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ON THE PROBLEM OF QUARK DEGREES OF FREEDOM IN NUCLEI

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Recent investigations have led to the conclusion that the consideration of nuclei as systems of quasi-independent nonrelativistic nucleons is incomplete and they require a relativistic description of the nucleon motion and taking into account quark degrees of freedom. These are first of all the prediction [1] and discovery [2] of the cumulative production of particles in hadron-nucleus and nucleus-nucleus collisions and change of an exponential fall-off at small momentum transfers of formfactors of light nuclei to a power-law fall-off at large momentum transfers [3] according to the quark counting rules [4].

Significant differences in the behaviour of quark structure function of nuclei of a different atomic number and a nontrivial A-dependence of cross sections of cumulative processes point to a different manifestation of quark degrees of freedom in these nuclei [5]. The same is confirmed by the results of recent deep-inelastic lepton nucleus experiments [6,7] (the so-called EMC-effect). Different models are suggested for the explanation of this phenomenon (see, e.g., ref. [8]). It seems that the above regularities are of the same nature [9] and are determined by the possibility of formation of multiquark configurations (multiquark bags) in nuclei. In the present paper we analyse the EMC-effect and show that the effect can be explained by taking into account the scattering on colourless multiquark configurations which are contained in the medium of spectator nucleons, over the relativistic motion [10] of which the corresponding average is taken.

Consider deep inelastic scattering of charged leptons on a nucleus A. We shall assume that in the nucleus, together with nucleons (three-quark bags), there are formed with definite probabilities, the configurations with six, nine, etc. quarks (see in this connection [11,12]), and leptons interact with the nucleus by the exchange of virtual photons with quarks from these bags. Then the nucleus structure function can be represented by the sum:

$$F_2(x, Q^2) = \sum_{K=1}^{A} N(A, K)\, F_2^K(x, Q^2)\,, \qquad (1)$$

where F_2^K is the structure function of the nucleus A which contains a 3K-quark bag and (A−K) nucleons. The coefficients N(A, K) of these structure functions have the meaning of the effective number of 3K-quark bags in the nucleus A and obey the normalization condition:

$$\sum_{K=1}^{A} K N(A,K) = A. \qquad (2)$$

We use the parametrization of N(A, K) in the form of the Bernoulli distribution:

$$N(A,K) = \frac{A!}{K!(A-K)!} P(A)^{K-1} [1 - P(A)]^{A-K} . \qquad (3)$$

For the parameter P(A) determining the probability of a three-quark nucleon to get into a 3K-quark bag we consider two possibilities [13]:

1. P(A) is determined by the ratio of the bag and nucleus volume:

$$P(A) = \frac{V_K}{V_A} = \frac{r_K^3}{R_A^3} = \frac{r_K^3}{(R_0 A^{1/3})^3} \sim A^{-1} . \qquad (4)$$

Taking the bag radius r_K close to the nucleon radius $r_K \approx 0.8$ fm and $R_0 = 1.4$ fm [14] we obtain $P(A) = 1.1867\, A^{-1}$.

2. P(A) is determined by the ratio of the bag and nucleus cross section

$$P(A) \sim \frac{r_K^2}{R_A^2} \sim A^{-2/3} . \qquad (5)$$

The coefficient of proportionality in formula (5) for this case has been obtained in ref.[15] by fitting the data on production of π-mesons with large transverse momenta in proton-nucleus scattering, and we shall also use this parametrization $P(A) = 0.085\, A^{-0.67}$.

In both the cases N(A,K) are fast decreasing functions of K and the main contribution to the structure function is given by first few terms of sum (1).

We proceed now to the calculation of structure functions F_2^K. As is well-known, the structure function F_2 appears in the decomposition of the deep-inelastic tensor $W_{\mu\nu}$ into the gauge-invariant structures. The tensor $W_{\mu\nu}$ itself is proportional to the imaginary part of the forward virtual Compton scattering amplitude: $T_{\mu\nu}(P_A, q) = i \int d^4z\, e^{iqz} \langle P_A | T(J_\mu(z) J_\nu(0)) | P_A \rangle$ that can be expressed through the two-photon vertex function and relativistic wave functions of composite systems [16]:

$$T_{\mu\nu}(P_A, q) = i \int \prod_{i=K+1}^{A} d^3\underline{p}_i\, d^3\underline{P}_K\, \delta^{(3)}(\underline{P}_A - \underline{P}_K - \sum_{i=K+1}^{A} \underline{p}_i) \times$$

$$\times \int \prod_{i=K+1}^{A} d^3\underline{q}_i\, d^3\underline{Q}_K\, \delta^{(3)}(\underline{P}_A - \underline{Q}_K - \sum_{i=K+1}^{N} \underline{q}_i) \times$$

$$\times \bar{\Psi}_{P_A}(\underline{P}_K, [\underline{p}_i]) \Gamma_{\mu\nu}(\underline{P}_K, [\underline{p}_i]; q; [\underline{q}_i], \underline{Q}_K) \Psi_{P_A}(\underline{Q}_K, [\underline{q}_i]) . \qquad (6)$$

