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SOFT PHOTOPRODUCTION IN PROTON-PROTON COLLISIONS AND CHARGED SUM RULES

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1. INTRODUCTION

In this paper the inclusive production of real soft photons in hadron-hadron collisions is considered. In accordance with Low's theorem[1] the first two terms in the expansion of the differential cross section of the soft photon emission in powers of the photon energy k_0 are expressed exactly in terms of the amplitude of the corresponding nonradiative process. Here, we shall consider the first term in this expansion. The inclusive photon spectrum in this approximation has the form: $k_0(d\sigma/dk) = a/k_0^2$. We shall show that in the general case the coefficient a is determined completely by the inclusive single- and two-particle densities of all charged particles created in hadron-hadron collisions. Further on we shall consider in more detail the process

$$p + p \to \gamma + X, \tag{1}$$

where X stands for whatever else is produced, but not observed in pp collisions.

We have performed numerical calculations for the cross section of process (1) at incident proton beam energy 70 GeV and photon scattering angle 8.4°. These values for the incident proton energy and photon scattering angle have been chosen in accordance with the possibilities of the experimental apparatus "Positronium" at IHEP[2].

The leading bremsstrahlung term of the considered process has been discussed earlier in a number of papers[3-7] attempting to understand the copious production of small mass e^+e^- pairs and the abnormally large γ/π^0 ratio, both measured in hadron-hadron collisions at high energies. A complete and detailed exposition of the contemporary state of the experimental situation can be found in review[7] (wherein the list of references of the original papers up to 1978 is given in full) and in refs.[8,9]. However, in papers [3-6] only fixed photon scattering angles ($\theta^* = 90°$ and $\theta^* = 0°$, θ^* - being the photon scattering angle in the c.m.s.) have been investigated. Restrictions on the values of the transverse momentum of the emitting hadrons have been imposed there too.

Below in this paper we suggest a general expression for the cross section of inclusive soft photon emission (in the leading $1/k_0^2$ approximation) for arbitrary scattering angles.

No constraints on the values of the hadron transverse momentum have been laied upon.

In the considered approximation the cross section of soft photoproduction is expressed through integrals over the single- and two-charged densities. These densities should obey certain sum rules following from charge conservation. The corresponding sum rule of the single-particle distributions is well known[10]. In the present paper analogous sum rules of the two-particle distributions have been derived.

Analytical expressions for the inclusive distributions discussed here have been obtained by fit to the experimental data and exist in different kinematical regions[11-13]. However, as we have convinced they do not satisfy the charged sum rules. It has been shown that in calculating the soft bremsstrahlung spectrum of the considered process, one must use only such charged particle distributions which exactly fulfil the charged sum rules. The charged sum rules, which ensure gauge invariance of inclusive soft photoproduction in the considered approximation, impose rigorous constraints on the particle distributions used in the calculations. Any deviations of the particle distributions from them lead to meaningless results.

So, we need to modify the existing parametrizations for the particle distributions in order to ensure fulfilment of the charged sum rules. In the present paper we point out a possibility how to do this making use of the existing parametrizations for the inclusive particle spectra. The distributions thus obtained have been used to carry out model calculations by a computer of the photon bremsstrahlung spectrum of process (1) at proton beam energy 70 GeV.

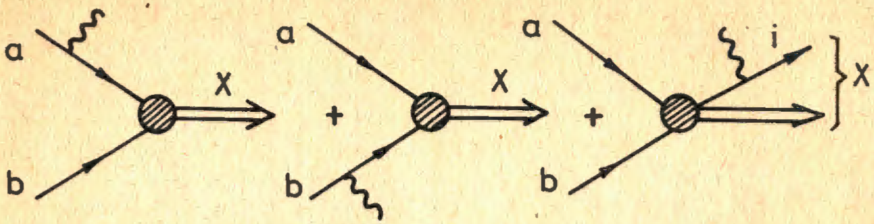
2. CROSS SECTION OF INCLUSIVE SOFT PHOTORPODUCTION IN HADRON-HADRON COLLISIONS

Here, we shall obtain the general expression for the first term in the k_0 expansion of soft bremsstrahlung of the inclusive process:

$$a + b \to \gamma + X, \tag{2}$$

where a and b stand for any hadrons. In the considered approximation only the diagrams drawn in the figure, in which the photon is emitted from all outer charged hadrons, should be taken into account.

