|:\Omega_i:|0\rangle$ together with the corresponding perturbative tails have been fitted. With the same notation as in Fig. 1 the Figs. 2-5 show the results for those combinations which provide the best fits with respect to the χ^2 value. The other combinations produce large condensate values differing from the quoted ones (cf. Table 1) by less than one order of magnitude and lead to worse χ^2 values. As for the $D = 6$ gluon condensate $\langle 0|:\Sigma:|0\rangle$ we find non-vanishing values of the $D = 8$ condensation parameters $\langle 0|:\Omega_i:|0\rangle$. The errors quoted for them have been estimated by the standard CERN library MINUIT procedure minimizing the χ^2 value and therefore correspond to the statistical errors of the Monte-Carlo data, only. Due to the poor knowledge of the perturbative tails our condensate values should contain yet systematic errors, which are very difficult to estimate.

It should be mentioned that we also tried to fit the data without the non-perturbative contribution, i.e., by a pure polynomial including one or two more perturbative coefficients. Then in the scaling region ($0.9 \lesssim 1/g_0^2 \lesssim 1$) the fits became definitely worse.

Finally, we determine non-invariant $D = 8$ gluon condensation parameters by

$$\langle 0|:O_i:|0\rangle = \sum_j C_{ij}^{-1}\, g(\mu)^{-\frac{2\gamma_j}{b}} \langle 0|:\Omega_j:|0\rangle ,$$

at subtraction scales μ relevant for phenomenological QCD sum rule investigations in the charmonium case (see Ref. /4/). We use the standard value $\alpha_s(2m_c) \equiv \frac{g^2(2m_c)}{4\pi} \simeq 0{,}2$ with $m_c \simeq 1.26$ GeV and

$$\alpha_s(\mu) = \frac{\alpha_s(\mu_0)}{1 + \alpha_s(\mu_0)\frac{11N_c}{6\pi}\log\frac{\mu}{\mu_0}} ,$$

The results are presented in Table 2. For comparison we also quote the numbers found with the factorization hypothesis $\langle G^4\rangle \sim \langle G^2\rangle^2$ /18,4/.

We used $\langle 0|\frac{\alpha_s}{\pi}:G^a_{\mu\nu}G^a_{\mu\nu}:|0\rangle \simeq 0.012\ \mathrm{GeV}^2 \simeq (9.1^{+6.0}_{-4.0})\ 10^7 \Lambda_L^4$ and denote $\mathcal{G} \equiv \langle 0|g^2:G^a_{\mu\nu}G^a_{\mu\nu}:|0\rangle$. At the given scales the lattice results are typically by 4 orders of magnitude larger than one would expect from simple factorization. Even if a suppression of systematic errors mentioned before could change the lattice extimates by one order of magnitude (and presumably change the erroneous sign of $\langle 0|:O_4:|0\rangle$), the $D = 8$ condensation parameters would be surprisingly larger.

The inclusion of dynamical quark degrees of freedom should not reduce the values in such a drastic manner that factorization becomes nearly satisfied.

Table 2

Renormalization non-invariant condensate parameters $\langle 0|:O_i:|0\rangle$ at different scales. For comparison the numbers due to factorization have been added

Renormalization-non-invariant condensate parameter	lattice results at scale $\mu = m_c$	$\mu = 2m_c$	$\mu = 4m_c$	factorization hypothesis /4/
$\langle 0|:O_1:|0\rangle$	$(10.0 \pm 0.7)10^{22}\ \Lambda_L^8$	$(3.4 \pm 0.3)10^{22}\ \Lambda_L^8$	$(1.4 \pm 0.1)10^{22}\ \Lambda_L^8$	$\frac{7}{24}\mathcal{G}^2 = (4^{+7}_{-3})\ 10^{18}\ \Lambda_L^8$
$\langle 0|:O_2:|0\rangle$	$(18. \pm 2.\ )10^{22}\ \Lambda_L^8$	$(6.2 \pm 0.6)10^{22}\ \Lambda_L^8$	$(2.7 \pm 0.3)10^{22}\ \Lambda_L^8$	$\frac{5}{16}\mathcal{G}^2 = (4^{+7}_{-3})\ 10^{18}\ \Lambda_L^8$
$\langle 0|:O_3:|0\rangle$	$(0.3 \pm 0.1)\ 10^{22}\ \Lambda_L^8$	$(0.10 \pm 0.06)10^{22}\ \Lambda_L^8$	$(0.04^{\pm 0.04})10^{22}\ \Lambda_L^8$	$\frac{1}{16}\mathcal{G}^2 = (0.8^{+1.4}_{-0.6})10^{18}\ \Lambda_L^8$
$\langle 0|:O_4:|0\rangle$	$(-9. \pm 1.)10^{22}\ \Lambda_L^8$	$(-2.5 \pm 0.5)10^{22}\ \Lambda_L^8$	$(-0.8 \pm 0.3)10^{22}\ \Lambda_L^8$	$\frac{5}{16}\mathcal{G}^2 = (4^{+7}_{-3})\ 10^{18}\ \Lambda_L^8$
$\langle 0|:O_5:|0\rangle$	$(3.3 \pm 0.3)10^{22}\ \Lambda_L^8$	$(1.07 \pm 0.09)10^{22}\ \Lambda_L^8$	$(0.43 \pm 0.04)10^{22}\ \Lambda_L^8$	$\frac{5}{72}\mathcal{G}^2 = (0.9^{+1.6}_{-0.6})10^{18}\ \Lambda_L^8$
$\langle 0|:O_6:|0\rangle$	$(3.6 \pm 0.5)10^{22}\ \Lambda_L^8$	$(0.9 \pm 0.2)10^{22}\ \Lambda_L^8$	$(0.2 \pm 0.1)\ 10^{22}\ \Lambda_L^8$	$\frac{35}{144}\mathcal{G}^2 = (3^{+6}_{-2})\ 10^{18}\ \Lambda_L^8$

