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MOTT MECHANISM AND THE HADRONIC TO QUARK MATTER PHASE TRANSITION

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Recently some effort has been made to investigate the behaviour of matter in the high temperature and/or high density region, and it is widely accepted that a phase transition from hadronic matter to a quark-gluon plasma should occur at a critical temperature of approximately 150–225 MeV/1/ for zero baryon number density. At this phase transition, quark deconfinement will take place, and the new state can be characterized as a gas of quarks and gluons which only weekly interact (asymptotic freedom). In the low density/low temperature region it is well believed that no free quarks and gluons can exist, and all quarks are confined in colour neutral bound states (hadrons).

There have been performed different model calculations to obtain the critical quantities which describe the phase border line. Within the two-phase approaches/2/ different descriptions for each phase are used to get an equation of state, and the phase transition is obtained by matching the pressure or the chemical potential of both phases. The quark-gluon plasma is described by perturbative QCD at finite temperature, but these approximations do not lead to bound states in the low density phase, which should be identified with the hadronic state of matter. Up to now, the only approach which describes both phases within the same theory are Monte-Carlo simulations of QCD on a lattice/3/, but here the inclusion of fermions remains a rather involved problem.

A relevant description of the hadronic – quark matter phase transition should give the correct low density phase of only bound states and the high density phase of interacting free particles. Furthermore, this phase transition should be described on the basis of a consequent quantum statistical approach, as given, for instances, by the method of thermodynamic Green functions.

The effect of destruction of bound states due to increasing particle density has been investigated in nuclear matter/4/ (Mott effect). Because of phase space occupation (Pauli blocking) the interaction becomes less operative, and the bound states merge in the continuum of scattering states. A similar phenomenon is also well known from plasma physics and semiconductor physics, see ref./5/, in order to describe the metal-insulator transition at high density, where the weakening of interaction is due to screening. We suggest that the Mott mechanism due to the Pauli blocking may serve as a physical background for the understanding of the deconfinement transition of hadrons into their constituent quarks. However, in order to apply the concept of the Mott mechanism to quark-hadron matter we have to deal with the special structure of QCD in contrast to QED.

We consider a model system of quarks and antiquarks with two flavour degrees of freedom (u,d) and three colour degrees of freedom. For the interaction we employ a standard string type potential with linear confinement[6,7] (only s-states are considered)

$$V_{q\bar{q}} = V_{QCD} + V_{conf} + V_{ss}\,, \qquad V_{qqq} = \frac{1}{2}\sum_{i,j} V_{q_i q_j}\,, \tag{1}$$

$$V_{q_i q_j}' = \frac{1}{2} V_{QCD} + V_{conf} + V_{ss}\,; \tag{2}$$

$$V_{QCD} = 4\alpha_s/3r\,, \quad V_{conf} = \lambda r + c\,, \quad V_{ss} = (6 m_q^2)^{-1}\sigma_1\sigma_2\nabla^2 V_{QCD}(r)\,. \tag{3}$$

Applying the variational method[6,7], we found the following common set of parameters α_s=0.52, λ=0.19 GeV^2, c = -0.568 GeV, m_q = 0.27 GeV by which the s-wave meson and s-wave baryon mass spectra are reproduced.

The quantum statistical description of a many quark system requires a special approach because of the divergent behaviour of V_{conf} at large distances. However, divergencies in the thermodynamic quantities do not occur because of the saturation of this interaction. This means, that the system of quarks must be divided into colourless clusters, and the interaction potential (1) acts only within these clusters. The configuration which gives the most important contribution to the partition function, i.e., with the smallest value of total energy, is characterized by interaction strings between nearest neighbours. We will consider only this configuration.

In the case of $q-\bar{q}$ pairing, the density of potential energy is given by

$$v_{pot} = \frac{1}{2}\{n_q \int V_{q\bar{q}}(r)\, w_{\bar{q}}(r)\, d^3r + n_{\bar{q}} \int V_{\bar{q}q}(r)\, w_q(r)\, d^3r\}\,, \tag{4}$$

where $w_{\bar{q}}(r)$ denotes the distribution function of the next $\bar{q}$ particle if a q-particle is situated at r = 0. The two-particle distribution function may be evaluated within a ladder approximation. In the case of a $q-\bar{q}$ bound state, $w_{\bar{q}}(r)$ is determined by the wave function of relative motion,

$$w^b_{\bar{q}}(r) = |\psi(r)|^2\,, \tag{5}$$

whereas for free states (statistically independent distribution) we have

$$w^{HF}_{\bar{q}}(r) = \frac{1}{3} n_{\bar{q}} \exp(-4\pi n_{\bar{q}} r^3/9)\,. \tag{6}$$

In fact, the interaction potential (1) becomes an effective density dependent potential for uncorrelated free quarks.

