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PROPERTIES OF A SCALAR GLUEBALL

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1. INTRODUCTION

An exciting prediction of QCD as the theory of strong interactions is the existence of glueballs, the bound states made up of gluons [1-4].However, a definite verification of the prediction is not yet done. There are announced glueball candidates [5-10], but different interpretations are possible as well [11-12]. Thus, it becomes more and more clear that for the identification of these states one should know not only their masses and quantum numbers but also their decays to ordinary hadrons. This is even more urgent because the glueball candidates [5-7,10] do not behave in their decays as one naturally expects [13]. Since the glueballs are flavour singlets, it is expected [13] that they are equally coupled to all flavours and so, their decays into, e.g., $\pi^+\pi^-$ and K^+K^- mesons should only differ by phase space factors increasing thus decay to pions. However, experimentally the opposite is seen for the glueball candidates [5-7,10,14].

Independent theoretical results [15,16] and, maybe, experimental indications [5-10] show that the scalar glueball is probably the lightest one with mass around 1 GeV. So, the number of its hadronic decay modes is limited; it decays only to the lighter pseudoscalar mesons. This suggests that in order to understand the decay properties of the scalar glueball, it is highly desirable to have a nontrivial model describing interactions between this glueball and pseudoscalar mesons. Moreover, one can hope that the main characteristics of the model can even be generally valid for interactions of glueballs with pseudoscalar mesons.

Recently, an effective Lagrangian model of this type has been suggested in our paper [17]. This model has been shown [17] to satisfy the anomaly relation of the trace of the energy - momentum tensor of QCD [18] and the important low-energy theorems of refs.[16,19,20]. Here (section 2) we want to present the model in more detail. We shall see that the part of the Lagrangian that describes the effective interaction between a scalar glueball and a pair of pseudoscalar Goldstone mesons is predicted if one specifies the mass of the scalar glueball. The model will be shown to be in a resonable agreement with the glueball assignment for the g_s (1240)[7] scalar meson. In this way it will be explicitly demonstrated that the coupling of the scalar glueball to pseudoscalar Goldstone bosons is only due to a chiral-symmetry-breaking quark-mass term in QCD Lagrangian, i.e., in the

SU(3)xSU(3) chiral symmetry limit the glueball does not decay to lighter pseudoscalars. In the case of exact SU(2)xSU(2) symmetry this glueball does not decay to pions while in the realistic world the width of such a decay is proportional to m_π^4 and is strongly suppressed. Thus, we cal this coupling the SU(2)xSU(2) rule. We shall also show that the SU(2)xSU(2) coupling rule explains the existing experimental data [6,14] for decays of a tensor glueball candidate $\theta(1640)$ into pseudoscalar pairs (section 3). In section 4 some conclusions are drawn.

2. AN EFFECTIVE LAGRANGIAN FOR A HYPOTHETICAL SCALAR GLUEBALL AND PSEUDOSCALAR GOLDSTONE MESONS

Let us begin our considerations by assuming that the low-energy dynamics of the octet of the pseudoscalar Goldstone mesons is described by the following effective Lagrangian (for further references see, e.g., ref. [21])

$$\mathcal{L} = \frac{1}{4}\,\mathrm{Tr}[(\partial_\mu U)(\partial^\mu U^+)] + \mathcal{L}_{SB}\,, \tag{1a}$$

where

$$\mathcal{L}_{SB} = -\,\mathrm{Tr}[M(U+U^+)]\,. \tag{1b}$$

Here the elements of the 3x3 field-matrix U(x) form the $(3,\bar{3})$ representation of the chiral SU(3)xSU(3) group, i.e., under chiral transformations U(x) transforms as follows

$$U \to A\,U\,B^+\,, \tag{2}$$

where A and B are unitary matrices of transformations. The matrix M in eq.(1b) is real diagonal one and is proportional to the mass matrix of light quarks. So, the explicit breaking of chiral invariance due to the quark masses is provided by $\mathcal{L}_{SB}$ term (eq.(1b)) representing the genuine $(3,\bar{3})+(\bar{3},3)$ model [22]. In the "current algebra" Lagrangian (1) the matrix U(x) satisfies the constraint [21]

$$U(x)\,U^+(x) = f_\pi^2 \tag{3}$$

and can be parametrized as

$$U(x) = f_\pi \exp\Big(i \sum_{j=1}^{8} \frac{\lambda_j \phi_j(x)}{f_\pi}\Big)\,, \tag{4}$$

