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B, H, H′, Q_1, AND Q_2 MESON DECAYS IN THE SUPERCONDUCTOR QUARK MODEL

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Recently[1,2], the model based on the four-quark interactions has been developed to describe the low-energy properties of mesons. Strong and electromagnetic interactions of four nonets $J^{PC} = 0^{++}$, 0^{-+}, 1^{--}, and 1^{++} have been studied using a Lagrangian with four different types of quark couplings (scalar, pseudoscalar, vector and axial-vector). In this paper we shall be concerned with properties of the fifth 1^{+-} meson nonet. For this purpose we introduce a new type of the four-quark interaction in the original Lagrangian of the model

$$\Delta \mathcal{L} = -\frac{G_3}{4}(\bar{q}\gamma_5 \lambda_a \overset{\leftrightarrow}{\partial}_\mu q)^2 ,$$

where $\bar{q} = (\bar{u}, \bar{d}, \bar{s})$ are coloured quark fields (summation over colour indices is assumed), λ_a are the Gell-Mann matrices $(0 \le a \le 8)$, G_3 is the coupling constant.

Decays of axial-vector mesons have been examined in the literature*. It is known[4,5] that the experimental data are well fitted by use of three usual-quark-model parameters. The first two parameters - g_A (f-coupling) and g_B (d-coupling) - are the S-wave coupling constants of the 1^{++} (A_1) and 1^{+-} (B) octets with vector (V) and pseudoscalar (P) octets. The third one - ϕ is the $Q_A - Q_B$ mixing angle. In Ref.[5] the following values are obtained for them: $g_A = 0.82 \pm 0.07$ GeV, $g_B = 0.59 \pm 0.07$ GeV, $\phi = 34° \pm 3°$.

In the superconductor quark model $g_A = \frac{g_\rho^2}{4} F_\pi = 0.88$ GeV, where g_ρ is the $\rho \to \pi\pi$ decay constant $(g_\rho^2/4\pi = a_\rho \approx 3)$, and F_π is the pion-weak-decay constant ($F_\pi = 93.3$ MeV). In this way there are only two (g_B and ϕ) unknown parameters. The d-coupling constant g_B can be determined from the $B \to \omega\pi$ decay. The mixing angle ϕ is estimated from the Q_1 and Q_2 meson decays.

Strong decays of axial-vector mesons are described using quark triangle diagrams. We need their contact terms only. When we concern ourselves with radiative decays of axial-vector mesons, we use these triangle diagrams as blocks. Dropping here terms higher than those of the second order in external particle momenta (the contact terms cancel one another) we obtain gauge-invariant amplitudes for these processes. The coup-

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*Particularly, the nonlocal quark model[3] yielded results which differ much from ours.

ling constants at the amplitudes are given by the model. In this paper we calculate radiative decays of the B(1235), H(1190), Q_1(1280), and Q_2(1400) mesons. Radiative decays of the A_1, D and E axial-vector mesons have been studied in Ref./6/.

Transition from the effective four-quark interaction to the phenomenological meson Lagrangian was thoroughly discussed in Refs./1/. Meson fields as composite two-quark states arise at the first stage of the transition in the form of collective variables in the generating functional. The 1^{+-} nonet coupling with the quarks is given by a Lagrangian

$$\Delta\mathcal{L}' = \frac{f_B}{4} B^\mu_a (\bar{q}\gamma_5 \lambda_a \overleftrightarrow{\partial}_\mu q), \qquad (1)$$

where $B^\mu_a(x)$ are the fields of the 1^{+-} meson nonet. The coupling constant f_B is determined by the B^μ_a field renormalization at the second stage of the transition, upon integration in the generating functional over the quark fields. The obtained phenomenological meson vertices are produced by quark loops.