We first discuss the case $n_q = n_{\bar{q}}$ where the baryon number density $n_q - n_{\bar{q}}$ -vanishes, and the chemical potential μ_B is equal to zero. We will estimate the free energy density $f^{HF}(T)$ of the free quark matter which is given by

$$f^{HF}(T) = f_q^{id}(T) + v_{pot}^{HF}\,,$$

$$f_q^{id}(T) = 2 g_q T \int \frac{d^3p}{(2\pi)^3} \ln(\exp(\epsilon_q(p)/T) + 1)\,, \tag{7}$$

$$\epsilon_q^2(p) = m_q^2 + p^2\,.$$

g_q = 12 is the quark degeneration factor, and particles and antiparticles are considered by factor 2. The potential energy in Hartree-Fock approximation (uncorrelated distribution of free quarks in $w_{\bar{q}}(r)$) is given by

$$v^{HF}_{pot} = \frac{1}{3} n_q n_{\bar{q}} \int V_{q\bar{q}}(r) \exp(-4\pi n_{\bar{q}} r^3/9)\, d^3r\,. \tag{8}$$

(The exchange contribution vanishes because only $q-\bar{q}$ pairs are interacting).

The formation of bound states where $w^b_{\bar{q}}(r)$ is determined from a ladder approximation leads to the two particle cluster binding energy contained in m_π. The free energy contribution of bound states is given by

$$f_h^{id}(T) = g_\pi T \int \frac{d^3p}{(2\pi)^3} \ln(\exp(\epsilon_\pi(p)/T) - 1)\,, \quad \epsilon_\pi^2(p) = m_\pi^2 + p^2\,. \tag{9}$$

In principle, the bound state energies m_π should also be shifted by density effects similar to the Hartree-Fock shift of free particles. As shown in detail in[4], the Hartree-Fock shift of the bound states consists of the single particle self-energy shift and a Pauli blocking term (phase space occupation). Both contributions act in opposite directions and compensate approximately each other. Therefore the bound state energy m_π remains nearly unshifted, whereas the free particle energies are shifted if the total quark density $n_q + n_{\bar{q}}$ is increasing. At a certain density n_q^{Mott} the bound state merge in the continuum of free particle states and disappear. This mechanism of dissolving bound states is suggested to be the microscopic reason for the quark deconfinement.

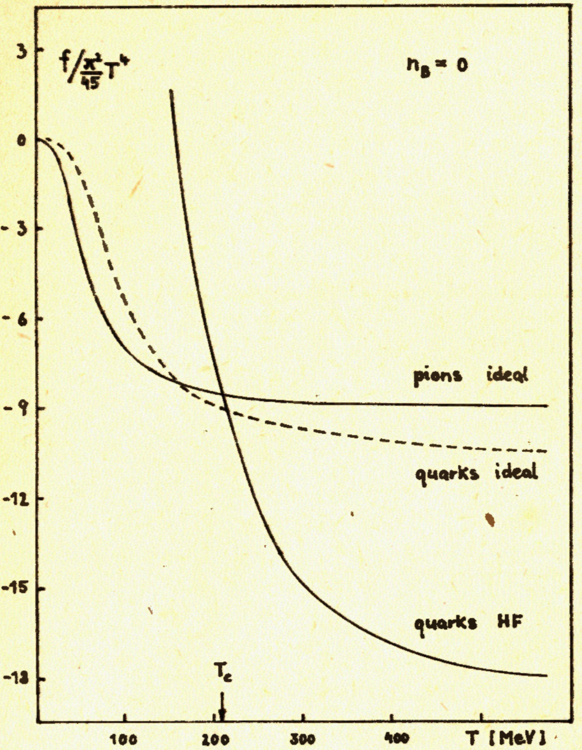

Fig.1. Free energy of the quark-hadron system in dependence on temperature for baryon density $n_B = 0$. Pions ideal: hadronic state of matter (pions only); quarks HF: free quark matter state, Hartree-Fock approximation; quarks ideal: free quark matter state (ideal system).

Macroscopically, we consider both the free energy densities for the quark matter and for the hadronic matter, see Fig.1. The stable phase is characterized by the lowest value of free energy. The critical temperature is shifted from T_c=150 MeV to T_c = 210 MeV if the potential energy in Hartree-Fock approximation is taken into account.

Let us now discuss the case of finite baryon density n_B. The equation of state $n_B(T,\mu_B)$ is obtained starting from the thermodynamic Green function, see /4/. After a cluster expansion of the self-energy, within a ladder-HF approximation we obtain

$$n_B = \frac{1}{3}\, n_q - \frac{1}{3}\, n_{\bar{q}} + \sum_h b_h (n_h - n_{\bar{h}}),$$

$$n_q = g_q \int \frac{d^3p}{(2\pi)^3} (\exp(\epsilon_q(p) - \mu_B/3)/T + 1)^{-1}, \tag{10}$$

$$n_h = g_h \int \frac{d^3p}{(2\pi)^3} (\exp(\epsilon_h(p) - \mu_h)/T \pm 1)^{-1}, \tag{11}$$

where b_h is the baryon number, g_h the degeneration factor of hadrons h, $g_q = 12$. In the relativistic case we have for the quasiparticle energies in HF approximation

$$\epsilon_q(p) = \Sigma_0(p) + \sqrt{(\underline{p} - \underline{\Sigma})^2 + (m_q + \Sigma_s)^2}, \tag{12}$$

where $\Sigma = (\Sigma_0, \underline{\Sigma})$ and Σ_s are the self-energy shifts in relativistic Hartree-Fock approximation. In the non-relativistic case taken here the usual Hartree-Fock shift is obtained from Σ_0.

