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R.S.Khalmuradov, A.V.Radyushkin

# ONE-LOOP CORRECTIONS TO PION FORM FACTOR IN QCD IN A LIGHT-LIKE GAUGE

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## I. INTRODUCTION

The behaviour of the pion electromagnetic form factor for asymptotically large momentum transfers Q can be calculated perturbatively within the QCD[1-4]. The calculations are based on the factorization theorem for the contributions related to long- and short-distance dynamics, i.e., on the representation[1,4,5-8]

$$F_\pi(Q^2) = \int_0^1 dx \int_0^1 dy\, \phi^*(y, \mu^2) E(Q^2, \mu^2, g(\mu); x, y) \phi(x, \mu^2) \cdot \{1 + O(1/Q^2)\} \tag{1.1}$$

that is valid in the $Q^2$-region where all possible power corrections (i.e., higher twists and soft contributions) are sufficiently small. Information about the long-distance dynamics is accumulated in eq.(1.1) by the wave functions $\phi(x, \mu^2)$, $\phi(y, \mu^2)$ corresponding to the probability amplitude for transition of the pion into its constituent quarks carrying in the infinite momentum frame the fractions x and $(1-x)$ (or y and $(1-y)$) of the pion longitudinal momentum. Another function $E(Q^2, \mu^2, g; x, y)$ present in eq.(1.1) is the amplitude of the short-distance parton subprocess $q\bar{q}\gamma^* \to q'\bar{q}'$. The representation (1.1) has the structure similar to that of the operator expansion for the virtual Compton amplitude, and in this connection $E(Q^2, \mu^2, g; x, y)$ is often referred to as the coefficient function.

The lowest order approximation for the coefficient function

$$E^{(0)}(Q^2, \mu^2, g; x, y) = 2\pi\alpha_S(\mu) \cdot \frac{C_F}{N_c} \cdot \frac{1}{xyQ^2} \tag{1.2}$$

is well-known starting from the pioniering papers[1-4]. In a paper by F.M.Dittes and one of the authors (A.R.) the one-loop contribution to E was also calculated[9]. An analogous calculation was performed independently also by R.D.Field et al.[10]. However, the diagram by diagram comparison of the results presented in refs.[9] and [10] (this is possible because both calculations were performed in the Feynman gauge) revealed that they differ for some contributions. After a thorough study of ref.[10] we established, however, that even the total result for $E^{(1)}(Q^2, \mu^2, g; x, y)$ given in ref.[10] does not coincide with the sum of diagram by diagram contributions presented in the same paper[10], probably, due to errors in summing the contributions or misprints. In ref.[11], on the basis of the results of ref.[9] the coefficient function $E(Q^2, \mu^2, g; x, y)$ was calculated in the framework of the mass singularity factorization approach proposed in ref.[12]. The use of this scheme guarantees the universality of the wave functions $\phi(x, \mu^2)$ (i.e., their process-independence), hence, the use of this scheme is, in principle, more preferable than the scheme of ref.[9] although the numerical difference between the results of refs.[9] and [11] is not very significant.

Our aim in the present paper is a check of the results of refs.[9-11] based on the calculation of $E^{(1)}(Q^2, \mu^2, g; x, y)$ in the light-like axial gauge* $P_\mu A^\mu = 0$ where $P$ is the momentum of the initial pion (we recall that in the lowest twist approximation considered here $P^2$ is treated as zero). It should be noted that our choice $n_\mu = P_\mu$ essentially simplifies the calculations compared to the more standard choice (used, e.g., in ref.[13]) that requires $(nP) \neq 0$, $(nP') \neq 0$. The necessity of such a check is motivated by the recently completed calculations of the two-loop corrections $V^{(2)}(x, y, g)$ to the evolution kernels for the pion wave function[13-16]. This opens the possibility of calculating the complete (renorminvariant) $O(\alpha_s)$ correction to the pion form factor asymptotics, provided, first, that one is sure that the results for $E^{(1)}$ and $V^{(2)}$ are correct (within the factorization schemes chosen) and, second, that one is able to calculate $E^{(1)}$ and $V^{(2)}$ within the same scheme, only in that case the final result for the $O(\alpha_s)$ correction to $F_\pi(Q^2)$ would be reliable, self-consistent and renorminvariant. Furthermore, in the present paper we describe also an algorithm for the one-loop calculations that can prove to be useful in other calculatuons of a similar type.