where f_π is the pion decay constant (f_π = 93 MeV), ϕ_i's (i = = 1,...8) are fields of the octet of the pseudoscalar Goldstone mesons and λ's are the Gell-Mann λ matrices normalized to $\mathrm{Tr}(\lambda_i\lambda_j) = 2\delta_{ij}$. The Lagrangian (1) combined with eq.(4) completely reproduces current algebra results for the system of pseudoscalar Goldstone mesons. We mention here that we neglect the pseudoscalar (non-Goldstone boson) singlet field (and, correspondingly, a term in eq.(1) that solves the U(1)-problem) since such a neglect is not essential in what follows provided the scalar glueball is light and cannot decay into the $\eta\eta'$ nor $\eta'\eta'$ systems.

An interesting and important result coming from eqs.(1) and (3) (or (4)) is the trace of the "improved" energy-momentum tensor $\theta_{\mu\nu}$ [23] which has the following form

$$(\theta^\mu_\mu)_1 = -\frac{1}{2}\,\mathrm{Tr}[(\partial_\mu U)(\partial^\mu U^+)] - 4\mathcal{L}_{SB}\,, \tag{5}$$

where index "1" labels the correspondence to eq.(1). To deduce eq.(5), it is useful to introduce the scalar u_j's and pseudoscalar v_j's (j = 0,1,...,8) fields by the relations

$$u_j = \frac{1}{4}\,\mathrm{Tr}[\lambda_j(U+U^+)]\,,\qquad v_j = \frac{1}{4i}\,\mathrm{Tr}[\lambda_j(U-U^+)]\,. \tag{6}$$

Then Lagrangian (1) can be rewritten in the form:

$$\mathcal{L} = \frac{1}{2}\sum_{i=0}^{8}[(\partial_\mu u_i)^2 + (\partial_\mu v_i)^2] + \mathcal{L}_{SB}\,. \tag{7}$$

Now let us assume that the fields u's and v's (and consequently the field-matrix U) have dimensions (conformal weights) equal to the number d, i.e., under dilatation transformations $x \to \rho x$ ($\rho > 0$ being an arbitrary number) one gets $U(x) \to \rho^{-d}U(x)$ and $U^+(x) \to \rho^{-d}U^+(x)$. It is an easy exercise to obtain the "improved" energy-momentum tensor [23] from eq.(7). We get

$$\theta_{\mu\nu} = \sum_{i=0}^{8}[(\partial_\mu u_i)(\partial_\nu u_i) + (\partial_\mu v_i)(\partial_\nu v_i)] - g_{\mu\nu}\mathcal{L} + \frac{d}{6}[g_{\mu\nu}\partial^\lambda\partial_\lambda - \partial_\mu\partial_\nu]\sum_{i=0}^{8}(u_i^2 + v_i^2)\,. \tag{8}$$

The trace of the $\theta_{\mu\nu}$ reads (after the use of equations of motion)

$$\theta^\mu_\mu = (d-1)\sum_{i=0}^{8}[(\partial_\mu u_i)^2 + (\partial_\mu v_i)^2] + (d-4)\mathcal{L}_{SB}\,, \tag{9a}$$

or, in a more compact form (using eqs.(6)):

$$\theta^\mu_\mu = \frac{d-1}{2}\,\mathrm{Tr}[(\partial_\mu U)(\partial^\mu U^+)] + (d-4)\mathcal{L}_{SB}\,. \tag{9b}$$

Due to condition (3) the dimension (conformal weight) d = 0 [24] and then eq.(9b) gives eq.(5).

On the other hand, in QCD the result for the trace of the energy-momentum tensor is given as [18]

$$(\theta^{\mu}_{\mu})_{QCD} = \frac{\beta(g)}{2g} F^{(a)}_{\mu\nu} F^{(a)\mu\nu} - (1+\gamma_m(g))\,\mathcal{L}^{QCD}_{SB}, \qquad (10)$$

where $F^{(a)}_{\mu\nu}$'s ($a = 1,\dots,8$) are gluon-field strength tensors, $\beta(g)$ is the Callan-Symanzik function and $\gamma_m(g)$ is the mass anomalous dimension. The term $\mathcal{L}^{QCD}_{SB}(x) = -\sum_i m_{q_i} \bar{q}_i(x) q_i(x)$ (m_{q_i}'s are quark masses, $q_i(x)$'s are quark fields, i is a given flavour) represents the chiral-symmetry-breaking term in the QCD Lagrangian.

