сообщения
объединенного
института
ядерных
исследований
дубна

E2-84-479

L.V.Avdeev, D.I.Kazakov, O.V.Tarasov

NO ANOMALY IS OBSERVED

1984

The problem of supersymmetric regularization is of great importance, especially, in connection with cancellations of divergences in extended super Yang-Mills (SYM) theories. As is shown in ref.[1], the conventional dimensional regularization (CDR)[2] breaks supersymmetry at the two-loop level. The regularization by dimensional reduction (RDR)[3] is a modification of CDR for supersymmetric theories. Although better than CDR, RDR is not without a flaw either. The scope of RDR is discussed in detail in ref.[4].

In a recent paper[5] R. van Damme and G. 't Hooft claimed that there is a clash between requiring simultaneously both unitarity and supersymmetry because it is impossible to transform the theory regularized by CDR into that regularized by RDR by means of finite local counterterms. This would cast doubt on the three-loop cancellations in N=2,4 SYM theories found out by using RDR[1,6,7]. We show in the present paper that the claim of ref.[5] is due to some errors in calculations. The correct results exhibit no anomaly.

The idea of RDR is the following. To regularize ultra-violet divergences, one goes from the dimension $d=4$ to $d=4-2\varepsilon$, thus changing the number of gauge fields. As far as in supersymmetric theories it is necessary to provide the equality of Bose and Fermi degrees of freedom, one is to add the so-called ε-scalars, the number of fermions remaining unchanged as in four dimensions. To find the contribution of these additional ε-scalars to the regularized diagrams, it is sufficient to increase the total number of scalars by 2ε everywhere it stands. After that, as in CDR, the divergences are minimally subtracted[8]. Then the difference between the two schemes should lie in the fact that the renormalized parameters are related by a finite transformation.

There are two ways to find this transformation. The first one is to relate the β-functions[9]. The second is to equalize in the different schemes some renormalization-invariant quantities, e.g., the Bogolubov - Shirkov invariant charges[10]. In the leading order we have to recalculate the two-loop β-functions or to extract finite parts of the one-loop diagrams. In ref.[5] it is claimed that these two ways lead to different results. Our aim is to verify this statememt.

RDR preserves supersymmetry at the two-loop level, as established in refs. /1,11,12/ . (Although in ref. /12/ there is a wrong statement that the scheme of ref. /1/ breaks supersymmetry while RDR does not: in fact, the scheme /1/ is RDR). The renormalized SYM theories remain single-charged with all the couplings equal:

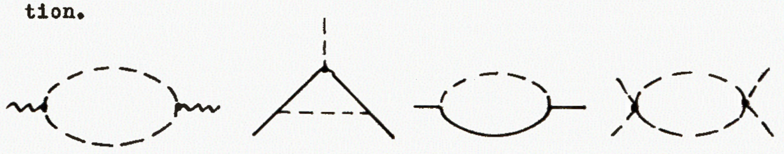
$$g_R = y_R = \rho_{1R} = \rho_{2R} \quad , \tag{1}$$

in the notation of ref. /5/ with the subscript "R" for RDR.

The CDR is non-invariant, so, in order to restore supersymmetry, the charges should be related through some finite coefficients A, B_1, B_2:

$$\begin{aligned} g_c &= y_c \left[1 + A y_c + O(y_c^2) \right] , \\ \rho_{1c} &= y_c \left[1 + B_1 y_c + O(y_c^2) \right] , \\ \rho_{2c} &= y_c \left[1 + B_2 y_c + O(y_c^2) \right] , \end{aligned} \tag{2}$$

with "C" for CDR. Equalizing the invariant charges of RDR to those of CDR we obtain the connection between the couplings. Only the diagrams with scalars (Figure) do contribute to the one-loop recalculation.



a) b) c) d)