Here $\Gamma_{\mu\nu}$ is the two-photon vertex function, Ψ_{P_A} is the relativistic wave function of the composite system consisting of a 3K-quark bag and (A - K) nucleons. In (6) the following notation is introduced $\underline{p} = (p_+, \vec{p}_\perp)$; $p_+ = p_0 + p_3$; $d^3\underline{p} = dp_+ d\vec{p}_\perp$; $\delta^{(3)}(\underline{p}) = \delta(p_+)\, \delta^{(2)}(\vec{p}_\perp)$. Integration $\int d^3\underline{p}_i$ is to be understood in the following sense:

$$\int d^3\underline{p}_i = \int_0^{P_{A,+}} dp_{i,+} \int d\vec{p}_{i,\perp}$$

$P_{A,\mu}$ ($\mu = 0,1,2,3$) is the 4-momentum of nucleus A, $P_{K,\mu}$ and $Q_{K,\mu}$ are the 4 momenta of the 3K-quark bag, $p_{i,\mu}$, $q_{i,\mu}$ are the 4-momenta of nucleons. The square brackets in the arguments of Ψ_{P_A} and $\Gamma_{\mu\nu}$ denote the sets of corresponding variables: $[\underline{p}_i] = \underline{p}_{K+1}, \underline{p}_{K+2}, \dots, \underline{p}_A$.

Now we represent the relativistic wave function $\Psi_{P_A}(\underline{P}_K, [\underline{p}_i])$ in the form:

$$\Psi_{P_A}(\underline{P}_K, [\underline{p}_i]) = \Psi_{A-K}([\underline{p}_i]) \int \prod_{j=1}^{3K} d^3\underline{k}_j\, \delta^{(3)}(\underline{P}_K - \sum_{j=1}^{3K} \underline{k}_j) \Psi_{3K}([\underline{k}_j]), \qquad (7)$$

where $\underline{k}_j = (k_{j,+}, \vec{k}_{j,\perp})$ are components of the quark momentum in the 3K-quark bag.

Calculating the function $\Gamma_{\mu\nu}$ in the lowest order in electromagnetic interaction, we get the following expression for the structure function F_2^K:

$$F_2^K(x, Q^2) = \frac{\pi}{(4\pi)^{A+2K}} \frac{M_A Q^2 (Q^2/\xi_A^2 - M_A^2)}{\xi_A^2 (Q^2/\xi_A^2 + M_A^2)^2} \times$$

$$\times \int_0^1 \prod_{i=K+1}^{A} \frac{dx_i}{x_i} Z_K dZ_K\, \delta(1 - Z_K - \sum_{i=K+1}^{A} x_i) \times$$

$$\times \int \prod_{i=K+1}^{A} d\vec{p}_{i,\perp} d\vec{P}_K\, \delta^{(2)}(\vec{P}_{K,\perp} + \sum_{i=K+1}^{A} \vec{p}_{i,\perp}) |\Phi_{A-K}([x_i, \vec{p}_{i,\perp}])|^2 \times \qquad (8)$$

$$\times \sum_{\ell=0}^{3K} e_\ell^2 \int_0^1 \prod_{j=1}^{3K} \frac{dz_j}{z_j} \delta(1 - \sum_{j=1}^{3K} z_j) \times$$

$$\times \int \prod_{j=1}^{3K} dk_{j,\perp}\, \delta^{(2)}(\vec{P}_{K,\perp} - \sum_{j=1}^{3K} \vec{k}_{j,\perp}) |\Phi_{3K}([z_j, \vec{k}_{j,\perp}])|^2 \times$$

$$\times [Q^2 - 2m_\ell^2 + \frac{6Q^2}{(\xi_A/Z_K)^2} z_\ell (z_\ell - \xi_A/Z_K)]\, \delta[\vec{k}_{\ell,\perp}^2 + m_\ell^2 - \frac{Q^2 z_\ell (z_\ell - \xi_A/Z_K)}{(\xi_A/Z_K)^2}] .$$