In the pseudoscalar Goldstone meson sector described by eqs.(1) and (4) the relation (10) is effectively represented by eq.(5). However, because of different dimensions (conformal weights) of the terms $\mathcal{L}_{SB}$ and $\mathcal{L}^{QCD}_{SB}$ in eqs.(5) and (10) chiral noninvariant pieces of these equations are formally different (naive comparison gives the unacceptable result $\gamma_m(g) = 3$). Although such a difference is allowed for effective Lagrangians nevertheless, being guided by eq.(10) we want to enlarge eq.(1) in a way to include a scalar field into it. In fact, to follow closer eq.(10), the improvement of the dimension of eq.(1b) is needed. This can be done by assuming the existence of a dimensional, flavour-independent scalar field $\sigma(x)$ (dimension $d_\sigma = 1$) which can be used to write down the following symmetry-breaking term

$$\mathcal{L}'_{SB}(x) = -[\sigma(x)]^{(3-\gamma_m)}\ \mathrm{Tr}[M(U(x)+U^{+}(x))] \qquad (11)$$

instead of eq.(1b). In eq.(11) γ_m is a parameter which will be specified later. We note here that since σ is flavour-independent, it is singlet under chiral (i.e., in the flavour space) transformations, and therefore $\mathcal{L}'_{SB}$ belongs again to the $(3,\bar{3})+(\bar{3},3)$ representation as it is required [22]. We also remark that consistency with spontaneous symmetry breaking (requiring VEV $\langle\sigma\rangle_0 = \sigma_0 \neq 0$) and correct behaviour of $\sigma(x)$ under dilatations ($x \to \rho x$, $\sigma(x) \to \rho^{-1}\sigma(x)$) need introduction of the actual physical field $\tilde{\sigma}(x)$ $(\langle\tilde{\sigma}\rangle_0 = 0)$ through the parametrization [25]

$$\sigma(x) = \sigma_0 \exp\left(\frac{\tilde{\sigma}(x)}{\sigma_0}\right), \qquad (12)$$

where $\tilde{\sigma}(x) \to \tilde{\sigma}(x) - \sigma_0 \ln\rho$ when $x \to \rho x$.

It should be stressed here that there is no need to change the dimension of the first, chirally invariant but dilatationaly noninvariant term in eq.(1a) since just this term gives a chirally symmetrical contribution to eq.(5) in agreement with the QCD trace anomaly, eq.(10). Moreover, in the chiral-symmetry limit it is this piece of the trace of the energy-momentum tensor (eq.(5)) which effectively represents the low-energy theorem of refs. [19,20]*

$$\langle P(p_1)\bar{P}(p_2)|\,(\theta^{\mu}_{\mu})\,|\,0\rangle\Big|_{\substack{\text{chiral}\\ \text{limit}}} = q^2, \qquad (13)$$

where $q^2 = 2p_1\cdot p_2 = (p_1+p_2)^2$ is the invariant (mass)^2 of the $P\bar{P}$ system.

Thus, a minimal enlargement of Lagrangian (1) including the σ-field is proposed to be of the following form

$$\mathcal{L}_{compl} = \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{4}\mathrm{Tr}[(\partial_\mu U)(\partial^\mu U^{+})] - V(\sigma) + \mathcal{L}'_{SB}, \qquad (14)$$

where U, σ and $\mathcal{L}'_{SB}$ are given by eqs.(4), (12), and (11), respectively, and $V(\sigma)$ is a chirally invariant potential and as such dependent only on the flavour-independent σ-field. The Lagrangian (14) gives

$$(\theta^{\mu}_{\mu})_{14} = -\frac{1}{2}\mathrm{Tr}[(\partial_\mu U)(\partial^\mu U^{+})] + 4V(\sigma) - \sigma\frac{dV(\sigma)}{d\sigma} - (1+\gamma_m)\,\mathcal{L}'_{SB}\ . \qquad (15)$$

