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SPONTANEOUS TOPOLOGICAL VACUUM DEGENERATION IN NON-ABELIAN THEORY

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1. It is to be noted that in classical electrodynamics the vacuum fields $A_i(\vec{x},t)=\partial_i\Lambda(\vec{x})$ are described by the Laplace eq. $\partial_i^2\Lambda(\vec{x})=0$ that has a unique nonsingular $(\Lambda(\infty)=0)$ solution: $\Lambda(\vec{x})\equiv 0$ (for example, see ref. /1/). This result does not change, if we choose a finite-volume space $|\vec{x}|\leq R$ and the boundary condition on $U(1)$-group element: $\exp\left(i\Lambda(R)\frac{e}{\hbar c}\right)=1$.

In this paper we would like to attract attention to the fact that the generalization of the transversal vacuum equation to non-Abelian fields in a finite-volume space has nontrivial solutions

$$\partial_i^2\Lambda^a_{(n)}(\vec{x})=0, \qquad \Lambda^a_{(n)}(\vec{x})=2\pi n\frac{x^a}{R}\cdot\frac{\hbar c}{e} \qquad n=\pm(0,1,2,\dots) \tag{1}$$

which satisfy the boundary condition

$$\lim_{|\vec{x}|\to R} e^{\hat\Lambda_{(n)}(\vec{x})}=\pm 1, \qquad \hat\Lambda=g\frac{\tau^a\Lambda^a}{2i}, \qquad g=\frac{e}{\hbar c}.$$

The transverse non-Abelian vacuum (unlike the electromagnetic one) degenerates, and the number of vacua coincides with the number of the homotopy group elements $\pi_3(SU(2))=Z$. It is easy to be convinced of that the degeneration has nontrivial consequences even in the limit of an infinite volume. Let us consider, for example, a quark wave function, which in the degenerate vacuum is given by a sum of product of phase factors $e^{\hat\Lambda_{(n)}(\vec{x})}$ and a solution of a "free" Dirac equation $\psi_0(\vec{x})$:

$$\psi(\vec{x})=\sum_{n=-\infty}^{\infty}e^{\hat\Lambda_{(n)}(\vec{x})}\psi_0(x)=\delta\left(\frac{|\vec{x}|}{R}\right)\psi_0(x)=R\,\delta(|\vec{x}|)\,\psi_0(\vec{x}).$$

Summation over the degeneration leads to the change of transformation properties of the "free" solution $\psi_0(\vec{x})$, which are not restored in the limit of an infinite volume. This is a typical example of the spontaneous symmetry breaking analogous to Bogolubov quasiaverages /2/.

Due to the interference of the infinite number of phase factors, the probability amplitude to find such a quark with momentum $(\vec{k})$ is equal to zero

$$\psi(\vec{k})\simeq\int d^3x\,\exp(i\vec{k}\vec{x})\,\delta(|\vec{x}|)\,\psi_0(\vec{x})=0$$

for the function class $\{\psi_0(\vec{x})\}$ by which "free" states are described. This purely quantum effect may be interpreted as a "color confinement".

2. The definition of physical variables permits also a dynamical arbitrariness. For example, in electrodynamics the electrical tension $E_i=\partial_0 a_i$ is defined up to an arbitrary gauge phase $\partial_0\Lambda(\vec{x},t)$ satisfying eq. $\partial_i^2\partial_0\Lambda(\vec{x},t)=0$. Just such an equation (in the lowest order in g) takes place for the non-Abelian theory in gauge *) $\nabla_i\partial_0\hat{A}_i=0$ ($\nabla_i=\partial_i+[\hat{A},\ ]$), where the fields $\hat{A}_i$ are defined nonuniquely, up to a gauge transformation $\hat{A}_i'=v(\hat{A}_i+\partial_i)v^{-1}$ satisfying eq. $\nabla^2(A)(v\partial_0 v^{-1})=\partial^2\partial_0\hat\Lambda+O(g)=0$ that has recently been called the Gribov equation /5/. Time $\left(-\frac{T}{2};+\frac{T}{2}\right)$ boundary conditions for $\Lambda^a(\vec{x},t)$ are phase values at $t=\pm\frac{T}{2}$ $\left(\Lambda^a(\vec{x},\pm\frac{T}{2})=\Lambda^a_{(\pm)}(\vec{x})\right)$. These functions $\Lambda^a_{(\pm)}(\vec{x})$ satisfy the vacuum equations $\partial_i^2\Lambda^a_{(\pm)}(x)=0$. It is clear that the spontaneous degeneration of both the vacua $\Lambda^a_{(\pm)}(\vec{x})$ with number $n_\pm$ leads to a nontrivial solution for the phase $\Lambda^a(\vec{x},t)$

$$\Lambda^a(\vec{x},t)=\Lambda^a_N(\vec{x},t)=N(t)\,2\pi\frac{x^a}{R}\cdot\frac{1}{g}, \qquad N(t)=\frac{t}{T}(n_+-n_-)+\frac{1}{2}(n_++n_-),$$

where $N(t)$ is a function that describes the interpolation between different vacua $\left(N(\pm\frac{T}{2})=n_\pm\right)$, and it represents a Goldstone mode of the topological degeneration /4/.

*) The quantization in such a gauge and its uniqueness are discussed in /3,4/.