The paper is organized in the following way. In Section II we discuss the general structure of the one-loop Feynman diagrams responsible for the radiative corrections to the coefficient function $E(Q^2, \mu^2, g; x, y)$. In Section III we describe the algorithm of the one-loop calculations, present the results of our calculations for separate diagrams and compare our final result with those obtained earlier in refs.[9-11]. In the concluding Section we summarize the paper.

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*After completing our computations we were informed by M.Sarmadi that he also calculated the one-loop corrections to $E(Q^2, \mu^2, g; x, y)$[13] using a light-like axial gauge $n^\mu A_\mu = 0$ ($n^2 = 0$) different from ours and the factorization scheme[12] used in ref.[11]. His result[13] coincides with that given in ref.[11].



## II. GENERAL STRUCTURE OF THE ONE-LOOP CONTRIBUTIONS

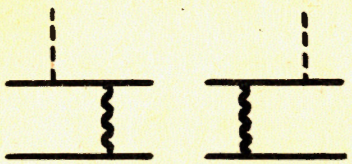
Contributions of the one-loop diagrams of massless QCD contain usually ultra-violet divergences as well as mass singularities (i.e., infra-red and collinear divergences). The most convenient regularization procedure for all these divergences in QCD is the dimensional one[17] which converts the above-mentioned divergences into poles $1/\epsilon$ where $\epsilon = (d-4)/2$ and $d$ is the space-time dimension. The poles $1/\epsilon_{IR}$ due to the infrared divergences should cancel for colourless states after summing the contributions of all one-loop diagrams[5-8], the ultra-violet poles $1/\epsilon_{UV}$ are removed by the R-operation (see, e.g.,[18]), while the poles corresponding to the collinear divergences are factorized out into the wave function renormalization constants by the factorization procedure[5-8].

In all the papers[9-16] mentioned above the $\overline{MS}$-convention[19] for the dimensional regularization was used, i.e., the change

$$\frac{d^4k}{(2\pi)^4} \to \frac{d^{4-2\epsilon}k}{(2\pi)^{4-2\epsilon}} (4\pi e^{-\gamma_E})^{\epsilon} (\mu^2)^{\epsilon} \qquad (2.1)$$

for the virtual momentum integration ($\gamma_E$ is Euler's constant). However, there are some differences between refs.[9-11] in details of the choice of the R-operation and of the factorization procedure. In paricular, the factorization scheme used in refs.[5] and [9,10] can be conventionally referred to as an E-oriented, since the recipe for calculating the coefficient function E within this scheme can be formulated in a more simple way than that of the wave function (or what is the same, of the evolution kernel V). Namely, to get E in this scheme one should calculate for massless quarks the on-mass-shell transition amplitude $q\bar{q}\gamma^* \to q'\bar{q}'$ regularized according to the recipe (2.1) and subtract the poles $1/\epsilon$ related to the collinear divergences. However, the recipes for removing the ultra-violet (UV) poles used in refs.[9] and [10] differ from each other. In particular, in ref.[9] for this purpose the standard R-operation[18] was used that is equivalent to a subtraction of the poles $1/\epsilon$ from the contributions of the divergent subgraphs, while in ref.[10] the UV poles are removed just like the collinear ones, i.e., by subtracting the pole part of the whole diagram contribution. On the one-loop level these recipes are identical only if the contribution of the corresponding tree (i.e., Born) diagram does not depend on $\epsilon$. However, in the problem considered the contribution of the simplest diagram (fig.1) is proportional to $(1-\epsilon)$ because