We see already formal consistency between eqs.(10) and (15) and we also expect that the parameter γ_m is approximately given by perturbation theory, i.e., $\gamma_m \doteq \gamma_m(g(\mu))$, where μ is some typical hadronic mass scale. We choose for definiteness $\alpha_s(\mu) = 0.7$ at $\mu = 0.2$ GeV [26] and then $\gamma_m \doteq 2\alpha_s/\pi + O(\alpha_s^2) = 0.5 + O(\alpha_s^2)$. To completely specify Lagrangian (14) it still remains to find the potential $V(\sigma)$. To do this, let us expand $V(\sigma)$ in the right field $\tilde{\sigma}$:

$$V(\sigma) = V(\sigma_0) + \left\langle\frac{dV}{d\tilde{\sigma}}\right\rangle_0 \tilde{\sigma} + \frac{1}{2}\left\langle\frac{d^2V}{d\tilde{\sigma}^2}\right\rangle_0 \tilde{\sigma}^2 + \dots\ . \qquad (16)$$

Using parametrizations (4) and (12) in eq.(14), and eliminating the term linear in $\tilde{\sigma}$ from (14) by requiring

$$\left\langle\frac{dV}{d\tilde{\sigma}}\right\rangle_0 = \frac{1}{2}\,\frac{3-\gamma_m}{\sigma_0}\,(2m_K^2 + m_\pi^2)\, f_\pi^2, \qquad (17)$$

we obtain Lagrangian (14) in a correct form. From this Lagrangian one easily finds, e.g., the σ-particle (mass)^2:

$$m_\sigma^2 = \left\langle\frac{d^2V}{d\tilde{\sigma}^2}\right\rangle_0 - \frac{3-\gamma_m}{\sigma_0}\left\langle\frac{dV}{d\tilde{\sigma}}\right\rangle_0 \qquad (18)$$

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* As usual, we shall calculate in tree approximation, and states will be normalized covariantly: $\langle p\,|\,p'\rangle = (2\pi)^3\, 2\omega_p\, \delta^{(3)}(p-p')$.

and the interaction term:

$$\mathcal{L}_{\sigma P\bar{P}}(x) = -\frac{1}{2}\frac{3-\gamma_m}{\sigma_0}\tilde{\sigma}(x)\sum_{i=1}^{8} m_i^2 \phi_i^2(x), \tag{19}$$

where m_i's ($i = 1,\dots,8$) are masses of the octet of the pseudoscalar mesons. It is seen from eqs.(10), (15), and (16) that the chirally invariant part of the trace anomaly is effectively given as

$$H(x) \equiv -\frac{\beta(g)}{2g}F^2(x) = H_0 + H_1\tilde{\sigma}(x) + H_2\tilde{\sigma}^2(x) + O(\tilde{\sigma}^3) + \frac{1}{2}\mathrm{Tr}[(\partial_\mu U)(\partial^\mu U^+)], \tag{20}$$

where

$$H_0 = -<\frac{\beta(g)}{2g}F^2>_0 = \sigma_0 <\frac{dV}{d\tilde{\sigma}}>_0 - 4V(\sigma_0),$$
$$H_1 = \sigma_0 <\frac{d^2V}{d\tilde{\sigma}^2}>_0 - 4<\frac{dV}{d\tilde{\sigma}}>_0, \tag{21}$$
$$H_2 = \frac{1}{2}[\sigma_0 <\frac{d^3V}{d\tilde{\sigma}^3}>_0 - 4<\frac{d^2V}{d\tilde{\sigma}^2}>_0], \text{etc.}$$

To find coefficients H_i ($i = 1,2,\dots$) one can use successively the following low-energy theorems [16] (valid in the chiral-symmetry limit):

$$i\int dx <o|T(H(x)H(o))|o> = 4H_0[1+O(m_q)],$$
$$i^2\int dx\int dy <o|T(H(x)H(y)H(o))|o> = 16H_0[1+O(m_q)], \text{etc.} \tag{22}$$

Combining eq.(20) and the first of eqs.(22) we get

$$H_1^2 = 4m_\sigma^2 H_0[1+O(m_q)]. \tag{23}$$

Analogously, eqs.(22) can be used to calculate all the coefficients H_i in terms of, e.g., m_σ and H_0. Moreover, from eqs.(17), (18), (21), and (23) one finds

$$m_\sigma^2\sigma_0^2 = 4H_0[1+O(m_q)]. \tag{24}$$

The value of H_0 is approximately given as follows (for the $SU(3)_c$-colour group and for three light flavours, $N_F = 3$):

$$H_0 = -<\frac{\beta(g)}{2g}F^2>_0 = \frac{9}{8}<\frac{\alpha_s}{\pi}F^2>_0 + O(\alpha_s^2), \tag{25}$$