We get, up to two-loop corrections,

$$\begin{aligned} &\text{(a)} \quad g_c = g_R \left(1 - \frac{2}{3} \frac{g_R}{16\pi^2} \right) , \\ &\text{(b,c)} \quad y_c = y_R \, , \\ &\text{(d)} \quad \rho_{1c} = \rho_{1R} \left(1 - 6 \frac{\rho_{1R}}{16\pi^2} + 4 \frac{\rho_{2R}}{16\pi^2} \right) , \\ &\qquad \rho_{2c} = \rho_{2R} \left(1 + 2 \frac{\rho_{2R}}{16\pi^2} \right) , \end{aligned} \tag{3}$$

both for N=2 and N=4 SYM theories in the case of the SU(2) gauge group. Using eqs. (1) and (3) we deduce:

$$A = -\frac{2}{3} \frac{1}{16\pi^2} \, , \quad B_1 = -2 \frac{1}{16\pi^2} \, , \quad B_2 = 2 \frac{1}{16\pi^2} \, , \tag{4}$$

in contradiction with ref. /5/ .

To find A, B_1, B_2 from the β -functions, we need the one-loop expressions for arbitrarily chosen g, y, ρ_1 and ρ_2 , and the two-loop ones in the supersymmetric case. We have obtained the following one-loop β - functions, the same in RDR and CDR:

for the N=2 SYM theory

$$\begin{aligned} 16\pi^2 \beta_g^{(1)} &= -4g^2 , \\ 16\pi^2 \beta_y^{(1)} &= 8y^2 - 12yg , \\ 16\pi^2 \beta_{\rho_1}^{(1)} &= 14\rho_1^2 - 12\rho_1\rho_2 + 3\rho_2^2 - 12g\rho_1 + 3g^2 + 8y\rho_1 - 8y^2 , \\ 16\pi^2 \beta_{\rho_2}^{(1)} &= 12\rho_1\rho_2 - 9\rho_2^2 - 12g\rho_2 - 3g^2 + 8y\rho_2 , \end{aligned} \tag{5}$$

and for the N=4 SYM theory

$$\begin{aligned} 16\pi^2 \beta_g^{(1)} &= 0 , \\ 16\pi^2 \beta_y^{(1)} &= 12y^2 - 12yg , \\ 16\pi^2 \beta_{\rho_1}^{(1)} &= 26\rho_1^2 - 20\rho_1\rho_2 + 3\rho_2^2 - 12g\rho_1 + 3g^2 + 16y\rho_1 - 16y^2 , \\ 16\pi^2 \beta_{\rho_2}^{(1)} &= 12\rho_1\rho_2 - 13\rho_2^2 - 12g\rho_2 - 3g^2 + 16y\rho_2 , \end{aligned} \tag{6}$$

in accordance with ref. /5/. In two loops we have computed only the gauge and Yukawa β - functions:

$$\begin{aligned} (16\pi^2)^2 \beta_g^{(2)} &= 0 , \quad \text{RDR and CDR} ; \\ (16\pi^2)^2 \beta_y^{(2)} &= \begin{cases} 0 , & \text{RDR} ; \\ -8y^3 , & \text{CDR} ; \end{cases} \end{aligned} \tag{7}$$

for both N=2 and N=4 SYM theories. Now the coefficient A can be found from the relation

$$\beta_g[g(y)] = \beta_y[y, g(y)] \frac{dg(y)}{dy} \qquad (8)$$

and it coincides with that of eq. (4) for CDR (for RDR, as mentioned above, eq. (1) is obtained).

Thus, we conclude that the two different ways yield the same results. Hence, no anomaly is observed: RDR and CDR are equivalent up to two loops. Most probably, their equivalence can be proved recursively in all orders because the introduction of the ε- scalars deforms the theory not stronger than a change of fermionic traces by $O(\varepsilon)$ /2/ .

Moreover, the true supersymmetry anomaly(an impossibility of renormalizing the theory in a supersymmetric way) is unlikely to be found at all. The renormalization in N=1 supersymmetric gauge theories has been studied in ref./13/ . The conclusion is that in the minimal-subtraction scheme (it can be constructed in any regularization and is gauge-independent for general gauge theories /14/) the counterterms are supersymmetric in any gauge. One only needs an invariant regularization to exist. For N=1 supersymmetric gauge theories such a regularization has been explicitly constructed in ref./15/ . If one uses a non-invariant regularization, then supersymmetry can be restored with finite counterterms by requirung single-chargeness. In ref./16/ it has been proved that the single-charged special solutions of the renormalization-groups equations necessarily exist in higher orders if they have been found in the one-loop approximation. The calculations of the present paper and of ref./9/ confirm this general statement.