The diagram in Fig.1 leads to the following kinetic term in the Lagrangian of the free B^μ_a field

$$\mathcal{L}_{kin} = -\frac{I_1}{4} \mathrm{Tr}(\bar{B}^2_{\mu\nu}), \qquad (2)$$

where $\bar{B}_{\mu\nu} = (\partial_\mu B^a_\nu - \partial_\nu B^a_\mu)\lambda_a$. On renormalizing the field $B = (f_B/2) B^R$ (in what follows the index R will be omitted), we find that $f_B = \sqrt{2/I_1}$. In this case the kinetic term (2) takes a standard form $\mathcal{L}_{kin} = -\frac{1}{8}\mathrm{Tr}(\bar{B}^2_{\mu\nu})$. Due to a derivative in Lagrangian (1) the coupling constant f_B is expressed by means of the quadratically divergent integral* $I_1 = -\frac{3i}{(4\pi)^2}\int_0^\infty \frac{dx}{x^2} R(x)$, where $R(x) = \int d\mu^2 \rho(\mu^2) e^{-i\mu^2 x}$. Let us remind that the renormalization constants of other meson nonets depend on the logarithmically divergent integral only** $I_2 = \frac{3}{(4\pi)^2}\int_0^\infty \frac{dx}{x} R(x)$.

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*Unlike the authors of Refs./1,2/ we use the gauge-invariant Pauli-Villars regularization /7/ to calculate the quark-loop diagrams.

** If we take the regularizator $\rho(\mu^2)$ in the form of a simple cut-off $\rho(\mu^2) = \delta(\mu^2 - m^2) - \delta(\mu^2 - \Lambda^2) - (\Lambda^2 - m^2)\frac{d}{d\mu^2}\delta(\mu^2 - \Lambda^2)$, then the integrals are equal to

$$I_1 = \frac{3m^2}{16\pi^2}\left(\frac{\Lambda^2}{m^2} - \ln\frac{\Lambda^2}{m^2} - 1\right), \quad I_2 = \frac{3}{16\pi^2}\left(\ln\frac{\Lambda^2}{m^2} - 1 + \frac{m^2}{\Lambda^2}\right), \qquad (3)$$

where m is the constituent quark mass.

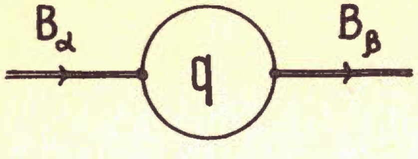

Fig.1

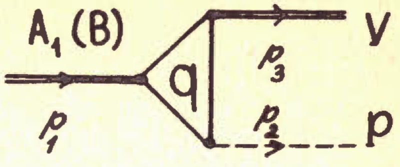

Fig.2

Now we proceed to the decays of the axial-vector mesons $A^a_{1\mu} \to V^\beta_\nu P^\gamma$ and $B^a_\mu \to V^\beta_\nu P^\gamma$. The corresponding quark triangle diagrams are presented in Fig.2. The amplitudes are given by

$$T_{A_1 \to VP} = -i g_A \mathrm{Tr}\{\lambda_a [\lambda_\beta, \lambda_\gamma]_-\} g_{\mu\nu}\epsilon^\mu(p_1)\epsilon^\nu(p_3),$$

$$T_{B \to VP} = g_B \mathrm{Tr}\{\lambda_a [\lambda_\beta, \lambda_\gamma]_+\} g_{\mu\nu}\epsilon^\mu(p_1)\epsilon^\nu(p_3).$$

We consider here the contact terms only. Diagram with A_1 meson is logarithmically divergent. The contact term of diagram with B meson is quadratically divergent. For the coupling constant g_B we obtain

$$g_B = \frac{g_\rho}{2}\sqrt{\frac{I_1}{2I_2}} \equiv \frac{g_\rho}{2}\kappa F, \qquad (4)$$

where κ is an arbitrary parameter.