$$\gamma^\mu \gamma_\nu \gamma_\mu = -2(1-\epsilon)\gamma_\nu . \qquad (2.2)$$



a) b)
*Fig.1. Lowest order diagrams.*

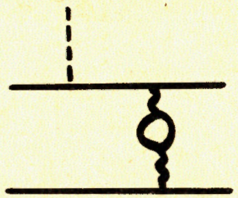


*Fig.2. Diagram containing a self-energy divergent subgraph.*

Hence, if the contribution of the divergent subgraph in a one-loop diagram (e.g., in that one due to the self-energy insertion into the gluonic propagator, fig.2) equals $(A/\epsilon + B)$, then the renormalized contribution to E calculated according to the first recipe [9] would be equal to $BE^{(0)}$, while the one calculated by the second recipe [10] would be equal to $(B - A)E^{(0)}$. It is obvious that one should prefer the first recipe rather than the second one, because the latter is equivalent to a rather exotic version of the R-operation with the counterterms depending on the physical process under consideration. On the other hand, the factorization scheme proposed in ref. [12] and incorporated in refs. [11,13] may be referred to as a V-oriented one, because the recipe of the evolution kernel calculation in this scheme is primary (and more simple) while the recipe of the coefficient function construction is secondary (and more complicated). The UV-poles are subtracted in refs. [11,13] in the same way as in ref. [9], i.e., by using the standard version of the R-operation.

It is to be noted here that in the light-like axial gauges ($n_\mu A^\mu = 0$, $n^2 = n_\mu n^\mu = 0$) used in refs. [11-13,15] and in the present paper, there appear new singularities of the $\ln(n^2)$-type that manifest themselves as new $1/\epsilon$ poles in dimensionally regularized integrals. These singularities may be divided into two types. The first type corresponds to new UV-divergences, and to remove them one should introduce n-dependent counterterms of $\bar\psi \hat{n}(n_\mu \partial^\mu)^{-1} \psi$-type [12]. These divergences, naturally, should cancel for physical quantities like $F_\pi(Q^2)$. However, for auxiliary objects like E and $\phi$ such a cancellation in general may not occur. Moreover, in the problem under consideration the new UV-poles in E are cancelled by those in $\phi$. To simplify the factorization scheme it was proposed in ref. [12] to subtract the new UV-poles just like the "old" ones. We also adhere to this recipe. However, these exist also the second type of the $\ln(n^2)$ singularities. They are related to the $\delta(x)$-parts of the contributions like



$$\left(\frac{1}{x}\right)_+ = \frac{1}{x} - \delta(x)\int_0^1 \frac{dz}{z} \qquad (2.3)$$

which provide a regularization of the soft part of the bremsstrahlung gluonic spectrum, i.e., the convergence of the integrals of the type of

$$\int_0^1 f(x)\left(\frac{1}{x}\right)_+ dx = \int_0^1 \frac{f(x) - f(0)}{x}\,dx \qquad (2.4)$$

in the small-x region. To avoid the divergent integrals of $\int dz/z$-type, a modified form of the denominators of the axial part of the gluonic propagator has been used in refs. [12,13]:

$$\frac{k_\mu n_\nu + k_\nu n_\mu}{(kn)} \to \frac{k_\mu n_\nu + k_\nu n_\mu}{2}\left(\frac{1}{(kn) + i\delta} + \frac{1}{(kn) - i\delta}\right) \qquad (2.5)$$

(principal value prescription). As a result, the divergent expressions are substituted by $\ln\delta$ terms that cancel after summing all the diagrams contributing to the one-loop coefficient function $E^{(1)}(Q^2, \mu^2, g; x, y)$. In the present paper we will not use the prescription (2.5), because the dimensional regularization (2.1) itself provides the finitess of results.