where $<(\alpha_s/\pi)F^2>_0$ is the familiar gluon-condensate term parametrizing nonperturbative effects of QCD [26]. Shifman, Vainshtein, and Zakharov were first [26] who estimated this condensate by

analysing the QCD sum rules for charmonium. They obtained

$$<\frac{\alpha_s}{\pi}F^2>_0 = 0.012\ \mathrm{GeV}^4. \tag{26}$$

However this value is not yet strictly determined, and a larger value than given in eq.(26) is called for (may be by a factor 2 ÷ 3) [27]. Thus, the only arbitrary parameter of our model (eq.(14)) remains the mass m_σ of the scalar σ-particle. Since the σ-particle dominates the scalar gluonic current (see eqs. (20), (22)-(24)), then this particle must be identified with a hypothetical scalar glueball. Such an identification is supported also by a large N_c-dynamics (N_c is a number of colours). For example, from eqs.(20) and (23) it is (as it must be for a true glueball, see ref. [16]) $<o|H(o)|\sigma> = 2m_\sigma\sqrt{H_0} \sim N_c$ in the large N_c limit because, as usual, $m_\sigma \sim N_c^0$, and from eq.(25) $H_0 \sim N_c^2$.

It is worth to note here that just the constructed Lagrangian (14) gives (combining eqs.(15)-(19)) a generalized version (for nonzero quark masses) of eq.(13), namely,

$$<P(p_1)\bar{P}(p_2)|(\theta^\mu_\mu)_{14}|0> = 2p_1\cdot p_2 + (3-\gamma_m)m_P^2\frac{m_\sigma^2}{m_\sigma^2 - q^2} + (1+\gamma_m)m_P^2, \tag{27}$$

which for higher σ-particle nass ($m_\sigma^2 > q^2 \geq 4m_P^2$) behaves as

$$<P(p_1)\bar{P}(p_2)|(\theta^\mu_\mu)_{14}|0> = q^2 + 2m_P^2 \tag{28}$$

in full accordance with such a generalization of the low-energy theorem in ref. [20]. Taking eqs.(27) and (28) to be valid for all eight pseudoscalar mesons (i.e., $P\bar{P} = \pi^+\pi^-$, K^+K^-, $\eta\eta$, etc.), we easily see that the present model suggests the bound $m_\sigma > 2m_\eta \doteq 1.1$ GeV for the mass of the scalar glueball.

Defining the decay amplitude $T_{i\to f}$ as

$$<f|S|i> = \delta_{if} + i(2\pi)^4\delta^{(4)}(p_f - p_i)T_{i\to f}, \tag{29}$$

where, as usual, $S = T\exp(i\int dx\,\mathcal{L}_{int}(x))$, then using the interaction term (19) and combining it with eqs.(24) and (25) one easily obtains the following formulae for the decay widths of σ into pseudoscalar pairs:

$$\Gamma_{\sigma\to\pi^+\pi^-} = 2\Gamma_{\sigma\to\pi^\circ\pi^\circ} = Am_\pi^4\left(1-\frac{4m_\pi^2}{m_\sigma^2}\right)^{1/2},$$

$$\Gamma_{\sigma\to K^+K^-} = \Gamma_{\sigma\to K^0\bar{K}^0} = A m_K^4 \left(1 - \frac{4m_K^2}{m_\sigma^2}\right)^{1/2}, \qquad (30)$$

$$\Gamma_{\sigma\to\eta\eta} = \frac{1}{2} A m_\eta^4 \left(1 - \frac{4m_\eta^2}{m_\sigma^2}\right)^{1/2},$$

where the overall factor A is

$$A = \left(1 - \frac{\gamma_m}{3}\right)^2 \frac{m_\sigma}{8\pi \langle \frac{\alpha_s}{\pi} F^2 \rangle_0}. \qquad (31)$$

The scalar glueball candidate $g_s(1240)$ /7/ satisfies the mass bound $m_{g_s} = 1.24$ GeV > 1.1 GeV and still is light enough to have dominant hadronic decay only into pseudoscalar pairs. Then to a good accuracy the total width Γ_{g_s} is given as

$$\Gamma_{g_s} \doteq \Gamma_{g_s\to\pi\pi} + \Gamma_{g_s\to K\bar{K}} + \Gamma_{g_s\to\eta\eta}. \qquad (32)$$