The decay widths can be written as

$$\Gamma_{B \to VP} = \frac{\alpha_\rho F^2}{m_B} a^2_{B \to VP}\left[2 + \left(\frac{m_B^2 + m_V^2 - m_P^2}{2 m_B m_V}\right)^2\right]\sqrt{\left[1 - \left(\frac{m_V + m_P}{m_B}\right)^2\right]\left[1 - \left(\frac{m_V - m_P}{m_B}\right)^2\right]}. \qquad (5)$$

Ours estimations and the coefficients $a_{B \to VP}$ are given in Table 1. The parameter κ can be determined from the $B \to \omega\pi$ decay. It is known that the D/S wave ratio in this process equals ≈0.29/8/ (i.e., the S-wave partial width for $B \to \omega\pi$ is $\Gamma^S_{B\to\omega\pi}$ = 126 MeV). Using (5) we obtain κ = 3.14. Now from (3) and (4) we can find the cut-off parameter $\Lambda_1 \approx$ 740 MeV in the integral I_1 (m = 234 MeV). This means that the range of the effective quark-antiquark interaction in the case of coupling with derivative is longer than that in the case of couplings without it ($\Lambda_2 \approx$ 1100 MeV)/1/.

Table 1

The decay widths for the B, H, H′, Q_1, and Q_2 axial-vector mesons obtained from formulae (5) and (6) ($F_K = 1.2\ F_\pi$)

Mode	$a_{B\to VP(SP)}$	$\Gamma^{\text{theor.}}$ (MeV)	$\Gamma^{\text{exp.}}$ (MeV)/8/
$B \to \omega\pi$	$\kappa/\sqrt{3}$	126	137 ± 10
$H \to \rho\pi$	$\kappa \cos\chi$	360	320 ± 50
$H' \to \bar{K}^*K + K^*\bar{K}$	$\sqrt{\frac{2}{3}}\kappa \cos\chi$	165	~ 165*
$Q_1 \to \rho K$	$\frac{1}{2}(\frac{g_\rho}{2}\cos\phi + \kappa\sin\phi)$	36	38 ± 6
$\omega\bar{K}$	$\frac{1}{2\sqrt{3}}(\frac{g_\rho}{2}\cos\phi + \kappa\sin\phi)$	7.4	9.9 ± 1.8
$K^*\pi$	$\frac{1}{2}(-\frac{g_\rho}{2}\cos\phi + \kappa\sin\phi)$	4.7	14.4 ± 4.5
$K\epsilon$	$(-\cos\phi + \frac{g_\rho}{6\kappa}\sin\phi)$	1.3	2.7 ± 1.8
$Q_2 \to \rho K$	$\frac{1}{2}(-\frac{g_\rho}{2}\sin\phi + \kappa\cos\phi)$	7.2	5.4 ± 5.4
ωK	$\frac{1}{2\sqrt{3}}(-\frac{g_\rho}{2}\sin\phi + \kappa\cos\phi)$	2.2	1.8 ± 1.8
$K^*\pi$	$\frac{1}{2}(\frac{g_\rho}{2}\sin\phi + \kappa\cos\phi)$	160	169 ± 11
$K\epsilon$	$(\sin\phi + \frac{g_\rho}{6\kappa}\cos\phi)$	8.7	3.6 ± 3.6

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*The estimation is taken from/5/.

To describe decays of the $Q_1(1280)$ and $Q_2(1400)$ strange axial-vector mesons it is necessary to take into account mixing of pure Q_A (1^{++}) and Q_B (1^{+-}) states $Q_1 = Q_A\cos\phi + Q_B\sin\phi$, $Q_2 = -Q_A\sin\phi + Q_B\cos\phi$.

If the two Q mesons did not mix, then both Q′s would have a reduced branching ratio $K^*\pi/\rho K$ equal to one after removing phase-space effects. However, the heavier $Q_2(1400)$ decays overwhelmingly into $K^*\pi$, and the lighter $Q_1(1280)$ into ρK. From our calculations we have $\Gamma_{Q_1\to K^*\pi} = 0$ at the mixing angle $\phi = 44°$. We obtain good experimental fit for $\phi = 34°$.

