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# N=4 SUPEREXTENSION OF THE LIOUVILLE EQUATION WITH QUATERNIONIC STRUCTURE

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1. Introduction. High N superextensions of the Liouville equation are interesting to study for several reasons. First, based on analogy with the ordinary (N=0) Liouville equation and its N=1 supersymmetric counterpart /1/ which describe the bosonic and simple fermionic strings in the Polyakov approach /2/, one may conjecture that the equations with N $>$ 1 correspond to more refined versions of the superstring models having additional internal degrees of freedom. Second, these equations would describe certain classes of solutions to four-dimensional supergravity and super-Yang-Mills theories which property generalize the cylindrically-symmetric solutions of the self--duality equations of usual Yang-Mills theory /3/. In this connection, recall that the N=2 and N=4 supersymmetries (SUSY) in $\mathcal{D}$ =2 are related to N=1 and N=2 SUSY's in $\mathcal{D}$ =4 via dimensional reduction. Superextensions of the Liouville equation are interesting also on their own range as nontrivial examples of supersymmetric integrable systems.

In the present paper we construct N=4 supersymmetric extension of the Liouville equation. The case N=4 in two dimension, in contrast to N=2, admits non-abelian automorphism groups of the fermion charges. We consider the minimal situation with the automorphism group $SU(2)$ and show that the basic object of the theory is a quaternionic superfield (SF) subjected to the irreducibility constraints of the hypermultiplet type /4/. It contains 4+4 independent components on--shell and 8+8 - off-shell. The irreducibility conditions together with the dynamical equations follow from the zero curvature representation on superalgebra (SA) $su(1,1|2)$. In the bosonic sector of the model, along with dilaton $u(x)$, three fields $\varphi^i(x)$ parametrizing the symmetric space SU(2)xSU(2)/SU(2) are also present. In the limit of vanishing fermion fields, the system reduces to the N =0 Liouville equation and a certain modification of the equations of motion of the SU(2)xSU(2)/SU(2) nonlinear $\sigma$-model. As in deriving the extension with N=2 SUSY /5/, we make use of general group-theoretic approach dealing with the nonlinear realizations of infinite parameter symmetries. This allows us to show that the invariance group of the system we have constructed is the superextension of two-dimensional conformal group associated with the SU(2) superstring SA of Ademollo et al. /6/ (at the classical level). We find an explicit realization of the corresponding transformations on the quaternionic SF and the N=4 superspace coordinates. Our notation and terminology are basically the same as in the previous papers /5,7/.

2. <u>The Cartan forms and the covariant reduction</u>. In two dimensions one may consider two versions of rigid N=4 SUSY with nonabelian automorphism groups. In one version, the spinor charges are assigned to two real vectors of SO(4) while in the other they form two complex SU(2) doublets*). An elementary analysis of the relevant massless representations tells us that the minimal supermultiplets in which one may place the dilaton under the natural assumption that it is a singlet of the automorphism group, contain, respectively, 8+8 and 4+4 physical components on-shell. Thus, the second possibility is more economic and we shall restrict ourselves to its study.

In our scheme the N=0, N=1 and N=2 Liouville equations result in a uniform way from nonlinear realizations of two-dimensional conformal group and its N=1 and N=2 superextensions /5,7/. A generalization to the case N=4 implies a passage to a wider supergroup encompassing both the conformal group and supergroup of rigid N=4 SUSY. The minimal enlargement of this kind is associated with the SA $\mathcal{G}^A = \mathbb{K}^A_+(1|2) \oplus \mathbb{K}^A_-(1|2)$, where we have introduced the notation $\mathbb{K}^A(1|2)$ for the SU(2) superextension of the contact algebra $\mathbb{K}(1)$. This SA enters as in essential building block into the SU(2) superstring SA /6/ and is defined by the following structure relations **):

$$
\begin{aligned}
& i[L^n_\pm, L^m_\pm] = (n-m) L^{n+m}_\pm , \\
& \{G^r_{\alpha\pm}, \bar G^{s\beta}_\pm\} = -2\delta^\beta_\alpha L^{r+s}_\pm + 2(r-s)(\sigma^k)_\alpha{}^\beta T^{r+s}_{k\pm} , \\
& i[L^n_\pm, T^p_{k\pm}] = -p T^{p+n}_{k\pm} , \\
& i[L^n_\pm, G^r_{\alpha\pm}] = \left(\tfrac{n}{2} - r\right) G^{r+n}_{\alpha\pm} , \\
& i[T^p_{k\pm}, G^r_{\alpha\pm}] = -\tfrac{1}{2}(\sigma_k)_\alpha{}^\beta G^{r+p}_{\beta\pm} , \\
& [T^p_{k\pm}, T^l_{i\pm}] = \varepsilon_{kij} T^{p+l}_{j\pm} , \\
& \{G^r_{\alpha\pm}, G^s_{\beta\pm}\} = \{\bar G^{s\beta}_\pm, \bar G^{r\alpha}_\pm\} = 0 .
\end{aligned} \tag{1}
$$

*) In fact, these automorphism groups can be extended to chiral ones $SU_+(2)\times SU_-(2)$ or $SO_+(4)\times SO_-(4)$, but in our approach just their diagonal subgroups are realized linearly and homogeneously.

**) We consider the classical theory and therefore disregard essentially quantum $c$-number central charges which are present in the full SU(2) superstring SA /6/.



Here $\sigma^k$ are the Pauli matrices and different indices run over the ranges: $n, m = -1, 0, 1, \dots;\ r, s = -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \dots;\ p, l = 0, 1, 2, \dots;\ i, j, k = 1, 2, 3;\ \alpha, \beta = 1, 2$. The SA $\mathbb{K}^A(1|2)$ takes a particular place among the contact superalgebras: it admits no further extension by spinor generators (with the automorphism groups SU($n$), $n>2$) and drops out of the Kac classification /8/.