## III. OUTLINE OF THE CALCULATIONS AND DISCUSSION OF THE RESULTS

The main object one should calculate is the one-loop $q\bar{q}\gamma^* \to q'\bar{q}'$ transition amplitude. The kinematics is the following: the initial state quarks have momenta $xP$ and $(1 - x)P \equiv \bar{x}P$, the final state quarks have momenta $yP'$ and $(1 - y)P' \equiv \bar{y}P'$. Furthermore, $P^2 = P'^2 = 0$. The form factor $F_\pi(Q^2)$ is defined by

$$\langle P' |\, J^\mu(0)\, | P \rangle = (P + P')^\mu F_\pi(Q^2). \qquad (3.1)$$

It is convenient to get rid of the $\mu$-index multiplying eq. (3.1) by $P'_\mu$, i.e., to define $F_\pi$ by

$$F_\pi(Q^2) = \frac{2P'_\mu}{Q^2}\langle P' |\, J^\mu(0)\, | P \rangle, \qquad (3.2)$$

where $Q^2 = 2(PP')$. If one adheres to this definition, then the total lowest-order contribution to E is given only by the diagram 1a, while the contribution of fig.1b is zero. The use of the definition (3.2) also reduces the number of the one-loop diagrams contributing to E.

The explicit calculations were performed in the light-like axial gauge $P_\mu A^\mu = 0$. The gluon propagator in this gauge is

$$D_{\mu\nu}(k) = \frac{1}{k^2}\left(g_{\mu\nu} - \frac{k_\mu P_\nu + k_\nu P_\mu}{(kP)}\right). \qquad (3.3)$$

To check the computer program we calculated simultaneously all the diagrams also in the Feynman gauge where $D_{\mu\nu} = g_{\mu\nu}/k^2$.

The computer program SCHOONSCHIP /20/ suited for analytic calculations was systematically and intensively used throughout our work. The algorithm adopted for this purpose is the following. The first step was to calculate the traces of the $\gamma$-matrices, with a proper account of the fact that the number of the space-time dimensions is $(4 - 2\epsilon)$. The scalar invariants of the $(a(Pk) + b(P'k) + ck^2)$ -type, resulted after the first step was performed ($k$ is the integration momentum), were expanded then over the structures present in the denominators of the corresponding Feynman integrals. This trick of "cancelling the denominator by the numerator" considerably reduces the number of essentially independent Feynman integrals. By further transformations (e.g., by shifting the integration momentum $k \to k' = k + \Delta$ combined with the subsequent removal of terms odd in $k'$) it is possible to reduce the number of basic integrals to 7 and in the case of the Feynman gauge even to 4. The explicit expressions for the basic integrals are given in the Appendix.

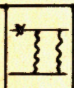
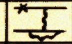
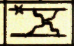
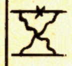
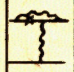
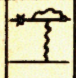
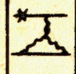
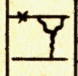
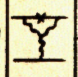
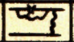
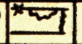
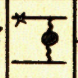
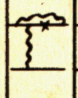
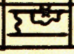
Final results for the contributions of particular diagrams have the structure $A/\epsilon^2 + B/\epsilon + C + O(\epsilon)$. Note, that the double poles do appear in a situation when the collinear divergence of the $k$-integral for $\epsilon = 0$ is accompanied either by an infrared divergence or by a $\ln(n^2)$-singularity of the second type (i.e., that related to integration over soft part of the gluonic spectrum). We observed, however, that the $1/\epsilon^2$-terms cancel for the sum of all relevant diagrams, in complete agreement with our expectation that both the infrared divergences and the $\ln(n^2)$-type singularities of the type mentioned above should cancel after the summation.