However, there might be a further source of an overestimation of the condensate values*. In accordance with Refs.[10,11] the leading non-perturbative contributions in expressions (6) have been read off from the classical (local) limits (1) of the operators (3). On the other hand the operators (3) themselves are non-local ones. Generally, they should be represented by operator expansions the coefficients of which containing all higher order perturbative corrections, as well. The latter could give rise to lower dimensional contributions (of the order $a^4 \langle g^2 \cdot g^2 GG \rangle$ and $a^6 \langle g^2 \cdot g^3 fGGG \rangle$) which have been omitted in Eqs.6 together with the higher dimensional, classical ones. We hope to come back to this point in the future.

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APPENDIX

The operator mixing under renormalization was studied[8] for the D=8 scalar operators given in the basis (SU(3) case)

$$R_1 \equiv (g^2 G^a_{\mu\nu} G^a_{\mu\nu})^2 \qquad R_2 \equiv (g^2 G^a_{\mu\nu} \tilde{G}^a_{\mu\nu})^2$$

$$R_3 \equiv (g^2 G^a_{\mu\nu} G^b_{\mu\nu})^2 \qquad R_4 \equiv (g^2 G^a_{\mu\nu} \tilde{G}^b_{\mu\nu})^2$$

$$R_5 \equiv (g^2 d^{abc} G^b_{\mu\nu} G^c_{\mu\nu})^2 \qquad R_6 \equiv (g^2 d^{abc} G^b_{\mu\nu} \tilde{G}^c_{\mu\nu})^2 ,$$

$\tilde{G}^a_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} G^a_{\rho\sigma}$ being the dual gluon field strength tensor.

They mix as follows $R_i(\mu') = R_i(\mu) + g^2(\mu) \cdot L \cdot \sum_j M_{ij} R_j(\mu)$

with $L = \frac{1}{16\pi^2} \log \frac{\mu'^2}{\mu^2}$ and

*One of the authors (M.M-P.) thanks J.Kripfganz for giving this argument.

$$M = \frac{1}{12} \begin{vmatrix} 30 & -42 & 139 & -89 & 37 & -47 \\ -42 & 30 & -89 & 139 & -47 & 37 \\ -60 & 84 & -482 & 118 & -26 & 46 \\ 84 & -60 & 118 & -482 & 46 & -26 \\ 90 & -126 & 201 & -51 & -171 & 9 \\ -126 & 90 & -51 & 201 & 9 & -171 \end{vmatrix} .$$

By diagonalizing this matrix we find the renormalization-invariant combinations Ω_i and the anomalous dimensions γ_i

$$\Omega_i = g^{\frac{2\gamma_i}{b}} \sum_{j=1}^{6} A_{ij} R_j ,$$

$$\gamma_i = -0.6134, -12.40, -31.82, 4.823, -20.84, -42.98,$$

$$A = \begin{vmatrix} -5.645 & -5.645 & -0.324 & -0.324 & 1 & 1 \\ 0.038 & 0.038 & 0.097 & 0.097 & 1 & 1 \\ 0.218 & 0.218 & -1.413 & -1.413 & 1 & 1 \\ 1.650 & -1.650 & -0.471 & 0.471 & 1 & -1 \\ -0.162 & 0.162 & -0.139 & 0.139 & 1 & -1 \\ 0.560 & -0.560 & -1.813 & 1.813 & 1 & -1 \end{vmatrix} .$$

For our purposes we have to reexpress the operators R_i in terms of the basis (1b). This can be done by consequent use of identities for the structure constants f^{abc}, d^{abc} and for $\epsilon_{\mu\nu\rho\sigma}$. The result is $R_i = \sum_j B_{ij} O_j$.

$$B = \begin{vmatrix} 0 & 0 & 0 & 1 & 0 & 3 \\ 0 & 2 & 4 & 0 & -12 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -4 & 1 & 0 & 1 & 0 & 3 \\ 0 & \frac{2}{3} & 0 & \frac{1}{3} & 0 & -1 \\ -\frac{8}{3} & 0 & \frac{4}{3} & 0 & 4 & 2 \end{vmatrix} .$$

Thus, matrix C in eq. (2b) is given by $C = A \cdot B$.

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Вычисление глюонных конденсатов размерностей шесть и восемь в решеточной КХД

Методом Монте-Карло вычислены глюонные конденсаты типа $\langle G^3 \rangle$ и $\langle G^4 \rangle$ в КХД на решетке в приближении отсутствия виртуальных кварк-антикварковых пар. Найденные значения для конденсатов $\langle G^4 \rangle$ указывают на сильное нарушение гипотезы факторизации $\langle G^4 \rangle \sim \langle G^2 \rangle^2$.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

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Six- and Eight-Dimensional Gluon Condensates from Lattice QCD

The $\langle G^3 \rangle$ and $\langle G^4 \rangle$-type gluon condensates are estimated from expectation values of appropriate lattice operators which have been evaluated by Monte-Carlo simulation of lattice QCD in quenched approximation. The $\langle G^4 \rangle$ values found indicate a strong violation of the factorization hypothesis $\langle G^4 \rangle \sim \langle G^2 \rangle^2$.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

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