Let us discuss in short the structure of SA(2). The generators $L^n_\pm$ form two commuting contact algebras $\mathbb{K}_\pm(1)$, whose sum is isomorphic to conformal algebra in two dimensions: $L^{-1}_\pm$ represent the light-cone coordinate translations, $U = L^0_+ - L^0_-$ is the SO(1,1)-pseudo-rotation generator and $D = L^0_+ + L^0_-$ is the dilatation generator, with respect to which the conformal weights of all other generators are specified. The generators $T^p_{i\pm}$ form two Kac-Moody algebras with the local part $su_+(2) \oplus su_-(2) = \{T^0_{i+}, T^0_{i-}\}$. The maximal finite dimensional subalgebra of each branch of $\mathcal{G}^A$ is SA $su(1,1|2)$:

$$
su_\pm(1,1|2) = \{G^{-\frac{1}{2}}_{\alpha\pm}, \bar G^{-\frac{1}{2}\beta}_\pm, L^{-1}_\pm, L^0_\pm, T^0_{i\pm}, L^1_\pm, G^{\frac{1}{2}}_{\alpha\pm}, \bar G^{\frac{1}{2}\beta}_\pm\} . \tag{2}
$$

Rigid N=4 SUSY is generated by

$$
\{G^{-\frac{1}{2}}_{\alpha\pm}, \bar G^{-\frac{1}{2}\beta}_\pm, L^{-1}_\pm, U, T^0_i = T^0_{i+} + T^0_{i-}\} . \tag{3}
$$

SA $\mathcal{G}^A$ contains also the infinite dimensional graded subalgebras of the N=1 and N=2-super-Liouville equations $\mathbb{K}_\pm(1|1)$ and $\mathbb{K}_\pm(1|2)$ /5/. There exists the chain of embeddings: $\mathbb{K}^A(1|2) \supset \mathbb{K}(1|2) \supset \mathbb{K}(1|1) \supset \mathbb{K}(1)$. One of possible reduction to $\mathbb{K}_\pm(1|2)$ is achieved by restricting the values of certain indices in (1) as follows $\alpha, \beta = 1;\ i, j, k = 3$.

Let us turn to constructing a nonlinear realization of supergroup $G^A$ associated (formally) with $\mathcal{G}^A$. As the stability subgroup we take $H = SO(1,1) \times SU(2)$ with the generators $U$ and $T^0_i$ (the relevance of this choice will be justified later). An element of the coset space $G^A/H$, where $G^A$ acts as left shifts, is represented by:

$$
g \equiv G^A/H = e^{ix^\pm L^{-1}_\pm} e^{\theta^\pm G^{-\frac{1}{2}}_\pm + \bar\theta^\pm \bar G^{-\frac{1}{2}}_\pm} e^{i z^\pm_1 L^1_\pm} e^{\xi^\pm G^{\frac{1}{2}}_\pm + \bar\xi^\pm \bar G^{\frac{1}{2}}_\pm} e^{v^{\pm i}_1 T^1_{i\pm}} \dots e^{iuD} e^{\varphi^k (T^0_{k+} - T^0_{k-})} \tag{4}
$$

Each $g$ is parametrized both by the coordinates $\{x^\pm, \theta^\alpha_\pm, \bar\theta_{\pm\beta}\}$, constituting the basis of N=4 superspace $\mathbb{R}^{2|8}$ and by an infinite set of the SF parameters $\{u, \varphi^i, z^\pm_n, \xi^\pm_r, v^{\pm i}_p\}$ $(n = 1, 2, \dots;\ r = \frac{1}{2}, \frac{3}{2}, \dots;\ p = 1, 2, \dots)$, defined on this superspace. Fortunately, according to the general theorem of ref. /9/, one may radically reduce the number of independent SF's by imposing covariant constraints on the Cartan forms. An analysis of the commutation relations (1) shows that the number of essential unremovable SF parameters is minimal just with our choice of the stability subgroup. Such are the dilaton SF $u(x, \theta, \bar\theta)$ and the SF's

$\varphi^i(x,\theta,\bar\theta)$, parametrizing the coset $SU_+(2)\times SU_-(2)/SU(2)$.

The Cartan forms on the infinite dimensional coset $G^A/H$ are introduced in a standard fashion[10].

$$\Omega = g^{-1}dg = \omega^{\pm} L_{\pm} + \rho^{\pm} T_{\pm} + \mu^{\pm} G_{\pm} + \bar\mu^{\pm} \bar G_{\pm} \tag{5}$$

(over $\pm$ the sum is taken, the rest of indices for simplicity are suppressed). The general 1 form $\Omega$ takes values in $\mathcal{G}^A$.

Our goal is to get the dynamical equations for $u$ and $\varphi^i$ with the zero curvature representation on some finite dimensional SA $\mathcal{G}_0^A \subset \mathcal{G}^A$, which properly extends the algebra $sl(2,R)$ of an analogous representation of the ordinary Liouville equation. Following the analogy with the case N=2[5] and keeping in mind that $\mathcal{G}_0^A$ should include the stability group algebra, the single choice is $\mathcal{G}_0^A = su(1,1|2)$, generated by the diagonal combinations of generators of $su_{\pm}(1,1|2)$ (2):

$$\left.\begin{cases} R_{\pm} = L^{-1}_{\pm} + m^2 L^{1}_{\mp}, & Q_{\alpha\pm} = G^{-\frac12}_{\alpha\pm} \pm m\, G^{\frac12}_{\alpha\mp}, \\ U,\ T^0_i, & \bar Q^{\alpha}_{\pm} = \bar G^{-\frac12\alpha}_{\pm} \pm m\, \bar G^{\frac12\alpha}_{\mp} \end{cases}\right\} = \mathcal{G}_0^A \tag{6}$$

$m$ being an arbitrary parameter of mass dimension. The generators $R_{\pm}, U$ form an $sl(2,R)$ subalgebra in $\mathcal{G}_0^A$.

Let us single out in (5) the components before the generators (6) and equate to zero the rest, that is the operation covariant with respect to the whole $G^A$. Then we automatically arrive at the zero curvature condition for the surviving 1-form $\Omega_0^{Red} \in \mathcal{G}_0^A$:

$$d^{ext}\,\Omega_0^{Red} = \Omega_0^{Red} \wedge \Omega_0^{Red}. \tag{7}$$

This follows from the original Maurer-Cartan equation for $\Omega$ and the dynamical constraints imposed by us on $\Omega$:

$$\Omega = \Omega_0^{Red}. \tag{8}$$

An implementation of such covariant constraints has been called by us in[5,7] the covariant reduction. In the present case there occurs the covariant reduction of the coset space $G^A/SO(1,1)\times SU(2)$ to its connected finite dimensional subspace SU(1,1|2)/SO(1,1)xSU(2), which can be interpreted as a N=4-superextension of pseudosphere $SL(2,R)/SO(1,1)$.