Let us consider the exclusive production of N arbitrary hadrons and a photon in pp collision. The matrix element $T(ab \to N\gamma)$ of this process in the discussed here $1/k_0^2$ approximation is expressed entirely in terms of the corresponding



The polar diagrams of process $a + b \rightarrow \gamma + X$.

nonradiative matrix element $T(ab \rightarrow N)$ in the form*:

$$T(ab \rightarrow N\gamma) = \epsilon^{\lambda}_{\mu}\Big(-e_a \frac{p_{a\mu}}{p_a \cdot k} - e_b \frac{p_{b\mu}}{p_b \cdot k} + \sum_{i=1}^{N} e_i \frac{p_{i\mu}}{p_i \cdot k}\Big)\, T(ab \rightarrow N) = \epsilon^{\lambda}_{\mu} \cdot M_{\mu}. \qquad (3)$$

Here p_a, p_b and e_a, e_b are the momenta and charges of the initial particles, p_i and e_i are the momentum and charge of the i-th final hadron, k and ϵ^{λ} are the momentum and polarization of the photon. Evidently, as a consequence of charge conservation, the matrix element (3) satisfies the condition of gauge invariance:

$$k_{\mu} M_{\mu} = 0. \qquad (4)$$

Using eq.(3) for the corresponding inclusive photon cross section of process (2), we obtain the following expression:

$$\begin{aligned} k_0 \frac{d\sigma}{d\vec{k}} = \frac{\alpha}{(2\pi)^2} \Big\{ & e_a^2 \frac{-m_a^2}{(k \cdot p_a)^2} + e_b^2 \frac{-m_b^2}{(k \cdot p_b)^2} + 2 e_a e_b \frac{(p_a \cdot p_b)}{(k \cdot p_a)(k \cdot p_b)} + \\ & + \sum_c \int \frac{d\vec{p}_c}{p_{c0}} \Big[e_c^2 \frac{-m_c^2}{(k \cdot p_c)^2} - 2 \frac{e_c}{(k \cdot p_c)} \Big(e_a \frac{p_a p_c}{k \cdot p_a} + e_b \frac{p_b p_c}{k \cdot p_b} \Big) \Big] \times \rho_c^{(1)}(\vec{p}_c) + \\ & + 2 \sum_{\substack{c,d \\ c \neq d}} \int \frac{d\vec{p}_c}{p_{c0}} \cdot \frac{d\vec{p}_d}{p_{d0}}\, e_c e_d \frac{p_c p_d}{(k \cdot p_c)(k \cdot p_d)}\, \rho^{(2)}_{cd}(\vec{p}_c, \vec{p}_d) + \\ & + \sum_c \int \frac{d\vec{p}_c}{p_{c0}} \frac{d\vec{q}_c}{q_{c0}}\, e_c^2 \frac{p_c q_c}{(k \cdot p_c)(k \cdot q_c)}\, \rho^{(2)}_{cc}(\vec{p}_c, \vec{q}_c) \Big\} \times \sigma_{tot}. \end{aligned} \qquad (5)$$

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*In the leading $1/k_0^2$ approximation considered here the spin of the hadrons can be neglected [14, 15].