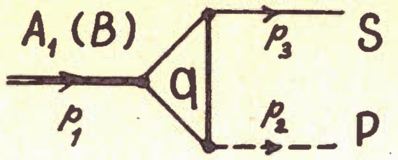
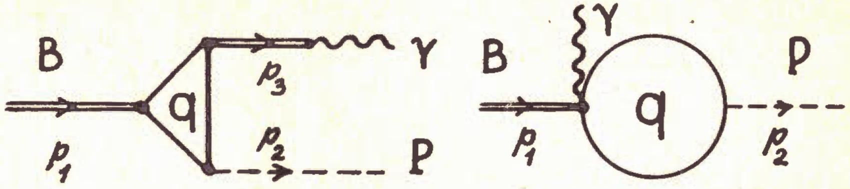


Fig.3

Fig.4.



It should be point out that the mixing angle θ of the $H_0 - H_8$ components in the H and H′ mesons differs from the ideal value θ_{id}. This deviation is characterized by the angle $\chi = \theta_{id} - \theta = 12°$.

The axial-vector meson decays $A^\alpha_{1\mu} \to S^\beta P^\gamma$ and $B^\alpha_\mu \to S^\beta P^\gamma$ are described by diagrams in Fig.3. The amplitudes for these process have the form

$$T_{A\to SP} = -i\frac{g_\rho}{4}\mathrm{Tr}\{\lambda_\alpha[\lambda_\beta,\lambda_\gamma]_+\}(p_3 - p_2)^\mu \epsilon_\mu(p_1),$$

$$T_{B\to SP} = \frac{g_\rho^2}{12\kappa}\mathrm{Tr}\{\lambda_\alpha[\lambda_\beta,\lambda_\gamma]_-\}p_2^\mu \epsilon_\mu(p_1).$$

The decay widths are given by

$$\Gamma_{B\to SP} = \frac{\alpha_\rho}{48} m_B a^2_{B\to SP}\left[\left(1+\frac{m_P^2 - m_S^2}{m_B^2}\right)^2 - \frac{4m_P^2}{m_B^2}\right]\sqrt{\left[1-\left(\frac{m_P+m_S}{m_B}\right)^2\right]\left[1-\left(\frac{m_S-m_P}{m_B}\right)^2\right]} \qquad (6)$$

(see also Table 1).

Let us discuss radiative decays of the B(1235), H(1190), $Q_1(1280)$, and $Q_2(1400)$ mesons. The $B\to\pi\gamma$ and $H\to\pi\gamma$ decay diagrams are presented in Fig.4. Calculating quark-loop integrals, we shall keep not only the contact terms but also the terms quadratic in momenta of particles. Quadratically divergent terms of these diagrams are cancelled. In front of the logarithmically divergent integral I_2 we have the gauge-invariant combination of momenta. The corresponding amplitude has the form

$$T_{B\to P\gamma} = \frac{e}{4\kappa F}\mathrm{Tr}\{Q[\lambda_\alpha,\lambda_\beta]_+\}(p_1^\nu p_3^\mu - g^{\mu\nu}p_1 p_3)\epsilon_\mu(p_1)\epsilon_\nu(p_3),$$

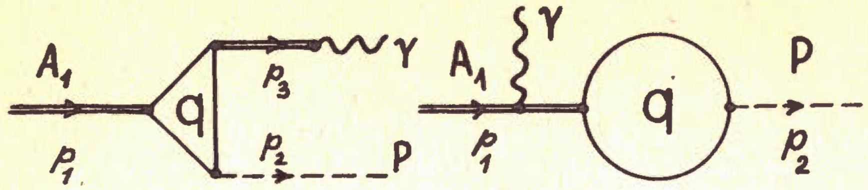


Fig.5

where $Q = \frac{1}{2}(\lambda_3 + \frac{\lambda_8}{\sqrt{3}})$ is the quark charge matrix. To obtain the $Q_1 \to K\gamma$ and $Q_2 \to K\gamma$ decay amplitudes one needs to take into account the diagrams of Fig.5 leading to f-type transitions, besides the diagrams already considered here. The total contribution of these diagrams to the amplitude is finite and satisfies the gauge-invariance requirement

$$T_{A\to P\gamma} = -\frac{ieg_\rho}{16\pi^2 F} \operatorname{Tr}\{Q[\lambda_\alpha , \lambda_\beta]_-\}(p_1^\nu p_3^\mu - g^{\mu\nu} p_1 p_3)\epsilon_\mu(p_1)\epsilon_\nu(p_3).$$