Labelling $x_\pi = \Gamma_{g_s\to\pi\pi}/\Gamma_{g_s}$, $x_K = \Gamma_{g_s\to K\bar{K}}/\Gamma_{g_s}$ and putting $m_\sigma = m_{g_s} =$ = 1.24 GeV we obtain $(x_\pi x_K)^{1/2} = 0.06$ from eqs.(30) and (32); and for $\gamma_m \doteq 0.5$, $\langle(\alpha_s/\pi)F^2\rangle_{0\,SVZ} = 0.012$ GeV^4 (see eq.(26)) we find $\Gamma_{g_s} = 270$ MeV while for $\langle\alpha_s F^2\rangle_0 = 2\langle\alpha_s F^2\rangle_{0\,SVZ}$ one gets $\Gamma_{g_s} = 135$ MeV. We see that the agreement with experimental values /7/ $(x_\pi x_K)^{1/2}_{\text{experiment}} = 0.04$ and $(\Gamma_{g_s})_{\text{experiment}}$ = = (140 ± 10) MeV is remarkable. Because of the lack of knowledge of precise values of the phenomenological parameters H_0 and γ_m it is difficult to say whether the consistency with experiment requires definitely a higher value of $\langle(\alpha_s/\pi)F^2\rangle_0$ although this seems to be the case when using reasonable approximations given by eqs.(23)-(25) and $\gamma/3 \ll 1$. We note also here that the decay pattern of another announced scalar glueball candidate G(1590) /10/ is not consistent with eqs.(30).

3. THE COUPLING OF A TENSOR GLUEBALL TO PSEUDOSCALAR MESONS

In the previous section we have explicitly illustrated (see eqs.(14), (19) and (30)) the SU(2)xSU(2) rule for the coupling of a scalar glueball to pseudoscalar mesons. Here we want to show that this rule is valid more generally, namely, for the coupling between the tensor glueball candidate $\theta(1640)$ and pseudoscalar mesons (for the original suggestion, see /28/).

So, let us label the field of the tensor glueball candidate $\theta(1640)$ as $\phi_{\mu\nu}(x)$, where

$$\partial^\mu \phi_{\mu\nu} = 0, \qquad g^{\mu\nu}\phi_{\mu\nu} = 0 \qquad (33)$$

and $\phi_{\mu\nu}$ is symmetrical in μ,ν (see, e.g., ref. /29/). Since $\phi_{\mu\nu}$ is flavour-blind, it is singlet under chiral (i.e., in the flavour space) transformations, and then besides U (eqs.(2) and (4)) the following derivative terms, for example, $\phi^{\mu\nu}(\partial_\mu\partial_\nu U)$, $\phi^{\mu\nu}(\partial_\mu U)(\partial_\nu U^+)U$, $\phi^{\mu\nu}U(\partial_\mu U^+)(\partial_\nu U)$, $\phi^{\mu\nu}U(\partial_\mu\partial_\nu U^+)U$, etc., satisfy eq.(2); thus, a linear combination of them can be used in eq.(1b) instead of U. However, not all these derivative terms are nontrivial and independent, because due to eq.(33) we have, e.g.,:

$$\partial_\mu(\phi^{\mu\nu}\partial_\nu U) = \phi^{\mu\nu}(\partial_\mu\partial_\nu U), \qquad (34a)$$

$$\partial_\mu[\phi^{\mu\nu}U(\partial_\nu U^+)U] = \phi^{\mu\nu}(\partial_\mu U)(\partial_\nu U^+)U + \phi^{\mu\nu}U(\partial_\mu\partial_\nu U^+)U + \phi^{\mu\nu}U(\partial_\nu U^+)(\partial_\mu U). \qquad (34b)$$

The l.h.s. of these relations are full derivatives and as such do not give nontrivial contributions to Lagrangian; then all three terms on the r.h.s. of eq.(34b) are not independent too. As a result (after the use of parametrization (4)), we choose

$$\mathcal{L}_{\theta P\bar{P}}(x) = g_1 \phi^{\mu\nu}(x) \sum_{i=1}^{8} m_i^2 (\partial_\mu\phi_i(x))(\partial_\nu\phi_i(x)), \qquad (35)$$

what is then the only nontrivial and independent Lagrangian term coming from the general effective quark-mass term and describing interaction between $\theta(1640)$ and pseudoscalar pair particles $P,\bar{P}$ ($P\bar{P} = \pi^+\pi^-$, K^+K^-, etc.). Here g_1 is some unknown constant and m_i's are masses of the pseudoscalar mesons. Using eqs.(29) and (35) it is easy to obtain explicitly the following partial decay widths /28/