Here m_a and m_b are the masses of the initial particles, $\alpha = e^2/4\pi$. The summation in eq. (5) is over all varieties c and d of charged particles created in ab collisions, m_c is the mass of the particle of type c, the integration is over the whole phase space. The densities $\rho_c^{(1)}(\vec{p}_c)$ and $\rho_{cd}^{(2)}(\vec{p}_c, \vec{p}_d)$ in eq.(5), are related to the invariant distribution functions (see the definitions for example ref. [16]) $F_c^{(1)}(\vec{p}_c)$ and $F_{cd}^{(2)}(\vec{p}_c, \vec{p}_d)$ of the single- and two-particle inclusive reactions:

$$a + b \rightarrow c + X, \qquad (6)$$

$$a + b \rightarrow c + d + X, \qquad (7)$$

by

$$\rho_c^{(1)}(\vec{p}_c) = \frac{F_c^{(1)}(\vec{p}_c)}{\sigma_{tot}}, \quad \rho_{cd}^{(2)}(\vec{p}_c, \vec{p}_d) = \frac{F_{cd}^{(2)}(\vec{p}_c, \vec{p}_d)}{\sigma_{tot}},$$

where σ_{tot} is the total cross section of the process $a + b \rightarrow X$.

As is seen from eq. (5), in order to compute the inclusive real photon spectrum in the considered here leading $1/k_0^2$ approximation, a complete display of the dependence on the momenta of single- and two-particle inclusive spectra of all charged particles of the corresponding nonradiative process is needed.

3. THE CHARGED SUM RULES

The single- and two-particle densities $\rho_c^{(1)}(\vec{p})$ and $\rho_{cd}^{(2)}(\vec{p}_c, \vec{p}_d)$ must obey sum rules following from charge conservation. For the considered here process of inclusive emission of soft photons these sum rules should be compulsorily taken into account even if a small deviation from them would mean violation of gauge invariance and would lead to unphysical results.

For the single-particle density distributions the charged sum rule is given by [10]

$$e_a + e_b = \sum_n e_c \vec{n}_c. \qquad (8)$$

Here $\vec{n}_c$

$$\vec{n}_c = \int \rho_c^{(1)}(\vec{p}_c) \frac{d\vec{p}_c}{p_{c0}} \qquad (9)$$

is the average multiplicity of the particle of type c measured in process (6).

In evaluating the cross section of process (1) we shall need also the sum rules based on charge conservation which should be satisfied by the two-particle density distributions. From the charge conservation law we obtain

$$(e_a + e_b) n_d^{(i)} = \sum_c e_c n_c^{(i)} n_d^{(i)}, \qquad (10)$$

where $n_c^{(i)}(n_d^{(i)})$ is the number of particles of type c(d) in the i-th final state. Multiplying eq.(10) by the value $\sigma^{(i)}/\sigma_{tot}$ ($\sigma^{(i)}$ is the cross section of the i-th final state) and summing over i the required sum rules are derived:

$$(e_a + e_b)\overline{n_d} = \sum_c e_c \overline{n_c n_d}. \qquad (11)$$

The two-particle inclusive densities are related to the quantity $\overline{n_c n_d}$ in eq. (11) by the following normalizing condition:

$$\int \frac{d\vec{p}_c}{p_{co}} \cdot \frac{d\vec{p}_d}{p_{do}} \rho_{cd}^{(2)}(\vec{p}_c, \vec{p}_d) = \overline{(n_c n_d - n_c \delta_{cd})}. \qquad (12)$$

In calculating the cross section of process (1) we shall take into account the sum rules (8) and (11).

4. CALCULATION OF THE CROSS SECTION OF PROCESS $p + p \to \gamma + X$

In this section we shall present the results of the calculations by a computer of the cross section of the inclusive photoproduction in proton-proton collisions. Our numerical results are relevant to the experimental apparatus "Positronium" in IHEP[2]. The energy of the incident proton beam was set equal to 70 GeV ($\sqrt{s}$ = 11.92 GeV), the angle of photon emission was set 8.4°.