The finite parts $C_i$ of the contributions of separate diagrams are given in the Table. Their sum corresponds to the coefficient function calculated within the simplest E-oriented scheme used in ref. /10/. However, as it was emphasized in Section II, this scheme does not satisfy the requirement that the effective coupling constant must be universal (i.e., independent of the process under consideration) because the corresponding counter terms in this scheme are different for different processes. It is easy to establish also that the pion wave functions are not universal within this scheme as well. Indeed, the universality of the wave functions implies that the poles $1/\epsilon$ related to the collinear divergences are absorbed by some



Table

Finite parts of the diagrams in the light-like axial gauge $(PA) = 0$. The following notation is used $\bar{x} = 1 - x$, $\bar{y} = 1 - y$, $\overline{xy} = 1 - x - y$, $D = [2\bar{x}\bar{y}(x - y)^2]^{-1}$, $Sp(y, \bar{y}) = Sp(-y/\bar{y}) - Sp(-\bar{y}/y)$ where $Sp(x)$ is the Spence function $Sp(x) = -\int_0^1 \frac{dz}{z}\ln(1 - xz)$, $L(a) = \ln(aQ^2/\mu^2)$

| | | |
|---|---|---|
| 1 |  | $C_F\{L^2(1) + (2\ln x + 2\ln y + \ln\bar{y} + 2)L(1) + \ln^2 x + (1/2)\ln^2 y + \ln^2\bar{y} + 2\ln x\ln y + \ln x\ln\bar{y} + 2\ln x + 2\ln y + \ln\bar{y} + Sp(y,\bar{y}) - Sp(1)\}$ |
| 2 |  | $-C_F(x\ln x/\bar{x})\{L(1) + (1/2)\ln x + \ln y - \ln\bar{y} + 1\}$ |
| 3 |  | $-(1/2)C_V\{L^2(1) + 2(\ln x + \ln y - 1)L(1) + \ln^2 x + \ln^2 y + 2\ln x\ln y - \ln x - \ln y - Sp(1) + 5\}$ |
| 4 |  | $C_V\{(1/2)\ln^2 y - (1/2)\ln^2\bar{y} - \ln y\ln\bar{y} - Sp(y,\bar{y}) - 2Sp(y) + 2Sp(1)\}$ |
| 5 |  | $C_V\{[(x/\bar{x})\ln x + \ln\bar{x}]L(1) + [x/(2\bar{x})]\ln^2 x + (1/2)\ln^2\bar{x} + [(2x-1)/\bar{x}]\ln x\ln y - (x/\bar{x})\ln x\ln\bar{y} + \ln\bar{x}\ln y + \ln y\ln\bar{y} + (x/\bar{x})\ln x + \ln\bar{x} - \bar{x}\bar{y}S(x,y) + 2Sp(y) - 2Sp(1)\}$ |
| 6 |  | $C_V\{-(1/2)L^2(1) - [\ln\bar{x} + (1/y)\ln\bar{y}]L(1) - (1/2)\ln^2\bar{x} - [1/(2y)]\ln^2\bar{y} - (\bar{y}/y)\ln x\ln\bar{y} + \ln y\ln\bar{y} - \ln\bar{x}\ln\bar{y} + Dx\bar{y}[2y\bar{y} - (x-y)(2-x)]\ln x + Dx\bar{x}[2y\bar{y} + (x-y)(3y-2)]\ln y + D\bar{x}\bar{y}[2y\bar{y} - (x-y)\bar{x}]\ln\bar{x} + D\bar{x}\bar{y}[2\bar{x}y + (x-y)(3x-1)]\ln\bar{y} - 2Dy^2\bar{x}\bar{y}^2 S(x,\bar{y}) + (2\bar{y}/y)Sp(\bar{y}) + [(5y-4)/(2y)]Sp(1) - 1\}$ |
| 7 |  | $C_V\{[(\bar{y}/y)\ln\bar{y} + 1/2]L(1) + [\bar{y}/(2y)]\ln^2\bar{y} + (\bar{y}/y)\ln x\ln\bar{y} - (\bar{y}/y)\ln y\ln\bar{y} + (1/2)\ln x + [(2-3y)/(2\bar{y})]\ln y - \ln\bar{y} - (2\bar{y}/y)Sp(\bar{y}) + (2\bar{y}/y)Sp(1) - 3/2\}$ |
| 8 |  | $-(3/4)C_A\{L^2(1) + (2\ln x + 2\ln y + 1)L(1) + \ln^2 x + \ln^2 y + 2\ln x\ln y + \ln x + \ln y - Sp(1) + 1\}$ |
| 9 |  | $C_A\{-(1/2)L^2(1) - [\ln x + \ln y + (1/2)\ln\bar{y} + 3/4]L(1) - (1/2)\ln^2 x - (1/4)\ln^2 y - (1/2)\ln^2\bar{y} - \ln x\ln y - (1/2)\ln x\ln\bar{y} - (3/4)\ln x + [y/(4\bar{y})]\ln y - (1/2)\ln\bar{y} - (1/2)Sp(y,\bar{y}) + (1/2)Sp(1) - 1/4\}$ |
| 10 |  | $(1/2)C_A\{-(1/2)L^2(1) + [(x/\bar{x})\ln x - \ln y + \ln\bar{y}]L(1) + [x/(2\bar{x})]\ln^2 x - \ln^2 y + \ln^2\bar{y} + (x/\bar{x})\ln x\ln y - (x/\bar{x})\ln x\ln\bar{y} + [(2y-3)/(2\bar{y})]\ln y + \ln\bar{y} + Sp(y,\bar{y}) + (1/2)Sp(1) - 1\}$ |
| 11 |  | $-(1/2)C_F[L(1) - (x/\bar{x})\ln x]$ |
| 12 |  | $(1/2)C_F[L(1) + \ln x]$ |
| 13 |  | $(1/2)C_A\{2L^2(1) + (4\ln x + 4\ln y + 1/3)L(1) + 2\ln^2 x + 2\ln^2 y + 4\ln x\ln y + (1/3)\ln x + (1/3)\ln y - 2Sp(1) + 34/9\} + (1/3)N_f[L(1) + \ln x + \ln y - 2/3]$ |
| 14 |  | $C_V\{(\ln\bar{y} - \ln y)L(1) - \ln^2 y + \ln^2\bar{y} + Dx^2\bar{y}(2-3x+y)\ln x + D\bar{x}[\bar{y}(x^2+2xy-y^2) - (x-y)y^2]\ln y + D\bar{x}^2\bar{y}(3x-y)\ln\bar{x} + D\bar{x}\bar{y}[2x\bar{y} + (x-y)(1-x-2y)]\ln\bar{y} + Sp(y,\bar{y}) - 2Dx^2\bar{x}^2\bar{y}S(\bar{x},y)\}$ |
| 15 |  | $C_F\{(\ln y - \ln\bar{y})L(1) + \ln^2 y - \ln^2\bar{y} + [(1-2y)/(2\bar{y})]\ln y - \ln\bar{y} - Sp(y,\bar{y})\}$ |