Here $q^2 = -Q^2$ is the 4-momentum transfer squared. The variable ξ_A is defined as

$$\xi_A = \frac{2x_A}{1 + (1 + 4M_A^2 x_A^2/Q^2)^{1/2}} = z_\ell Z_K = \frac{k_{\ell,+}}{P_{A,+}},$$

where $x_A = Q^2/2M_A\nu$, $\nu = (P_A q)/M_A$, M_A is the nucleus mass. The variable x_A varies in the interval $0 < x_A < 1$ and is related to the Bjorken variable $x = Q^2/2M\nu$ by $x_A = \frac{M}{M_A}x$ (M is the nucleon mass). It is evident that $0 < x < \frac{M_A}{M} \approx A$ and

$$\xi_A = \frac{M}{M_A}\xi \approx \frac{\xi}{A}, \quad \xi = \frac{2x}{1 + (1 + 4M^2x^2/Q^2)^{1/2}}.$$

In (8) e_ℓ and m_ℓ are the electric charge and mass of an ℓ -th quark, respectively.

The wave functions $\Phi_{A-K}([x_i, \vec{p}_{i,\perp}])$ and $\Phi_{3K}([z_j, \vec{k}_{j,\perp}])$ are related to the functions $\Psi_{A-K}([\underline{p}_i])$ and $\Psi_{3K}([\underline{k}_j])$ by the following formulas:

$$\Phi_{A-K}([x_i, \vec{p}_{i,\perp}]) = (P_{A,+})^{A-K}\left(\prod_{i=K+1}^{A} x_i\right)\Psi_{A-K}([\underline{p}_i]), \qquad (9a)$$

$$\Phi_{3K}([z_j, \vec{k}_{j,\perp}]) = (P_{K,+})^{3K-1}\left(\prod_{j=1}^{3K} z_j\right)\Psi_{3K}([\underline{k}_j]). \qquad (9b)$$

The variables x_i and z_j are defined as:

$$x_i = p_{i,+}/P_{A,+}, \qquad 0 < x_i < 1, \quad \sum_{i=K+1}^{A} x_i = 1 - Z_K,$$

$$z_j = k_{j,+}/P_{K,+}, \qquad 0 < z_j < 1, \quad \sum_{j=1}^{3K} z_j = 1.$$

The variable Z_K is the ratio of "+"-components of the 4-momenta of the $3K$-quark bag and nucleus A, $\vec{p}_{i,\perp}$ and $\vec{k}_{j,\perp}$ are transverse momenta of the nucleons in nuclei and of a quarks in the bag, respectively.

Let us now choose the wave functions $\Phi_{A-K}([x_i, \vec{p}_{i,\perp}])$ and $\Phi_{3K}([z_j, \vec{k}_{j,\perp}])$ in the following form:

$$\Phi_{A-K}([x_i, \vec{p}_{i,\perp}]) \sim \exp\left[-a_A \sum_{i=K+1}^{A} \frac{\vec{p}_{i,\perp}^{\,2} + M_i^2}{x_i}\right], \qquad (10a)$$

$$\Phi_{3K}([z_j, \vec{k}_{j,\perp}]) \sim \exp\left[-\beta_K \sum_{j=1}^{3K} \frac{\vec{k}_{j,\perp}^{\,2} + m_j^2}{z_j}\right], \qquad (10b)$$

M_i are the nucleon masses, m_j are the quark masses.

In the deep inelastic limit ($Q^2 \gg M_i^2, m_j^2$; $\xi_A \to x_A$; $\xi \to x$) the Q^2-dependence in the structure functions F_2^K disappears, and after the corresponding calculations from formula (8) we obtain:

$$F_2^K = \frac{3K}{A}\,\frac{x_A I_K(x_A)}{\int_0^1 dx_A\, x_A I_K(x_A)}; \quad x_A = \frac{x}{A}, \qquad (11)$$

where

$$I_K(x_A) = \int_{x_A}^{1} dZ_K \frac{(1 - Z_K)^{A-K-1}}{1 + \frac{\beta_K}{a_A}(1 - Z_K)}\left(1 - \frac{x_A}{Z_K}\right)^{3K-2} = \qquad (12)$$

$$= \frac{x_A(1 - x_A)^{(A-K)+(3K-2)}}{1 + \frac{\beta_K}{a_A}(1 - x_A)} B(A-K, 3K-1) \times$$

$$\times\ {}_2F_1\left(3K-1, A-K, 1, A+2K-1; 1-x_A; \frac{(1+\beta_K/a_A)(1-x_A)}{1+(\beta_K/a_A)(1-x_A)}\right). \qquad (13)$$

Here B is the Euler beta-function, ${}_2F_1$ is the hypergeometric function of two variables (the Appel function).