It is easy to verify that the classical interpolating field

$$\hat{V}_i(\vec{x}/N(t)) = v(\vec{x}/N)\,\partial_i\, v(\vec{x}/N)^{-1}, \qquad v(\vec{x}/N(t)) = e^{\lambda_N(\vec{x},t)} \tag{2}$$

is an analog of the instanton field /6/ in Minkowski space with the topological index $\nu = n_+ - n_-$ and the action

$$S(R,T/\nu) = \frac{1}{2}\int\limits_{-T/2}^{T/2} dt \int\limits_{|x|\le R} d^3x\,(\partial_0 V_i^a)^2 \equiv \frac{1}{2}\int\limits_{-T/2}^{T/2} dt\,\frac{\dot{N}^2}{2} I^{\Phi} = \frac{8\pi^2}{g^2}\nu^2\left(\frac{2\pi R}{T}\right), \quad \left(I^{\Phi} = \frac{16\pi^3}{g^2}R\right)$$

which is finite at $R \sim T \to \infty$. We may calculate the θ-vacuum energy spectrum by using the quasiclassical approximation /7/. The same spectrum appears at a direct quantization of the Goldstone variable

$$\delta S/\delta\dot{N} = \mathcal{K} \qquad [\mathcal{K}, N] = i \qquad H = \frac{\mathcal{K}^2}{2I^{\Phi}}$$

$$H\psi = \frac{(2\pi l + \theta)^2}{2I^{\Phi}}\psi \qquad (\psi = e^{i\mathcal{K}N})$$

if one takes into account that points N and $N+1$ are physically equivalent: $\psi(N+1) = e^{i\theta}\psi(N)$ ($\mathcal{K}\psi = (2\pi l + \theta)\psi$, θ is a parameter of the homotopy group representation). The eigenstates of the operator $N(t)$ are defined by the infinite sum

$$\hat{N}/N\rangle_\theta = \sum_{n=-\infty}^{\infty} e^{in\theta}(N+n)/N+n\rangle$$

and therefore the non-Abelian field $\hat{A}_i = \hat{V}_i + a_i^{\Phi}$ contains a large component $\hat{V}_i(\vec{x}/N)$.

3. Perturbation theory over the vacuum field $V(\vec{x}/N)$ and with exact quantization of the Goldstone mode is formulated in detail in author's ref. /8/. Here we give a final result of the expansion of the Lagrangian in the lowest order in a_μ^{Φ} :

$$L^{\Phi} = \frac{\dot{N}^2}{2} I^{\Phi} + \int\limits_{x \le R} d^3x\left[-\frac{1}{4}\left(\nabla_\mu(V)a_\nu^{\Phi} - \nabla_\nu(V)a_\mu^{\Phi}\right)^2 + \bar{\psi}\,\nabla_\mu(V)\gamma_\mu\psi\right],$$

where $\nabla_\mu(V)$ is the covariant derivative in the purely gauge field $V_\mu = v(\vec{x}/N)\,\partial_\mu\, v(\vec{x}/N)^{-1}$. When calculating the colorless Green functions like the electromagnetic and weak current correlators $\langle J^{\mathcal{M}}_{(x)} J^{\mathcal{M}}_{(y)}\rangle$ (when color field sources are absent) the dependence on the variable N is completely removed by the gauge transformation $v(\vec{x}/N)$, except for the terms determining the conserving topological momentum.

$$\mathcal{K} = \frac{\delta L^{\Phi}}{\delta\dot{N}} = \dot{N}\Phi^{\Phi} + \xi \qquad \xi = \frac{2\pi g}{R}\int d^3x\, x^a\left(\bar{\psi}\gamma_0\frac{\tau^a}{2}\psi + \epsilon^{abc}a_i^b\,\partial_0 a_i^c\right).$$

As a result, for the finite-volume space the Hamiltonian of the theory acquires the additional low-energy interaction

$$\Delta H = \frac{\dot{N}^2}{2} I^{\Phi} = \frac{(\theta - \xi)^2}{2I^{\Phi}}, \qquad \left(I^{\Phi} = \frac{16\pi^3}{g^2}R\right) \tag{3}$$

which might be worth being taken into account in the QCD-bags model. The interaction (3) is renormalizable; it breaks chiral $U(1)$-symmetry and CP-symmetry (with breaking parameter θ)and leads to the oscillator-like potential between quarks and to dynamical quark (ψ) and gluon (a^{Φ}) masses which disappear in a large-momentum limit.

The Hamiltonian (3) disappears in the infinite-volume limit $R \to \infty$, and in the limit the theory with the spontaneous vacuum degeneration in colorless sector $(\langle J^{\mathcal{M}}_{(x)} J^{\mathcal{M}}_{(y)}\rangle)$ coinsides with the usual QCD.

In ref. /9/, where the spontaneous vacuum degeneration in the Schwinger model was considered, it is shown that the analogous low-energy interaction in the limit $R \to \infty$ leads to the Coleman θ-vacuum Hamiltonian.

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Спонтанное топологическое вырождение вакуума
в неабелевой теории

Показано, что неабелевые поля в пространстве конечного объема топологически вырождены в низшем порядке теории возмущений. Обсуждаются физические следствия такого вырождения.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Spontaneous Topological Vacuum Degeneration
in Non-Abelian Theory

It is shown that the transversal non-Abelian fields in a finite-volume space can topologically be degenerated in the lowest order of perturbation theory. Physical consenquences of such a degeneration are discussed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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