The widths are given by

$$\Gamma_{B\to P\gamma} = \frac{\alpha}{24F^2} a^2_{B\to P\gamma}\left(\frac{m_B^2 - m_P^2}{m_B}\right)^3 . \qquad (7)$$

Table 2 presents values of the widths and expressions for the constants $a_{B\to P\gamma}$; $\alpha = e^2/4\pi = 1/137$.

Table 2

The radiative decay widths of the B, H, Q_1, and Q_2 mesons obtained from formula (7) ($F_K = 1.2\ F_\pi$).

Mode	$a_{B\to P\gamma}$	$\Gamma^{\text{theor.}}$ (keV)
$B \to \pi\gamma$	$1/6\kappa$	177
$H \to \pi\gamma$	$1/2\kappa$	$1.43\cdot10^3$
$Q_1 \to K^+\gamma$	$-\frac{g_\rho}{8\pi^2}\cos\phi + \frac{1}{6\kappa}\sin\phi$	37
$K^\circ\gamma$	$-\frac{1}{3\kappa}\sin\phi$	107
$Q_2 \to K^+\gamma$	$\frac{g_\rho}{8\pi^2}\sin\phi + \frac{1}{6\kappa}\cos\phi$	351
$K^\circ\gamma$	$-\frac{1}{3\kappa}\cos\phi$	355

To sum up, we may conclude that the quark model of the superconductor type can be expanded to include the 1^{+-} axial-vector meson nonet by adding to the original Lagrangian /2/ four-quark terms with derivatives. The new constant f_B depends on the quadratically divergent integral I_1 and has no unambiguous correspondence to old constants g (for scalar mesons) and $g_\rho = \sqrt{6}g$ (for vector mesons). To determine this constant we can use the experimental width of the $B \to \omega\pi$ decay. The fundamental parameters of the expanded model are the constituent quark masses and cut-off parameters Λ_i which characterize the bounds of quark-antiquark interactions in the case with couplings without derivatives ($\Lambda_2 \approx 1100$ MeV) and with them ($\Lambda_1 \approx 740$ MeV).

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Волков М.К., Осипов А.А. E2-84-298

Распады B, H, H′, Q_1 и Q_2 мезонов в кварковой модели сверхпроводящего типа

Феноменологический мезонный лагранжиан, полученный на основе эффективного четырехкваркового взаимодействия, применяется для описания двухчастичных распадов B /1235/, H /1190/, H′ /1480/, Q_1 /1280/ и Q_2 /1400/ аксиально-векторных мезонов. Даны оценки ширин всех основных мод распадов этих мезонов. Для угла смешивания $Q_A - Q_B$ состояний получено значение $\phi = 34^\circ$, а отклонение угла смешивания синглет-октетных компонент H /1190/ и H′ /1480/ мезонов от идеального составляет 12°. Предсказаны вероятности радиационных распадов B/1235/, H /1190/, Q_1 /1280/ и Q_2 /1400/ мезонов.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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B, H, H′, Q_1, and Q_2 Meson Decays in the Superconductor Quark Model

A phenomenological meson Lagrangian, based on the effective four-quark interaction, is used to describe two-particle decays of the B/1235/, H/1190/, H′ /1480/, Q_1 /1280/, and Q_2 /1400/ axial-vector mesons. The widths of all basic decay modes of these mesons are estimated. The value of the $Q_A - Q_B$ mixing angle $\phi = 34^\circ$ is obtained. The deviation of the singlet-octet states mixing angle for H /1190/ and H′ /1480/ mesons from its ideal value equals 12°. The radiative decay widths of the B /1235/, H/1190/, Q_1 /1280/, and Q_2 /1400/ mesons are obtained.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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