universal renormalization factors

$$Z_\phi = (1 + \frac{V_1}{\epsilon} + O(1/\epsilon^2)), \qquad (3.4)$$

where $V_1$ is the one-loop evolution kernel. The relation between the transition amplitude $T(Q^2/\mu^2, g, \epsilon)$ and the coefficient function E in the one-loop approximation is then given by

$$T_0(\epsilon) + (\frac{B}{\epsilon} + C) + \ldots = (1 + \frac{V_1}{\epsilon} + \ldots) \otimes [E_0(\epsilon) + E_1(\epsilon) + \ldots] \otimes (1 + \frac{V_1}{\epsilon} + \ldots). \qquad (3.5)$$

In the case when $E(\epsilon)$ has a nontrivial dependence on $\epsilon$ (i.e., if $E_0(\epsilon) = E_0 + \epsilon E_0' + \ldots$ ), instead of a simple relation $E_1(0) = C$ corresponding to the factorization scheme used in refs.[9] and [10] we get a more complicated one

$$E_1(0) = C - V_1 \otimes E_0' - E_0' \otimes V_1 . \qquad (3.6)$$

In this situation the requirement that the one-loop coefficient function be given by the sum of finite parts C can be fulfilled only by adding $E_0'/E_0$-dependent (i.e., process-dependent) terms to eq.(3.4).