3. <u>The N=4 super-Liouville equation</u>. The constraints (8) contain an infinite set of the Pfaff's equations on the SF parameters of $G^A/H$. The most (infinite) part of these equations serves merely for expressing higher parameters and the 1-form $\Omega_0^{Red}$ in terms of the essential SF's $u, \varphi^i$ and their derivatives (the inverse Higgs phenomenon[9]). The remaining constraints have the dynamical character: they yield the differential restrictions on $u$ and $\varphi^i$:

$$\mathcal{D}_-^{(\alpha} q_{\gamma}{}^{\beta)} = 0, \quad \bar{\mathcal{D}}_{+(\alpha}\, q_{\gamma)}{}^{\beta} = 0 \tag{9}$$



$$\begin{cases} \mathcal{D}_-^{\alpha}(q\,\mathcal{D}_+^{\gamma} q^{-1})_{\gamma}{}^{\beta} = 0 \\ \mathcal{D}_+^{\alpha}(q^{-1}\bar{\mathcal{D}}_{-\gamma} q)_{\beta}{}^{\gamma} + 4im\,\bar q_{\beta}{}^{\alpha} = 0. \end{cases} \tag{10}$$

Here, $u$ and $\varphi^i$ were combined into a single quaternionic SF

$$q_{\alpha}{}^{\beta} \equiv (e^{-u-i\varphi\cdot\sigma})_{\alpha}{}^{\beta}, \tag{11}$$

satisfying the reality condition

$$\bar q_{\alpha}{}^{\beta} \equiv (e^{-u+i\varphi\cdot\sigma})_{\alpha}{}^{\beta} = -\varepsilon_{\alpha\gamma}\,\varepsilon^{\beta\delta}\, q_{\delta}{}^{\gamma}, \tag{12}$$

and N=4 covariant spinor derivatives were introduced:

$$\mathcal{D}^{\alpha}_{\pm} = i\theta^{\alpha\pm}\frac{\partial}{\partial x^{\pm}} + \frac{\partial}{\partial\bar\theta^{\pm}_{\alpha}}, \quad \bar{\mathcal{D}}_{\pm\alpha} = i\bar\theta^{\pm}_{\alpha}\frac{\partial}{\partial x^{\pm}} + \frac{\partial}{\partial\theta^{\alpha\pm}}, \quad \{\mathcal{D}^{\alpha}_{\pm}, \bar{\mathcal{D}}_{\pm\beta}\} = 2i\,\delta^{\alpha}_{\beta}\frac{\partial}{\partial x^{\pm}}. \tag{13}$$

The system (9), (10) is just the desirable N=4 extension of the Liouville equation written in an explicitly covariant SF representation. This system, by construction, is equivalent to the zero curvature condition for the 1-form $\Omega_0^{Red}$ and hence can be looked upon as the integrability condition for some linear problem (sect. 5). The relations (9) are the irreducibility conditions for $q_{\alpha}{}^{\beta}$ and directly generalize the Grassmann analyticity constraints of the case N=2[5]. One may easily check their compatibility with the dynamical equations (10). Taking into account the reality of $q_{\alpha}{}^{\beta}$, it follows also from (9): *)

$$\bar{\mathcal{D}}_-^{(\alpha} q_{\gamma}{}^{\beta)} = 0, \quad \mathcal{D}_{+(\alpha}\, q_{\gamma)}{}^{\beta} = 0. \tag{9'}$$

The constraints of the type (9), (9') are well known in $\mathcal{D}=4$; they single out there a simplest N=2 representation, the hypermultiplet[4].

To expose the irreducible component field content of $q_{\alpha}{}^{\beta}$, we apply the standard method of defining the components via a successive action on $q_{\alpha}{}^{\beta}$ by covariant spinor derivatives[11]. The irreducible basis set of SF's is as follows

$$q_{\alpha}{}^{\beta},\ \mathcal{D}_+^{\alpha}(q)_{\alpha}{}^{\beta},\ \bar{\mathcal{D}}_{-\alpha}(q)_{\beta}{}^{\alpha},\ \mathcal{D}_{\alpha-}\mathcal{D}_+^{\beta}(q)_{\beta}{}^{\alpha},\ \bar{\mathcal{D}}_{\alpha-}\mathcal{D}_+^{\beta}(q)_{\beta}{}^{\alpha}. \tag{14}$$

All other SF's of this type reduce upon using the constraints (9), (9') to $x^{\pm}$-derivatives of the basis set (14). So, we may define the irreducible components as $\theta,\bar\theta$ -independent parts of SF's (14). More convenient is the slightly different definition:

$$(q_0)_{\alpha}{}^{\beta} = q_{\alpha}{}^{\beta}\big|_{\theta=\bar\theta=0}, \quad C_1 = \mathcal{D}_{-\alpha}\mathcal{D}_+^{\beta} q_{\beta}{}^{\alpha}\big|_{\theta=\bar\theta=0}, \quad C_2 = \bar{\mathcal{D}}_{-\alpha}\mathcal{D}_+^{\beta} q_{\beta}{}^{\alpha}\big|_{\theta=\bar\theta=0},$$
$$\psi_-^{\alpha} = -(\bar q\,\mathcal{D}_-^{\beta}\bar q^{-1})_{\beta}{}^{\alpha}\big|_{\theta=\bar\theta=0}, \quad \chi_+^{\alpha} = (\mathcal{D}_+^{\beta} q\, q^{-1})_{\beta}{}^{\alpha}\big|_{\theta=\bar\theta=0}. \tag{15}$$

*) As usual, SU(2)-indices are raised and lowered with the help of tensors $\varepsilon^{\alpha\beta}$ and $\varepsilon_{\alpha\beta}$.

Thus $q_\alpha^{\ \beta}$ contains off-shell 8+8 independent components. The bosonic subset consists of physical real SU(2)-singlet and triplet $(q_0)_\alpha^{\ \beta}$ and two auxiliary complex SU(2)-singlets $C_1$ , $C_2$ . The fermions comprise two SU(2)-doublets. On-shell this set reduces to 4+4, that coincides with the on-shell content of hypermultiplet /4/ and agrees with the analysis in the beginning of Section 2.