The structure function F_2^A corresponding to the last term on the sum (1), i.e., to the $3A$ -quark bag is of the form

$$F_2^A(x) = 9A(3A-1)\,x_A(1-x_A)^{3A-2}. \qquad (14)$$

A characteristic feature of structure functions F_2^K is a possible existence in a nucleus of a superfast quark which in extreme situation takes all the momentum of nucleus. The structure functions (11) and (14) are normalized to the number of quarks in the corresponding bag

$$\int_0^A dx\, F_2^K(x) = 3K. \qquad (15)$$

This automatically leads (taking into account (2)) to the normalization of the total structure function

$$\int_0^A dx\, F_2(x) = 3A. \qquad (16)$$

One can relate the parameters α_A and β_K of the relativistic wave functions $\Phi_{A-K}(\{x_i, \vec{p}_{i,\perp}\})$ and $\Phi_{3K}(\{z_j, \vec{k}_{j,\perp}\})$ to the radii of the nucleus A and 3K-quark bag, respectively

$$R_A^2 \sim 8A\alpha_A , \tag{17a}$$

$$r_K^2 \sim 24K\beta_K . \tag{17b}$$

As is seen from formulas (11)-(13), the structure functions F_2^K depend on the ratio

$$\frac{\beta_K}{\alpha_A} = \frac{A}{3K}\frac{r_K^2}{R_A^2} = \frac{r_K^2}{3R_0^2}\frac{A^{1/3}}{K} . \tag{18}$$

Choosing $r_K = 0.8$ fm, $R_0 = 1.4$ fm we get $\beta_K/\alpha_A = 1.109A^{1/3}/K$.

In the experiments [6,7] the ratio of structure functions $F_2(Fe)/F_2(D)$ is measured in the region $x<1$ that corresponds to the region of small $x_A < 1/A$. It is evident from formulas (12), (13) and it can be checked by straight-forward numerical calculations that the structure functions weakly depend on the parameter β_K/α_A in this region. That is why in the numerical calculations we have supposed that the ratio β_K/α_A is the same for all bags. For the iron nucleus we have taken $(\beta_K/\alpha_A)_{Fe} = 0.1$. But for the deuteron (considering that the average distance between nucleons in the deuteron is somewhat larger than in other nuclei) we have chosen the value $(\beta_K/\alpha_A)_D = 0.05$.

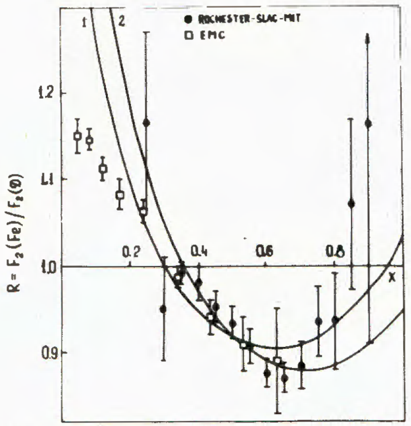

Fig.1. The ratio of the structure functions of iron and deuteron. 1 and 2 - calculations with parametrizations $P(A) \sim A^{-1}$ and $P(A) \sim A^{-2/3}$, respectively.

Experimental data on the ratio $F_2(Fe)/F_2(D)$ and the curves calculated by the formulas (1), (3), (11)-(14) with two parametrizations for P(A) are shown in Fig.1. The curves reproduce rather well the experimental data in the region $x > 0.2$ and somewhat differ from the data in the region $x < 0.2$. This difference is probably due to the fact that we restrict our consideration by the valence quarks and do not take into account contributions of the sea quarks and gluons.

The analysis performed shows that deviation of the ratio $F_2(Fe)/F_2(D)$ from unity is due to a larger contribution of the multiquark configurations in

the iron nucleus as compared with the deuteron. Note that according to (3) the probabilities of formation of six,- nine-, etc., quark configurations in heavy nuclei are larger than in light nuclei. In the deuteron the contribution of a six-quark state is only a small admixture to the contribution of a two-nucleon state. The deviation from unity of the ratio of the sum of two-nucleon and six-quark contributions to the contribution of a pure two-nucleon state does not exceed 5% for the deuteron in the shole region $0 < x < 2$ (in this connection see also [17]).

As has already been mentioned, the main contribution to the structure function $F_2(x)$ is given by several first terms of sum (1). Fig.2 shows how the ratio $F_2(Fe)/F_2(D)$ changes by a subsequent addition to the structure function of six-, nine-, etc., quark-bag contributions. (The curves correspond to the parametrization $P(A) \sim A^{-1}$. The similar picture holds for the second parametrization). It is seen that the "saturation" of the curve occurs upon adding the 9-quark-bag contribution, and the addition of other terms of sum (1) practically does not change the curve.