Since up to now no complete theory of inclusive processes exists, in our calculations for the inclusive distribution functions we have employed model analytical expressions. In the summation over the particle types c and d in eq. (5), we retained only charged pions and protons. The average multiplicities of the remaining charge particles in pp collisions are much less than those of the pions and protons. For example, at $\sqrt{s}$ = 3 GeV the average multiplicities in pp collisions are[17] $\langle n_{\pi^+}\rangle = 3.25$; $\langle n_{\pi^-}\rangle = 2.84$; $\langle n_p\rangle = 1.46$; $\langle n_{k^+}\rangle = 0.29$; $\langle n_{k^-}\rangle = 0.18$; $\langle n_{\bar{p}}\rangle = 0.07$. As our numerical calculations showed the analytic expressions for the inclusive spectra of the charged particles existing in the literature do not satisfy sum rules (8) and (11). As we have pointed out earlier any deviation from the charged sum rules leads to violation of gauge invariance and unphysical results. So, we had to modify the existing inclusive distributions in order to provide an exact fulfilment of relations (8) and (11). This has been achieved by investigating normalization coefficients in the expressions for the inclusive single- and two-particle distributions.

For the single-particle distributions for π^+, π^- and protons we used the following analytical expression proposed on the base of thermodynamical and parton models[11]

$$F_c^{(1)}(\vec{p}_c) = N_c [A_c^{(1)} \exp(-B_c p_\perp) \exp(-D_c y^2) + A_c^{(2)} (1 - \frac{p_\perp}{E_a})^{\ell_c} (p_\perp^2 + M_c^2)^{-n_c}]. \qquad (13)$$

Table

c	$A_c^{(1)}$	$A_c^{(2)}$	B_c	D_c	ℓ_c	M_c	n_c	N_c
π^+	210.0	10.7	7.58	0.20	10.9	1.03	4.0	2.1
π^-	205.0	12.8	7.44	0.21	13.1	1.08	4.0	2.0
p	5.3	16.0	3.80	-0.20	0.	1.20	7.5	5.1

The values of the parameters of the inclusive distributions $F_c^{(1)}(\vec{p}_c)$ in eq.(13).

Here $p_\perp$ is the transverse momentum of the final hadron, y is its rapidity, E_a is the energy of the incoming proton, N_c is the introduced normalizing coefficient, $A_c^{(1)}$, $A_c^{(2)}$, B_c, D_c, ℓ_c, M_c, n_c are parameters, obtained through fit to the data, the subscript c labels the particle type π^+, π^-, p. The numerical values of the parameters $A_c^{(1)}$, $A_c^{(2)}$... (listed in the Table) were taken from reference[11], where inclusive single particle spectra in pp collisions were investigated at $\sqrt{s}$ = 23-63 GeV. Using the scaling hypotheses[18] we extrapolated the values of the parameters of ref.[11] to the considered here energies $\sqrt{s}$ = 11.9 GeV.

We fixed the normalizing coefficients N_c using the following values for the average multiplicities:

$$\langle n_{\pi^+}\rangle = 3.2; \ \langle n_{\pi^-}\rangle = 2.8; \ \langle n_p\rangle = 1.6 \qquad (14)$$

which within the experimental errors coincide with the values, presented in ref.[17] and exactly satisfy the sum rule (8). Using eqs. (9), (13) and (14) we get the values of the coefficients N_c listed in the last column of the Table. Note that the normalizations N_c significantly differ from one*.

For the two-particle densities we have assumed the following expression

$$\rho^{(2)}_{cd}(\vec{p}_c, \vec{p}_d) = N_{cd}\rho^{(1)}_c(\vec{p}_c)\rho^{(1)}_d(\vec{p}_d), \qquad (15)$$

where N_{cd} is the normalization coefficient introduced for the purpose of satisfying the sum rules (11). Usually in computations N_{cd} are set equal to one and thus the two-particle correlations are neglected. For the problem dealt with in the present paper such an approximation is too rough, it violates the sum rules (11). Charged two-particle correlations should be compulsorily taken into account. In order to satisfy eq.(11) the following values for the coefficients N_{cd} have been admitted:

$$N_{cd} = \begin{cases} 1, & c \neq d \\ \dfrac{\overline{n_c(n_c-1)}}{(\bar{n}_c)^2}, & c = d. \end{cases} \qquad (16)$$