The component equations can be derived most easily with using the property that the l.h. sides of the system (10) satisfy the same irreducibility conditions as $q_\alpha^{\ \beta}$ itself. Then the full set of essential equations is obtained by extracting $\theta$ , $\bar\theta$ -independent terms in the corresponding irreducible SF's of the type (14), which are equal to zero in virtue of (10). For auxiliary fields one gets in this way the expressions

$$\begin{aligned} C_1 &= -(\chi_+ q_0 \psi_-) \\ C_2 &= (\chi_+ q_0 \bar\psi_-) + 4im\, \mathrm{Sp}(\bar q_0 q_0), \end{aligned} \tag{16}$$

whereas the physical components obey the system:

$$\begin{cases} \partial_+ (q_0^{-1}\partial_- q_0)_\alpha^{\ \beta} = -m^2 (\bar q_0 q_0)_\alpha^{\ \beta} - \frac{im}{8}\big[\delta_\alpha^\beta\, \mathrm{Sp}(\psi_- \bar q_0 \bar\chi_+) - (\bar q_0 \chi_+)_\alpha \bar\psi_-^\beta - \\ \qquad\qquad - \psi_{-\alpha}(\bar\chi_+ q_0)^\beta\big], \\ \partial_+ \psi_-^\alpha = -m(\chi_+ q_0)^\alpha, \\ \partial_- \chi_+^\alpha = m(\psi_- q_0)^\alpha. \end{cases} \tag{17}$$

The bosonic sector of eqs. (17), with the fermionic fields omitted, is described by the equation

$$\partial_+ (q_0^{-1}\partial_- q_0)_\alpha^{\ \beta} = -m^2 (\bar q_0 q_0)_\alpha^{\ \beta} \tag{18}$$

After separating the ordinary Liouville equation for the dilaton $u(x) = u(x,\theta,\bar\theta)\big|_{\theta=\bar\theta=0} = -\frac{1}{4}\mathrm{Sp}\ln(\bar q_0 q_0)$ (it corresponds to the trace part of (18)), one leaves with three equations on the fields $\varphi^i(x)$:

$$\frac{\partial}{\partial x^+}\mathrm{Sp}\Big(q_0 \sigma^i \frac{\partial}{\partial x^-} q_0^{-1}\Big) = 0 . \tag{19}$$

These provide a certain modification of the standard equations of motion of the nonlinear $\sigma$-model on the homogeneous space $SU_+(2)\times SU_-(2)/SU(2)$. A detailed analysis of eq. (19) is performed in Appendix.

The equation (19) and, respectively, the system (17) as a whole belong to the non-Lagrangian type. Nevertheless, as is shown in Appendix, eq. (19) together with the Maurer-Cartan equations for the coset $SU_+(2)\times SU_-(2)/SU(2)$ determine a Lagrangian system after passing to some new variables.*) It is plausible, that an analogous redefinition exists for the full system (17).

***) L.D.Faddeev paid our attention to the fact that eqs.of the type (19) have specific (singular) Lagrangians even in the original variables /21/ .**



Since the N=4, N=2 and N=1 super-Liouville equations have purely group theoretic origin, it is natural to study the interplay between them at the level of their SA's $\mathbb{K}^A_\pm(1|2)$, $\mathbb{K}_\pm(1|2)$ and $\mathbb{K}_\pm(1|1)$. As has been mentioned above, $\mathbb{K}_\pm(1|2)$ follow from $\mathbb{K}^A_\pm(1|2)$ at $\alpha,\beta=1;\ i,j,k=3$ in (1). To this restriction there corresponds the following reduction of parameters of the coset elements (4):

$$\theta^{\pm 2} = \bar\theta^\pm_2 = 0,\ \xi^{\pm 2}_s = \bar\xi^\pm_{2s} = v^{\pm\rho 1} = v^{\pm\rho 2} = \varphi^1 = \varphi^2 = 0 . \tag{20}$$

The number of essential SF's decreases to two $u(x,\theta^1,\bar\theta_1)$ and $\varphi^3(x,\theta^1,\bar\theta_1)$, depending on a lesser number of Grassmann variables. Respectively, the constraints on the Cartan forms are also relaxed. As a result, the equation (9), (10) reduces to the system

$$\begin{cases} \mathcal{D}_-^1 q_1^{\ 1} = 0 \qquad , \quad \bar{\mathcal{D}}_{+1} q_1^{\ 1} = 0 \\ \mathcal{D}_+^1 \bar{\mathcal{D}}_{-1}(\ln q_1^{\ 1}) + 4im\, \bar q_1^{\ 1} = 0 \quad , \quad q_1^{\ 1} = e^{-(u+i\varphi^3)}, \end{cases} \tag{21}$$

which is easily recognized as the N=2 super-Liouville equation /5/. To go over to the case N=2 in the component equations (16), (17), one must put there $\psi_-^\alpha = \delta^{\alpha 1}\psi_-^1$, $\chi_+^\alpha = \delta^{\alpha 1}\chi_+^1$, $(q_0)_\alpha^{\ \beta} = \delta_{\alpha 1}\delta^{\beta 1}(q_0)_1^{\ 1}$ . It immediately follows from this consideration, that the solutions to the N=2 Liouville equation form a subset of those to the N=4 equation.