In refs. [5,18] the possibility was established for extracting information on the quark parton functions of nuclei from the data on cumulative pion production, and in ref. [9] similarity in the x-behaviour of the ratio of pion production cross sections on different nuclei and the ratio of the deep inelastic structure functions of the same nuclei was pointed out. It should be noticed that the data on cumulative production allow us to investigate the region $x > 1$ (which is not yet reached in the experiments analysed). We have calculated the ratios $F_2(Fe)/F_2(D)$, $F_2(Fe)/F_2(He)$, $F_2(Fe)/F_2(Al)$ in the whole region of x ($0 < x < A$). The curves of these calculations in the parametrization $P(A) \sim A^{-1}$ in the double logarithmic presentation are given in Fig.3. A similarity in the behaviour of these curves and the ratios of cross sections of cumulative pion production [9] is observed.

The analysis performed shows that the explanation of the observed deviation from unity of the ratio $F_2(Fe)/F_2(D)$ requires the consideration of multiquark configurations in nuclei. It seems very interesting to analysis the A-dependence of the ratio $F_2(A)/F_2(D)$ in formulas (1), (3), (11)-(13) in view of the new data on this dependence [19] and experimental investigations of deep inelastic lepton-nucleus scattering in the region $x > 1$ (it seems that until now in the region $1 < x < 1.4$ only the structure function of ^{12}C nucleus has been measured [20]).

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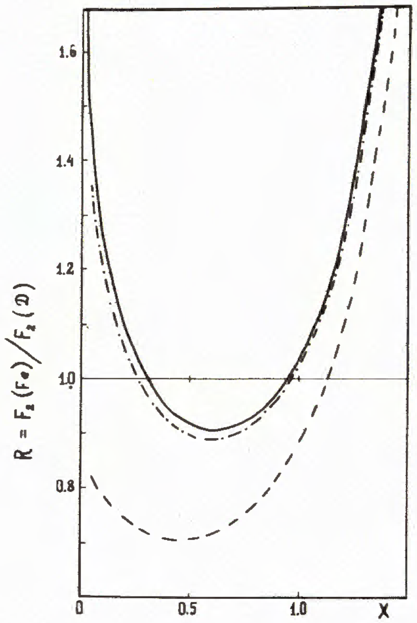
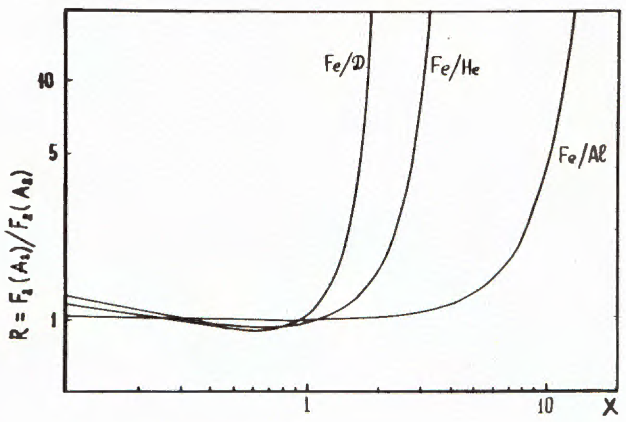


Fig.2. The ratio of structure functions of iron and deuteron. - - - - - three quark bags only; — · — · - three and six quark bags; ——— - three, six and nine quarks bags.

Fig.3. The ratio of structure functions of different nuclei in the whole region of x (0 < x < A).



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К вопросу о кварковых степенях свободы в ядрах

Проведен анализ ЕМС-эффекта и показано, что эффект может быть объяснен учетом рассеяния на бесцветных многокварковых конфигурациях в ядрах, находящихся в среде непровзаимодействовавших нуклонов ядра, по релятивистскому движению которых проводится соответствующее усреднение. Характерной особенностью подхода является возможность существования в ядре сверхбыстрого кварка, несущего в экстремальном случае весь импульс ядра. Прослежены общие черты процессов глубоконеупругого рассеяния лептонов и кумулятивного рождения частиц на ядрах.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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On the Problem of Quark Degrees of Freedom in Nuclei

Analysis of the EMC-effect is performed and it is shown that the effect can be explained by taking into account scattering on the colourless multiquark configurations in nuclei, which are contained in the medium of spectator nucleons of the nucleus and the corresponding average over the relativistic motion of these nucleons is taken. A characteristic feature of the approach is a possible existence in a nucleus of a superfast quark which in extreme situation takes all the momentum of nucleus. Similar features of lepton deep-inelastic scattering and cumulative particle production on nuclei are traced.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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