In the problem under consideration $E_0(\epsilon) = E_0(1-\epsilon)$ (see eq. (2.2)) and, hence, in the scheme defined by eq. (3.4) we have

$$E_1(0) = C + V_1 \otimes E_0 + E_0 \otimes V_1 . \qquad (3.7)$$

Note, that according to eq.(3.5), the combination $V_1 \otimes E_0 + E_0 \otimes V_1$ coincides with the $1/\epsilon$ coefficient or, what is the same, with the coefficient in front of the collinear logarithm $\ln(\mu^2/Q^2)$ resulted from the expansion $(\mu^2/Q^2)^\epsilon = 1 + \epsilon \ln(\mu^2/Q^2) + \ldots$ The explicit form of the combination can be found, e.g., in ref.[9] where the collinear $\ln(\mu^2/Q^2)$ and renormgroup $\ln(\mu_R^2/Q^2)$ logarithms are separated. Using the results of ref.[5] we find out that to get $E^{(1)}$ within the scheme proposed in ref.[12] and utilized in refs.[11] and [13], one should add the term

$$\Delta E^{(1)\,coll} = -\frac{\alpha_s}{2\pi} C_F (4 + \ln x + \ln y) E^{(0)} \qquad (3.8)$$

to the sum of finite parts C.

In a similar way, the use of the standard (i.e., maintaining the universality of the effective coupling constant) R-operation corresponds to adding to C of the coefficient related to the renormgroup logarithm $\ln(\mu_R^2/Q^2)$. The latter, according to ref.[9] is

$$\Delta E^{(1)\,RG} = \frac{\alpha_s}{2\pi} (C_F + \frac{11}{6} C_A - \frac{1}{3} N_f) E^{(0)}. \qquad (3.9)$$



Using for C the expression dictated by the Table, we established that the sum $C + \Delta E^{(1)\,RG}$ coincides with the result of ref.[9] while the sum $C + \Delta E^{(1)\,RG} + \Delta E^{(1)\,coll}$ with that of refs.[11,13]. We conclude then that the results of refs.[9,11,13] and of the present paper, with a proper account of the schemes used, all agree with each other. On the other hand, the disagreement between our expression for the sum of finite parts C and the result for E given in ref.[10] means that the latter is incorrect.

## CONCLUSIONS

In this paper we described an effective algorithm for the computation of the one-loop corrections to the asymptotic behaviour of the pion form factor in QCD adapted for computer calculations using the analytic calculation program SCHOONSCHIP. The computations were performed simultaneously in the Feynman and a light-like axial gauge $P_\mu A^\mu = 0$. Furthermore, in the latter case to regularize the singularities due to the denominator of the axial part of the gluonic propagator we incorporated the dimensional regularization that essentially simplified the calculations compared to the standard trick based on the $\delta$-regularization (2.5). The results obtained in the Feynman gauge coincided diagram by diagram with the results presented in ref.[9]. Furthermore, the total contribution given by the sum of all diagrams in the light-like gauge coincided with its Feynman gauge analogue. We investigated also the dependence of the coefficient function on the choice of the factorization and renormalization schemes. In particular, we found the explicit formulas (3.8), (3.9) for getting $E_1$ in the most natural scheme[11-13], in which the coupling constant g and the pion wave functions $\phi$ are the universal, process-independent quantities. It is worth emphasizing that it is this scheme that has been used in the recent computations[13-16] of the two-loop contribution to the evolution kernel $V(x, y; g)$.