$$\Gamma_{\theta\to\pi^+\pi^-} = 2\Gamma_{\theta\to\pi^0\pi^0} = C m_\pi^4 \left(1 - \frac{4m_\pi^2}{m_\theta^2}\right)^{5/2},$$

$$\Gamma_{\theta\to K^+K^-} = \Gamma_{\theta\to K^0\bar{K}^0} = C m_K^4 \left(1 - \frac{4m_K^2}{m_\theta^2}\right)^{5/2}, \qquad (36)$$

$$\Gamma_{\theta\to\eta\eta} = \frac{1}{2} C m_\eta^4 \left(1 - \frac{4m_\eta^2}{m_\theta^2}\right)^{5/2},$$

where an unknown overall constant C depends only on g_1 and m_θ. We see from eqs.(36) that the decay of $\theta(1640)$ into pions is naturally suppressed due to smallness of the pion mass. Eqs.(36) give (for $m_\theta = 1.64$ GeV):

$$\Gamma_{\theta\to\pi\pi}/\Gamma_{\theta\to\eta\eta} = 0.05, \quad \text{(experiment: < 1)},$$

$$\Gamma_{\theta\to\eta\eta} / \Gamma_{\theta\to K\bar{K}} = 0.26 , \quad \text{(experiment: } 0.33\pm0.2), \qquad (37)$$

where the experimental data are from ref.[14].

It is interesting to note here that $\mathcal{L}'_{SB}$ term (eq.(11)) has the dimension corresponding to the quark-mass term one if one assumes $\gamma_m = 0$ while in the case of a tensor particle coupled to pseudoscalar mesons (eqs.(34)-(35)) due to derivative couplings this property is automatically satisfied. Then the overall factor A in eq.(31) is specified by specifying m_σ ($\gamma_m/3 \ll 1$ and can be neglected) while the factor C in eq.(36) remains unknown and only the ratios (37) can be predicted. We see (eq.(37)) that this prediction is in a remarkable agreement with existing experimental data [14].

4. CONCLUSION

The Lagrangian (14) has been constructed as a minimal enlargement of eq.(1) so as to lead to eqs.(15) and (20). These equations effectively represent the important low-energy theorems of refs.[19,20] thus justifying the starting Lagrangian (14). The Lagrangian (14) contains besides the pseudoscalar octet fields the only scalar glueball field σ, i.e., other possible quarkonium scalar mesons and their eventual mixing with σ-glueball are neglected. However, this does not mean that there is no mixing between gluon and quark degrees of freedom. It fact, the present model realizes strong mixing of this type, as one can see from eq.(20), having on the r.h.s. large and unsuppressed pseudoscalar meson (i.e., quark) contributions too. It is just this type of mixing[16] that explicitly gives not only low-energy theorems of refs.[19,20] but also is consistent with the SU(2)xSU(2) coupling rule. This rule (see eqs.(19), (30), (35), and (36)) is in a good agreement with the existing experimental data on $g_s(1240)$[7] abd $\theta(1640)$[6,14] glueball candidates. However, any definite conclusions need further experimental work, namely, the confirmation of $g_s(1240)$[7] particle is urgently required.

Note added: after this work was finished the paper[30] has appeared in which the coupling of the type of eq.(19) (between a scalar glueball and mesons) has been independently mentioned.