4. <u>Transformation laws</u>. The equations (9), (10) are definitely invariant with respect to $G^A$ transformations, since they have been obtained by imposing $G^A$ covariant constraints on parameters of the coset $G^A/H$. An explicit realization of $G^A$ on $x, \theta, \bar\theta$ and $u(x,\theta,\bar\theta)$, $\varphi^i(x,\theta,\bar\theta)$ can be easily found by using the fact, that these quantities parametrize the coset elements (4), on which $G^A$ acts via left multiplications:

$$g^A \cdot g = g' \cdot h' . \tag{22}$$

Making use of the structure relations (1) leads to

$$\delta x^\pm = e^{i(\theta^\pm\bar\theta^\pm)\partial_\pm}\big[\tfrac{1}{2}f^\pm + i\theta^\pm\bar\zeta^\pm\big] + e^{-i(\theta^\pm\bar\theta^\pm)\partial_\pm}\big[\tfrac{1}{2}f^\pm - i\zeta^\pm\bar\theta^\pm\big], \tag{23}$$

$$\delta\theta^{\alpha\pm} = e^{i(\theta^\pm\bar\theta^\pm)\partial_\pm}\big[\zeta^{\alpha\pm} + \tfrac{1}{2}f^{\pm\prime}\theta^{\alpha\pm} + 2i(\theta^\pm\bar\zeta^{\pm\prime})\theta^{\alpha\pm} + \tfrac{i}{2}a^{k\pm}(\theta^\pm\sigma^k)^\alpha\big], \tag{24}$$

$$\begin{cases} \delta q_\alpha^{\ \beta} = W_\alpha^{+\rho} q_\rho^{\ \beta} + q_\alpha^{\ \gamma} W_\gamma^{-\beta}, \\ W_\alpha^{+\rho} = \frac{1}{2}\big(\mathcal{D}^{+\rho}\delta\bar\theta^+_\alpha - \bar{\mathcal{D}}^+_\alpha \delta\theta^{+\rho} - \delta_\alpha^\rho \mathcal{D}^{+\lambda}\delta\bar\theta^+_\lambda\big), \\ W_\gamma^{-\beta} = \frac{1}{2}\big(\bar{\mathcal{D}}^-_\gamma \delta\theta^{-\beta} - \mathcal{D}^{-\beta}\delta\bar\theta^-_\gamma - \delta_\gamma^\beta \bar{\mathcal{D}}^-_\rho \delta\theta^{-\rho}\big), \end{cases} \tag{25}$$

where $f^\pm(x^\pm)$, $\zeta^{\alpha\pm}(x^\pm)$, $a^{k\pm}(x^\pm)$ are infinitesimal parameters-functions of conformal, local supersymmetric and local $SU_\pm(2)$ transformations. To get a particular realization of the $\mathcal{G}^A$ generators, one should single out in these functions different monomials in

$x^+$, $x^-$ *). The coordinate transformation laws (23), (24), when written through complex variables $\xi^\pm = x^\pm + i\theta^\pm\bar\theta^\pm$ or $(\xi^\pm)^+$, coincide with those obtained in /12/. It is a simple task to check, that under these transformations, the supersymmetric covariant differentials

$$\Delta x^\pm = dx^\pm + i(\theta^\pm d\bar\theta^\pm - d\theta^\pm\bar\theta^\pm)$$

are simply multiplied by some superfunctions, in accordance with the definition of contact SA's /8/.

The transformation laws of the components of $q_\alpha^\beta$ are derived straightforwardly. They have a rather complicated form, therefore we shall give them only for physical components $(q_o)_\alpha^\beta$, $\psi^{-\beta}$, $\chi^{+\beta}$ on-shell (with $C_1$, and $C_2$ eliminated) and with the restriction to the plus branch of $\mathcal{G}^A$.

Conformal group:

$$\begin{cases} \delta(q_o)_\alpha^{\ \beta} = -\frac{1}{2} f^{+\prime}(q_o)_\alpha^{\ \beta} - f^+\partial_+(q_o)_\alpha^{\ \beta}, \\ \delta\chi^{+\beta} = -\frac{1}{2} f^{+\prime}\chi^{+\beta} - f^+\partial_+\chi^{+\beta}, \\ \delta\psi^{-\beta} = -f^+\partial_+\psi^{-\beta}. \end{cases} \qquad (26)$$

Local SUSY:

$$\begin{cases} \delta(q_o)_\alpha^{\ \beta} = \frac{1}{2}(q_o)_\alpha^{\ \beta}(\zeta^+\bar\chi^+) + \frac{1}{2}\bar\chi_\alpha^+(\zeta^+q_o)^\beta - \frac{1}{2}\bar\zeta_\alpha^+(\chi^+q_o)^\beta, \\ \delta\chi^{+\beta} = -4i\partial_+\zeta^{+\beta} - 4i(\zeta^+\partial_+q_o q_o^{-1})^\beta - \frac{1}{2}\chi^{+\beta}(\zeta^+\bar\chi^+) - \frac{1}{2}\zeta^{+\beta}(\chi^+\bar\chi^+) + \\ \qquad + \frac{1}{2}(\chi^+\bar\zeta^+)\chi^{+\beta}, \\ \delta\psi^{-\alpha} = 4im(\zeta^+q_o)^\alpha. \end{cases} \qquad (27)$$

Local $SU_+(2)$:

$$\begin{cases} \delta(q_o)_\alpha^{\ \beta} = \frac{i}{2}a^{+k}(\sigma^k)_\alpha^{\ \rho}(q_o)_\rho^{\ \beta}, \\ \delta\chi_+^{\ \beta} = -\frac{i}{2}a^{+k}(\chi_+\sigma^k)^\beta, \quad \delta\psi^{-\alpha} = 0. \end{cases} \qquad (28)$$

Note that the supertransformation of $\psi^{-\alpha}$ begins with a pure shift by $\zeta^{+\alpha}$ (because $(q_o)_\alpha^\beta$ begins with $\delta_\alpha^\beta$). Since, the $x^+$-independent piece of $\zeta^{+\alpha}$ is just the parameter of rigid N=4 SUSY, the above property means the spontaneous breaking of N=4 SUSY in the present case, with $\psi^{-\alpha}$ playing the role of the relevant Goldstino ( $\chi^{+\beta}$ is the Goldstino, with respect to the transformations with $\zeta^{-\beta}$). The strength of this breaking is measured by the scale parameter $m$. Let us adhere to an analogy with the ordinary Liouville equation, where the Poincare translations are also broken, with the same breaking parameter $m$, because of lack of the translation-invariant classical solutions /13/. The

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*) In fact, $\mathcal{G}^A$ may now be enlarged to include the generators with an arbitrary negative conformal weights, by allowing the infinitesimal parameters to be decomposable into the Laurent type series (this SA exactly coincides with the SU(2) superstring SA without $c$ -number terms /6/).



true invariance group of classical solutions is the diagonal $SL(2,R)$, which thus serves as the vacuum stability group. Likely, in the N=4 case unbroken symmetries are associated with the diagonal supergroup SU(1,1|2), generated by generators (6).