Finally, using eqs. (13) and (15) we come to the following soft bremsstrahlung spectrum in inclusive proton-proton collisions at lab. energy of the proton beam 70 GeV and photon scattering angle $\theta = 8.4°$ in $1/k_0^2$ approximation:

$$k_0 \frac{d\sigma}{d\vec{k}}\Big|_{\theta=8.4°, \phi=0°} = \frac{A}{k_0^2}\sigma_{tot}, \qquad (17)$$

where

$$A = \frac{\alpha}{(2\pi)^2} \cdot 10.8. \qquad (18)$$

At the end, we would like to stress that because of the lack of unique parametrization of the single- and two-charged particle densities in the whole phase space, the computation of the coefficient A in eq.(17) is very much model dependent.

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*Such high values of N_c are surely partly due to the fact that the parametrizations (13), which give a satisfactory fit only to data in the central region, have been extrapolated in our calculations to the whole phase space. Such an extrapolation appears especially rough for the proton as a leading particle.

5. CONCLUSIONS

In the present paper the leading term in the expansion in powers of the photon energy k_0 of inclusive production of soft photons in hadron-hadron collisions has been considered. It has been shown that this term is completely determined by the inclusive single- and two-charged particle densities of the corresponding nonradiative process. So, studying the radiative inclusive production of soft photons, one obtains an additional information about the charged-particle distribution functions in hadron-hadron interactions.

Using a computer, the cross section of inclusive soft photoproduction in proton-proton collisions at incident proton energy 70 GeV and laboratory photon scattering angle 8.4° has been evaluated. In the numerical calculations model-dependent expressions for the particle densities modified, however, in order to satisfy the sum rules following from charge conservation have been used. Our calculations showed that any deviations from the charged sum rules lead to unphysical results as soon as gauge invariance is violated. This should be taken into account in calculating the soft bremsstrahlung spectrum, using inclusive single- and two-particle distributions from experiment.

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Биленькая С.И., Христова Е.Х. E2-84-349
Инклюзивное рождение мягких фотонов
в протон-протонных соударениях и зарядовые правила сумм

Рассматривается инклюзивное рождение мягких фотонов в протон-протонных соударениях. В полюсном приближении дифференциальное сечение процесса полностью определяется одночастичными и двухчастичными функциями распределения всех заряженных частиц. Получены основанные на калибровочной инвариантности зарядовые правила сумм для двухчастичных функций распределения. Показано, что сечение тормозного излучения может быть вычислено только с такими функциями распределения, которые строго удовлетворяют этим правилам сумм. Такие функции распределения найдены в работе. С их помощью вычислено дифференциальное сечение инклюзивного рождения мягких фотонов при энергии налетающих протонов 70 ГэВ и угле вылета γ-кванта $8{,}4^{\circ}$. Показано, что при анализе мягкой части спектра фотонов в протон-протонных соударениях при высоких энергиях необходимо учитывать вклад тормозного излучения.

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Bilenkaya S.I., Christova E.Ch. E2-84-349
Soft Photoproduction in Proton-Proton Collisions
and Charged Sum Rules

Inclusive production of real soft photons in high-energy proton-proton collisions is considered. The cross section for the process in the leading bremsstrahlung approximation is completely determined by single and two-particle density distributions of all charged particles created in pp collisions. Charged sum rules of the two-particle densities are obtained. It is shown that the photon bremsstrahlung spectrum may be evaluated only by means of such particle densities which exactly fulfil the charged sum rules. Charged particle densities obeying these requirements have been found. These densities have been used to calculate the real photon bremsstrahlung spectrum at incident proton beam energy 70 GeV and photon scattering angle 8.4°. Numerical results show that in analysing the low energy photon spectrum in high energy proton-proton collisions it is necessary to take into account the contribution of the bremsstrahlung emission from charged particles.

The investigation has been performed at the Laboratory of Theoretical Physics and the Laboratory of Nuclear Problems, JINR.

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