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## APPENDIX

Basic integrals ($\tilde{d}k = \frac{d^{4-2\epsilon}k}{(2\pi)^{4-2\epsilon}}(4\pi e^{-\gamma_E})^{-\epsilon}(\mu^2)^{\epsilon}$):

a) Feynman gauge

$$1)\ \int \frac{\tilde{d}k}{(k-ap-bp')^2(k-cp-dp')^2} = \frac{1}{(4\pi)^2}\frac{1}{\epsilon}\{1+\epsilon[2-\ln\frac{(b-d)(c-a)Q^2}{\mu^2}]+O(\epsilon^2)\}$$

$$2)\ \int \frac{\tilde{d}k}{k^2(k-xp)^2(k-p')^2} = -\frac{1}{(4\pi)^2}\frac{1}{xQ^2}\frac{1}{\epsilon^2}\{1-\epsilon\ln\frac{xQ^2}{\mu^2}+\frac{\epsilon}{2}[\ln^2\frac{xQ^2}{\mu^2}-Sp(1)]+$$
$$+O(\epsilon^2)\}$$

$$3)\ \int \frac{\tilde{d}k}{(k-xp)^2(k-p')^2(y-yp')^2} = -\frac{1}{(4\pi)^2}\frac{\ln y}{x\bar{y}Q^2}\cdot\frac{1}{\epsilon}\{1-\epsilon\ln(x\sqrt{\bar{y}}\frac{Q^2}{\mu^2})+O(\epsilon^2)\}$$

$$4)\ \int \frac{\tilde{d}k}{(k-p)^2(k-p')^2(k-xp-yp')^2} = \frac{1}{(4\pi)^2}\frac{1}{(1-x-y)Q^2}\{Sp(x)+Sp(y)-Sp(\bar{x})-$$
$$-Sp(\bar{y})+\ln\bar{x}\ln\bar{y}-\ln x\ln y\}$$

b) Light-like axial gauge $P_\mu A^\mu = 0$.

$$5)\ \int \frac{\tilde{d}k}{(k-aP)^2(k-cP-dP')^2(kP)} = -\frac{2}{(4\pi)^2}\frac{1}{dQ^2}\frac{1}{\epsilon^2}\{1-\epsilon\ln(\frac{d(a-c)Q^2}{\mu^2})+$$
$$+\frac{\epsilon^2}{2}[\ln^2\frac{d(a-c)Q^2}{\mu^2}-Sp(1)]+O(\epsilon^2)\}.$$

$$6)\ \int \frac{\tilde{d}k}{(k-aP-bP')^2(k-cP-dP')^2(kP)} = \frac{2}{(4\pi)^2}\frac{1}{Q^2}\frac{\ln(b/d)}{b-d}\frac{1}{\epsilon}$$
$$\times\{1-\epsilon[\frac{Sp(d/b)-Sp(b/d)}{\ln(b/d)}+\ln(bd\frac{a-c}{b-d}\frac{Q^2}{\mu^2})]+O(\epsilon^2)\}$$

$$7)\ \int \frac{\tilde{d}k(kP)(kP')}{(k-xP)^2(k-yP')^2} = \frac{xyQ^4}{12(4\pi)^2}\frac{1}{\epsilon}\{1+\epsilon[\frac{13}{6}-\ln\frac{xyQ^2}{\mu^2}]+O(\epsilon^2)\}$$

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Халмурадов Р.С., Радюшкин А.В. E2-84-606
Однопетлевые поправки к формфактору пиона в КХД в светоподобной калибровке

В рамках пертурбативной хромодинамики в светоподобной аксиальной калибровке произведен расчет однопетлевых поправок к коэффициентной функции операторного разложения для асимптотики формфактора пиона. Исследована зависимость результатов от выбора ренормировочной и факторизационной схем.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Khalmuradov R.S., Radyushkin A.V. E2-84-606
One-Loop Corrections to Pion Form Factor in QCD in a Light-Like Gauge

In the framework of the perturbative QCD in a light-like axial gauge the one-loop corrections are calculated for the coefficient function of the operator expansion for the asymptotic behaviour of the pion form factor. The dependence of the results on the choice of renormalization and factorization schemes is investigated.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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