5. The linear problem. We demonstrate here, that the system (9), (10) has an interpretation as the consistency condition of certain linear set. Generally speaking, this follows already from the zero curvature representation (7) for the 1-form $\Omega_o^{Red}$. However, just as in the case N=2 /5/, the minimal linear problem can be constructed merely with the help of spinor components of $\Omega_o^{Red}$. Let us define the "lengthened" spinor derivatives

$$\nabla_\pm^\alpha = \mathcal{D}_\pm^\alpha + \Omega_\pm^\alpha, \qquad \bar\nabla_{\pm\alpha} = \bar{\mathcal{D}}_{\pm\alpha} + \bar\Omega_{\pm\alpha}, \qquad (29)$$

where $\Omega_\pm^\alpha$, $\bar\Omega_{\pm\alpha}$ are the coefficients of $d\bar\theta_\alpha^\pm$ and $d\theta^{\pm\alpha}$ in $\Omega_o^{Red}$:

$$\Omega_o^{Red} = d\theta^{\pm\alpha}\bar\Omega_{\pm\alpha} + d\bar\theta_\alpha^\pm\Omega_\pm^\alpha + \Delta x^\pm\Omega_\pm^x, \qquad (30)$$

$$\begin{cases} \Omega_+^\alpha = -\frac{i}{4}(q\mathcal{D}_+^\beta q^{-1})_\beta^{\ \alpha}U + i(\tilde q\mathcal{D}_+^\alpha\tilde q^{-1}\sigma^k)T_o^k + (\bar{\tilde q})_\beta^{\ \alpha}\bar Q^{+\beta}, \\ \bar\Omega_{+\alpha} = \frac{i}{4}(\tilde q^{-1}\bar{\mathcal{D}}_{+\beta}\tilde q)_\alpha^{\ \beta}U + i(\tilde q\bar{\mathcal{D}}_{+\alpha}\tilde q^{-1}\sigma^k)T_o^k + (\tilde q)_\alpha^{\ \beta}Q_\beta^+, \\ \Omega_-^\alpha = \frac{i}{4}(\tilde q\mathcal{D}_-^\beta\tilde q^{-1})_\beta^{\ \alpha}U + i(\tilde q^{-1}\mathcal{D}_-^\alpha\tilde q\sigma^k)T_o^k + (\tilde q)_\beta^{\ \alpha}\bar Q^{-\beta}, \\ \bar\Omega_{-\alpha} = -\frac{i}{4}(\tilde q^{-1}\bar{\mathcal{D}}_{-\beta}q)_\alpha^{\ \beta}U + i(\tilde q^{-1}\bar{\mathcal{D}}_{-\alpha}\tilde q\sigma^k)T_o^k + (\bar{\tilde q})_\alpha^{\ \beta}Q_\beta^-, \end{cases} \qquad (31)$$

$$\tilde q_\alpha^{\ \beta} = \left(\exp\left(-\tfrac{1}{2}(u + i\varphi\cdot\sigma)\right)\right)_\alpha^{\ \beta}. \qquad (32)$$

One may be convinced, that the equations (9), (10) are equivalent to the following constraints on the forms (31)

$$\{\nabla_-^\alpha, \nabla_-^\beta\} = \{\nabla_-^\alpha, \bar\nabla_{+\beta}\} = \{\nabla_-^\alpha, \nabla_+^\beta\} = 0 \qquad (33)$$

Anticommutators $\{\nabla_\pm^\alpha, \bar\nabla_{\pm\beta}\}$ serve to define the "lengthened" vector derivative:

$$\{\nabla_\pm^\alpha, \bar\nabla_{\pm\beta}\} = 2i\delta_\beta^\alpha\nabla_\pm^x = 2i\delta_\beta^\alpha(\partial_\pm^x + \Omega_\pm^x). \qquad (34)$$

All of the remaining commutators and anticommutators vanish as a consequence of (33). Note that the operators $\nabla_-^\alpha$, $\bar\nabla_{+\beta}$ are given not on the whole SA $su(1,1|2)$, but on its graded subalgebra with the generators $\{U, T_o^k, Q_{\alpha+}, \bar Q_-^\beta\}$:

$$\begin{cases} \{Q_{+\alpha}, \bar Q_-^\beta\} = 2m\delta_\alpha^\beta U - 2m(\sigma^k)_\alpha^{\ \beta}T_o^k \\ \{Q_{+\alpha}, Q_{+\beta}\} = \{\bar Q_-^\alpha, \bar Q_-^\beta\} = 0 \end{cases} \qquad (35)$$

which is a SU(2)-extension of subalgebra of the minimal zero-curvature representation of the N=2 super-Liouville equation /5/. The first two constraints in eq. (33) are just a generalization of analogous constraints of the case N=2 /5/. However, in contrast to N=2, there appears now one more independent essential constraint (at N=2, it is a consequence of first two ones). The operator $\nabla_+^{\alpha}$ is defined on the SA $\{U, T_\circ^{\kappa}, \bar{Q}_+^{\alpha}, Q_{-\beta}\}$, which is conjugated to (35) and closes with (35) on the full $su(1,1|2)$. However, the evaluation of $\{\nabla_-^{\alpha}, \nabla_+^{\beta}\}$ requires to know only the anticommutator

$$\{ Q_{+\alpha}, Q_{-\beta} \} = 0 ,$$

which does not lead one out of the above super subalgebras.

Now it becomes clear, that the linear problem can be written as

$$\nabla_\pm^{\alpha} V = \bar{\nabla}_{\pm\alpha} V = 0 , \qquad (36)$$

with $V$ being a row of four complex N=4 SF's which belong to the fundamental representation of SU(1,1|2). It is not difficult to put (36) into a more explicit form, using a realization of SU(1,1|2)-generators by 4x4-matrices with zero supertrace. The component linear problem can be obtained from the SF one by successive applying the spinor derivatives to eqs. (36).