The bound state energies $\epsilon_h(p)$ are nearly unshifted in HF-approximation if the density is increasing. If the bound state energy merge the continuum edge of scattering states which are shifted in HF-approximation, they are dissolved and give no contribution to the equation of state (10). This behaviour is denoted as Mott mechanism. The similarity of the quark deconfinement and the metal-insulator transition has also been pointed out recently by Satz /11/.

The evaluation of the equation of state is shown for T = 0 in Fig.2, where the contribution of the interaction part V_{QCD} to v_{pot}^{HF} is taken in different approximations. A weakening of the potential V_{QCD} at high densities (asymptotic freedom) is obtained with a running coupling parameter $\alpha_s(n_B)$ as given, for instance, in refs. /9,10/. Furthermore, the potential V_{QCD} is screened in the static approximation for the longitudinal part of the polarization function of matter, see ref. /8/.

It is shown in Fig.2, that the occurrence of stable hadronic matter at T = 0 depends very sensitively on the special form of the potential. For instence, a reasonable value of the phase transition density obtained from the Maxwell construction is seen only in case a of fig.2. Therefore the parameter set of a quark model potential must be determined also by requesting the stability of hadronic matter at zero temperature and low matter density values.

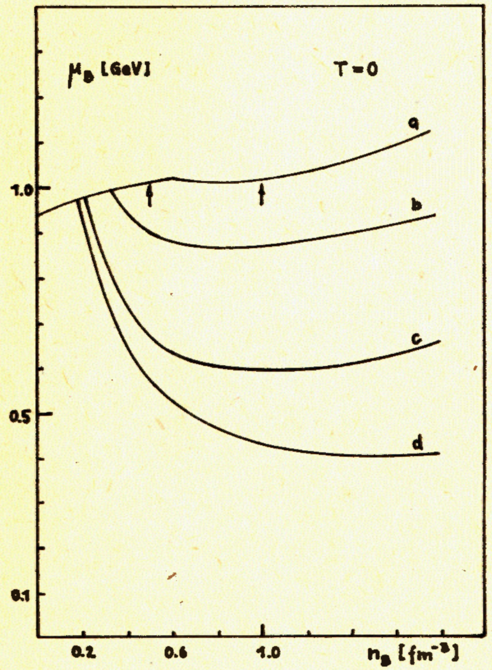

Fig.2. Equation of state $\mu_B(n_B)$ for the quark-hadron system in dependence on the baryon number density n_B for temperature T = 0. Different forms of $V_{QCD}(r)$ are considered: a – V_{QCD} ommitted, arrows denote the region of phase instability; b – screened V_{QCD}, running coupling parameter /9,10/; c – unscreened V_{QCD}, running coupling parameter; d – unscreened V_{QCD}, constant coupling parameter.

Thermodynamic instability $(\partial\mu_B/\partial n_B)_T<0$ occurs also at finite temperature and will give a phase transition region. It is the aim of the present work to point out the possibility of describing a phase transition from bound quarks (hadrons) to free quark matter, starting from a quantum statistical approach using a model potential. A reasonable value for the critical temperature T_c is obtained. More detailed results regarding the choice of parameters of the model potential and the resulting phase transition region are in progress.

From the equation of state the hadron-quark matter phase transition may be achieved on a common basis for both phases. Using the quantum statistical approach, the formation of bound states is obtained in a ladder approximation for the single particle self-energy, and the quark deconfinement is obtained in a natural way from the Mott mechanism which gives a different behaviour of free particle energies and bound state energies (Pauli quenching) if the total quark density is increasing.

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Блашке Д. и др. E2-84-572
Механизм Мотта и фазовый переход адронной материи в кварковую плазму

Единое описание адронной и кварковой материи получается с помощью техники термодинамических функций Грина. Связанные состояния исчезают /деконфайнмент кварков/ вследствие механизма Мотта. Этот эффект дает различные поведения для энергии свободных и связанных частиц при увеличении плотности материи. Простые модельные оценки даны для критической температуры кварк-адронного фазового перехода.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Mott Mechanism and the Hadronic to Quark Matter Phase Transition

A unified description of both the hadronic and quark matter phase can be found using the technique of thermodynamic Green functions. The destruction of bound states (quark deconfinement) is related microscopically to the Mott mechanism which leads to a different behaviour of free particle energies and bound state energies if the particle density is increasing. A simple model calculation is performed to obtain a rough estimate for the critical temperature of the hadronic-quark matter phase transition.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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