For the N=2,4 extended SYM theories considered in terms of N=1 superfields and in the Wess-Zumino gauge, the preservation of N=1 supersymmetry and of the global SU(N) invariance, together with gauge-independence /14/, will ensure /6,11/ single-chargeness. However, a substantiation of their higher-loop finiteness /7/ requires some further arguments, e.g., the existence of an N=2 supersymmetric regularization.

References

1. Avdeev L.V., Tarasov O.V., Vladimirov A.A. Phys.Lett., 1980, 96B, p.94.

2. 't Hooft G., Veltman M. Nucl. Phys., 1972, B44, p.189; Leibbrandt G. Rev.Mod.Phys., 1975, 47, p.849.
3. Siegel W. Phys.Lett., 1979, 84B, p.193.
4. Avdeev L.V., Vladimirov A.A. Nucl. Phys., 1983, B219, p.262.
5. van Damme R., 't Hooft G. A two-loop anomaly in supersymmetric gauge theories. Corrected version, UTRECHT preprint, June, 1984.
6. Grisaru M., Rocek M., Siegel W. Nucl. Phys., 1981, B183, p.141.
7. Caswell W.E., Zanon D. Nucl. Phys., 1981, B182, p.125; Avdeed L.V., Tarasov O.V. Phys.Lett., 1982, 112B, p.356.
8. 't Hooft G. Nucl. Phys., 1973, B61, p.455.
9. Maison D. Determination of correct β- functions for SYM theories using a supersymmetry violating renormalization scheme. Max-Planck-Inst. preprint MPI-PAE/PTh 43/84, May 1984.
10. Bogolubov N.N., Shirkov D.V. Introduction to the theory of quantized fields. Interscience Pub., N.Y., ch.VIII, 1959.
11. Avdeev L.V. Phys.Lett., 1982, 117B, p.317.
12. van Damme R. Nucl. Phys., 1983, B227, p.317.
13. Tyutin I.V. Yad.Fiz., 1983, 37, p.761.
14. Voronov B.L., Lavrov P.M., Tyutin I.V. Yad.Fiz., 1982, 36, p.498.
15. Krivoshchekov V.K. Teor.Mat.Fiz., 1978, 36, p.291.
16. Tyutin I.V. Sov.Phys. Lebedev Inst.Reps., 1978, No. 8, p.3.
17. Grisaru M.T., Siegel W. Nucl. Phys., 1982, B201, p.292; Howe P.S., Stelle K.S., Townsend P.K. Miraculous ultraviolet cancellations in supersymmetry made manifest. Preprint ICTP/82-83/20, London, 1983; Mandelstam S. Nucl. Phys., 1983, B213, p.149.

Received by Publishing Department
on July 6, 1984.

Авдеев Л.В., Казаков Д.И., Тарасов О.В. E2-84-479
Аномалия не наблюдается

Рассматривается двухпетлевая аномалия в суперсимметричных калибровочных теориях, обсуждаемая в недавней работе Р.ван Дамма и Ж.'т Хоофта. Показано, что она является результатом ошибки в вычислениях. В действительности возможно перейти от теории, регуляризованной с помощью обычной размерной регуляризации, к теории, регуляризованной с помощью размерной редукции, с использованием конечных локальных контрчленов. Следовательно, аномалия не наблюдается.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1984

Avdeev L.V., Kazakov D.I., Tarasov O.V. E2-84-479
No Anomaly is Observed

The two-loop anomaly in supersymmetric gauge theories discussed in a recent paper by R.van Damme and G.'t Hooft is considered. We find that this is due to some mistakes in calculations. In fact, it is possible to transform the theory regularized by the conventional dimensional regularization into the dimensionally reduced one by means of finite local counterterms. So, no anomaly is observed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1984