Finally, let us make a comment concerning the spectral parameter. By analogy with the cases N=0, N=1 and N=2 /5,7/ it is natural to introduce this parameter into the 1-form $\Omega_\circ^{Red}$ by a constant right transformation from the stability subgroup $H$ :

$$\Omega_\circ^{Red}(\lambda) = g_H^{-1}(\lambda)\, \Omega_\circ^{Red}\, g_H(\lambda) , \quad g_H(\lambda) \in H . \qquad (37)$$

In the N=4 case $H = SO(1,1) \times SU(2)$, so the spectral parameters, in fact, form a 4-dimensional manifold and may be joined into a real quaternion:

$$\lambda = \lambda_\circ + i \lambda^{\kappa} \sigma^{\kappa} . \qquad (38)$$

6. Conclusion. In the present paper we have constructed the N=4 supersymmetric extension of the Liouville equation with gauge $SU_+(2) \times SU_-(2)$ -symmetry, discussed its invariance properties and have shown, that it can be treated as the integrability condition of certain linear set in N=4 superspace. The obtained system yields a new realization of the $SU(2)$ superstring SA (on the classical level), different from the realization given by Ademollo et al./12/. Though the formal component content of the N=4 Liouville supermultiplet coincides with that of the free N=4 supermultiplet underlying the SU(2) superstring in the formulation of ref. /12/, the transformation properties of the component fields, with respect to the automorphism diagonal group SU(2), essentially differ. The authors of /12/ use the SU(2) scalar chiral N=4 SF, subjected to the additional irreducibility constraints of the fourth order in spinor derivatives. In our paper the basic object is the quaternionic N=4 SF $q_{\alpha}^{\beta}$ with the analyticity constraints of the first order in derivatives. Respectively, the physical bosons in /12/ are SU(2)-singlets, while in the present



scheme they are placed in the multiplet $1 \oplus 3$ of SU(2), and have a clear group and geometric meaning: the singlet field is dilaton, whereas the triplet one parametrizes the symmetric space $SU_+(2) \times SU_-(2)/SU(2)$ (transformation properties of auxiliary bosons are also different). These novel amusing properties of the model presented make it urgent to examine its quantum structure. It would happen, e.g., that the quantization of it will lead to the version of SU(2)-superstring, which is free of the main difficulty of the existing formulation/12/, the presence of ghosts at any space-time dimension. It is worth noting, that in the cases N=1 and N=2 the Liouville supermultiplets and those of corresponding superstrings/14/ fully coincide.

There remain many unsolved problems at the classical level. One of them is how to transform the system (9), (10) to the Lagrangian form. This is important for quantization. Another problem is to find the explicit form of general solution. Likely, it can be solved with the help of general method/3/, which successfully works in the cases N=1 and N=2 /5,7/. The less straightforward but, perhaps, more simple way implies the use of infinite parameter symmetry, with respect to the transformations (23)-(25). As is known, the general solutions of the N=0 and N=1 Liouville equations can be obtained by action of conformal group and its N=1 superextension on some particular solution (for N=1 it has been recently demonstrated by Arvis /15/). One may expect, that the manifolds of classical solutions of the N=2 and N=4 Liouville equations possess analogous transitivity properties.

In conclusion, let us briefly comment on the super extensions of the Liouville equation with internal SO(N) symmetry (N $\geqslant$ 3), which are related to contact SA's $K(1|N)$. It is easy to see, that the relevant Liouville supermultiplets inevitably contain, in their physical sector in parallel with the dilaton and the fields "living" in the coset $SO_+(N) \times SO_-(N)/SO(N)$, also the components with anomalous conformal weights and unclear geometric meaning. It is difficult to expect to get good equations for such components. In this sense, the N=4 Liouville supermultiplet, we have considered here, is the maximally possible one, containing dilaton and including no anomalous fields*).

As a final remark, we stress that the N=1, N=2 and N=4 super-Liouville equations are formulated most naturally in terms of the

*) Another type of extensions of the N=1 Liouville equation is provided by the supersymmetrized Toda lattices /16/. They proceed by increasing the number of N=1 supermultiplets and exhibit in the explicit form only simple N=1 SUSY. On the contrary, the extensions considered here and in /5/ deal with a single supermultiplet, irreducible with respect to corresponding extended SUSY.

real, complex and quaternionic SF's subjected, in the last two cases, to proper analyticity constraints. This can be regarded as one more argument in favour of the intimate connection between supersymmetries and systems of hypercomplex numbers, which is frequently discussed in literature (see, e.g. /17/).

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## Appendix

We present here some properties of the bosonic equation (19). It can be put in the form

$$\partial_+ \left( g(x)\, \partial_- g^{-1}(x) \right) = 0 , \tag{A.1}$$

where $g(x) = e^{i\varphi^k(x)\sigma^k}$ is the "principal chiral field" on the group SU(2). The equations of this type can be written for the principal field on an arbitrary internal symmetry group $G^r$, with the generators $T_i$ ($i = 1, 2, \dots r = \dim G^r$):

$$\partial_+ (g^r \partial_- g^{r-1}) = 0 \; ; \; g^r = e^{2i\pi^k(x) T^k} . \tag{A.2}$$

The Euclidean analogs of eqs. (A.2) appear with the choice of certain ansatzes for self-dual gauge fields in $D=4$ /3,18/.

As opposed to the ordinary equation of the nonlinear $\sigma$-model for the principal field:

$$\partial_+ (g^r \partial_- g^{r-1}) + \partial_- (g^r \partial_+ g^{r-1}) = 0 , \tag{A.3}$$

eq. (A.2) may be explicitly integrated:

$$g^r = e^{ia(x^-)} e^{ib(x^+)} , \tag{A.4}$$

with $a(x^-)$, $b(x^+)$ being arbitrary $G^r$-algebra valued functions. This simple appearance of the general solution is the consequence of gauge group of eq. (A.2):

$$g^r(x) \to e^{i\lambda(x^-)} g^r(x) e^{i\beta(x^+)} . \tag{A.5}$$

The general solution (A.4) can be constructed by the pure group-theoretic manner, by applying the transformations (A.5) to some particular solution, say to $g^r = I$.