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REFERENCES

1. Fritzsch H., Gell-Mann M. Proc.16-th Int.Conf.on High Energy Physics, vol.2, Batavia, 1972, p.135.
2. Fritzsch H., Minkowski P. Nuovo Cim., 1975, 30A, p.393.
3. Jaffe R., Johnson K. Phys.Lett., 1976, 60B, p.201.
4. Freund P.G.O., Nambu Y. Phys.Rev.Lett., 1975, 34, p.1645.
5. Scharre D.L. et al. Phys.Lett., 1980, 97B, p.329.
6. Edwards C. et al. Phys.Rev.Lett., 1982, 48, p.458.
7. Etkin A. et al. Phys.Rev., 1982, D25, p.2446.
8. Etkin A. et al. Phys.Rev.Lett., 1982, 49, p.1620.
9. Etkin A. et al. Preprint BNL-33384, 1983.
10. Binon F. et al. Nuovo Cim., 1983, 78A, p.313.
11. Chanowitz M.S. In: Particles and Fields - 1981: Testing the Standard Model. Proc.Meeting of the Division of Particles and Fields of the APS, Santa Cruz, California (ed.by C.A. Heusch and W.T.Kirk). AIP, New York, 1982, p.85.
12. Cohen J., Isgur N., Lipkin H.J. Phys.Rev.Lett., 1982, 48, p.1074; Filippov A.T. Pis'ma Zh.Eksp.Teor.Fiz., 1982, 36, p.96.
13. Lipkin H.J. Phys.Lett., 1981, 106B, p.114; ibib., 1982, 109B, p. 326.
14. Bloom E.D. XXI Int.Conf.on High Energy Physics, Paris, 1982, p.407.
15. Barnes T., Close F.E., Monaghan S. Nucl.Phys., 1982, B198, p.380; Berg B., Billoire A. Nucl.Phys., 1983, B221, p.109; Shifman M.A. Z.Phys.C - Particles and Fields, 1981, 9, p.347; Pascual P., Tarrach R. Phys.Lett., 1982, 113B, p.495; Cornwall J.M., Soni A. Phys.Lett., 1983, 120B, p.431; Balâzs L.A.P. Phys.Lett., 1983, 120B, p.426.
16. Novikov V.A. et al. Nucl.Phys., 1981, B191, p.301.
17. Lânik J. JINR, E2-84-266, Dubna, 1984.
18. Crewther R.J. Phys.Rev.Lett., 1972, 28, p.1421; Chanowitz M.S., Ellis J. Phys.Rev., 1973, D7, p.2490; Collins J.C., Duncan A., Joglekar S.D. Phys.Rev., 1977, D16, p.438.
19. Voloshin M., Zakharov V. Phys.Rev.Lett., 1980, 45, p.688.
20. Novikov V., Shifman M. Z.Phys. C - Particles and Fields, 1981, 8, p.43.
21. Witten E. Annals of Phys., 1980, 128, p.363; P.di Vecchia et al. Nucl.Phys., 1981, B181, p.318; Volkov M.K. Particles and Nuclei, 1982, 13, p.1070.
22. Gell-Mann M., Oakes R.J., Renner B. Phys.Rev., 1968, 175, p.2195; Glashow S.L., Weinberg S. Phys.Rev.Lett., 1968, 20, p.224.
23. Callan C.G., Coleman S., Jackiw R. Annals of Phys., 1970, 59, p.42.
24. Ellis J. Nucl.Phys., 1970, B22, p.478.
25. Salam A., Strathdee J. Phys.Rev., 1969, 184, p.1760; Migdal A.A., Shifman M.A. Phys.Lett., 1982, 114B, p.445.

26. Shifman M.A., Vainshtein A.I., Zakharov V.I. Nucl.Phys., 1979, B147, p.385,448.
27. Bell J.S., Bertlmann R.A. Nucl.Phys., 1981, B177, p.218; ibid., 1981, B187, p.285; Bradley A., Langensiepen C.S., Shaw G. Phys.Lett., 1981, B102, p.359.
28. Lánik J. JINR, E2-84-331, Dubna, 1984.
29. Gasiorowicz S. Elementary Particle Physics, Wiley and Sons, Inc., New York, 1966, p.432.
30. Cornwall J.M., Soni A. Phys.Rev., 1984, D29, p.1424.

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Ланик Й. E2-84-437
Свойства скалярного глюбола

Детально анализируется эффективно-лагранжевская модель /предложенная нами прежде/ для связи между скалярным глюболом и псевдоскалярными мезонами. Показано, что эта связь удовлетворяет SU(2) x SU(2) правилу. Модель находится в согласии с представлением скалярной частицы $g_s(1240)$ в качестве глюбола. Более того, показано, что SU(2) x SU(2) правило связи объясняет также существующие экспериментальные данные о распаде возможного тензорного глюбола $\theta(1640)$ на псевдоскалярные мезоны.

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Properties of a Scalar Clueball

A detailed analysis of a previously suggested effective Lagrangian model for coupling between a scalar glueball and pseudoscalar mesons is given. This coupling is shown to satisfy the SU(2) x SU(2) rule. The model is consistent with the glueball assignment for the scalar $g_s(1240)$ particle. Moreover, the SU(2) x SU(2) coupling rule explains also the existing experimental data for decays of the tensor glueball candidate $\theta(1640)$ into pseudoscalar mesons.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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