The equation (A.2), as it stands, has no proper Lagrangian (in contrast to eq. (A.3)) but acquires it after transition to some new variables. To demonstrate this, we represent $G^r$ as the diagonal subgroup in the direct product $G_1^r \times G_2^r$ and rewrite (A.2) in terms of



the Cartan forms on the symmetric homogeneous space $G_1^r \times G_2^r / G^r$. These forms are introduced by

$$\begin{cases} \omega_\pm(x) = \frac{1}{2}\left[ e^{-i\pi T} \partial_\pm e^{i\pi T} - e^{i\pi T} \partial_\pm e^{-i\pi T} \right] , \\ \theta_\pm(x) = \frac{1}{2}\left[ e^{-i\pi T} \partial_\pm e^{i\pi T} + e^{i\pi T} \partial_\pm e^{-i\pi T} \right] . \end{cases} \tag{A.6}$$

Here, $\omega_\pm(x)$ have the meaning of covariant derivatives of the coset parameters $\pi^i(x)$ and $\theta_\pm(x)$ are the components of $G^r$-connection on the coset. They satisfy the Maurer-Cartan equations

$$\begin{cases} \nabla_+ \omega_- - \nabla_- \omega_+ = 0 \\ \partial_+ \theta_- - \partial_- \theta_+ + [\theta_+, \theta_-] + [\omega_+, \omega_-] = 0 \end{cases} \tag{A.7}$$

where $\nabla_\pm = \partial_\pm + [\theta_\pm ,$ . Taking into account (A.7), one may represent (A.2) as follows

$$\nabla_+ \omega_- + [\omega_+, \omega_-] = 0 . \tag{A.8}$$

The equation (A.3) in the similar notation differs from (A.8) by absence of the last term. Note, however, that these equations have common subclasses of solutions, which are selected by the "self-duality" conditions:

$$\omega_+ = 0 \qquad \text{or} \qquad \omega_- = 0 \tag{A.9}$$

and correspond, respectively, to the restriction of (A.4) to $e^{ia(x^-)}$ or $e^{ib(x^+)}$.

Following Faddeev and Semenov-Tyan-Shansky /19/, we treat (A.7), (A.8) as the closed system of equations for the vector fields $\omega_\pm$, $\theta_\pm$. Introducing new connection

$$\tilde\theta_\pm = \theta_\pm \pm \omega_\pm , \tag{A.10}$$

one may equivalently write this system as follows

$$\begin{cases} \partial_+ \tilde\theta_- - \partial_- \tilde\theta_+ + [\tilde\theta_+, \tilde\theta_-] = 0 & \text{(A.11a)} \\ \tilde\nabla_+ \omega_- - \tilde\nabla_- \omega_+ = 0 & \text{(A.11b)} \\ \tilde\nabla_+ \omega_- = 0 . & \text{(A.11c)} \end{cases}$$

The 1-form $\tilde\theta_\pm$, owing to (A.11a), reduces to a pure gauge, therefore the rest of eqs. (A.11) are gauge equivalent to the system

$$\begin{cases} \partial_+ \tilde\omega_- - \partial_- \tilde\omega_+ = 0 \\ \partial_+ \tilde\omega_- = 0 \end{cases} \qquad \left( \begin{array}{l} \tilde\omega_\pm = g_0\, \omega_\pm\, g_0^{-1} \\ \tilde\theta_\pm = g_0^{-1} \partial_\pm g_0 \end{array} \right) , \tag{A.12}$$

whence

$$\tilde\omega_\pm = \partial_\pm \lambda(x) \quad , \quad \partial_+ \partial_- \lambda(x) = 0 . \tag{A.13}$$

Thus, the nonlinear non-Lagrangian equation (A.2), combined with the Maurer-Cartan equations (A.7) has been reduced, by a redefinition of variables, to the free equation (A.13), which is evidently of Lagrangian type.

Let us mention, finally, a deep analogy between (A.2) and the Liouville equation. They both possess infinite-parameter symmetry groups, which act transitively in the space of the relevant classical solutions. Just as the Liouville equation, Eq. (A.2) can be obtained by our method via a nonlinear realization of its invariance group. It is seen from the transformation law (A.5), that this group is given by the direct product of gauge groups $G^{\tau}_{1(+)} \times G^{\tau}_{2(-)}$ , which are obtained by gauging $G^{\tau}_{1}$ and $G^{\tau}_{2}$ by functions of $x^{+}$ and $x^{-}$ , respectively. Treating $G^{\tau}_{(\pm)}$ in the spirit of ref.[20], as abstract groups with constant parameters and as infinite number of generators (actually forming two commuting Kac-Moody algebras), one may construct their nonlinear realization, with the diagonal $G^{\tau}$ as the stability subgroup. Further, one may equate to zero all of the Cartan forms, except for those on $G^{\tau}$ . As a result, all higher parameter fields are eliminated in terms of the essential ones $u^{i}(x)$, which, in virtue of the same constraints, turn out to satisfy Eq. (A.2). The surviving Cartan form exactly coincides with $\mathcal{E}_{\pm}$ (A.10). The condition of its zero curvature (A.11a) is just equivalent to Eq. (A.2) (with taking account of the Maurer-Cartan equation (A.7)).

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Иванов Е.А., Кривонос С.О. E2-84-290
N=4 суперрасширение уравнения Лиувилля с кватернионной структурой

Построено N=4 суперсимметричное расширение уравнения Лиувилля. Оно обладает внутренней калибровочной SU(2)xSU(2) симметрией и адекватно формулируется в терминах вещественного кватернионного N=4 суперполя, подчиненного определенным условиям грассмановой аналитичности. Как динамические уравнения, так и условия аналитичности следуют из представления нулевой кривизны на супералгебре su(1, 1|2). Мы показываем, что полученная система инвариантна относительно преобразований бесконечномерной супералгебры SU(2) - суперструны, причем их реализация отлична от ранее известных. Обсуждается возможная связь N=4 уравнения Лиувилля с теорией SU(2) - суперструны.

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Ivanov E.A., Krivonos S.O. E2-84-290
N = 4 Superextension of the Liouville Equation with Quaternionic Structure

We construct an N=4 superextension of the Liouville equation with gauge SU(2)xSU(2) symmetry. It has an adequate formulation in terms of real quaternionic N=4 superfield subjected to certain Grassmann analyticity conditions and possesses a zero-curvature representation on superalgebra su(1, 1|2). We show that the obtained system exhibits invariance under transformations of infinite dimensional SU(2)-superstring superalgebra whose realization proves to be different from those known before. A possible relation with the theory of SU(2)-superstring is discussed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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