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A POSSIBLE COUPLING OF A SCALAR GLUEBALL TO PSEUDOSCALAR GOLDSTONE MESONS

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Independent theoretical results (for reviews and more references see, e.g., ref. [1,9]) indicate that a scalar glueball is probably the lightest one with the mass around 1 GeV. There are announced candidates for this state [2,3], however, not only their existence but also the glueball interpretation are to be confirmed.

Experimentally, the scalar glueball candidates [2,3] are not equally decaying (modulo phase space) to, e.g., $\pi^+\pi^-$- and K^+K^--systems as one naturally expects [4] for an SU(3)-symmetric coupling of SU(3)-flavour singlet glueball to ordinary hadrons.

Thus, for a clear identification of glueball states one should know not only their masses and quantum numbers but also their decay properties. Since the scalar glueball is the lightest one its hadronic decays are limited to lighter pseudoscalar mesons only, and thus to understand its decay properties it is desirable to have a model describing interactions between this glueball and pseudoscalar Goldstone bosons. There has been an attempt in this direction and a simple Lagrangian model has been suggested [5] which unfortunately does not satisfy the important low-energy theorems found in refs. [6,7].

The purpose of this note is to present another effective lagrangian model satisfying not only the anomaly relation of the trace of the energy-momentum tensor of QCD [8] but also low-energy theorems of refs. [6,7,9]. It will be shown that an important part of the Lagrangian describing couplings between a hypothetical scalar glueball and pseudoscalar Goldstone bosons is predicted if one knows the glueball mass. We shall see that these couplings obey the SU(2) x SU(2) chiral symmetry pattern rather than generally assumed SU(3)-flavour symmetry rule [4], thus naturally suppressing decay to pions (proportional to m_π^4). The model will be shown to be in a reasonable agreement with the glueball assignment for the $g_s(1240)$ scalar meson [2].

We shall start our considerations assuming that the low-energy dynamics of the octet of pseudoscalar Goldstone mesons is described by the following generally accepted nonlinear effective

Lagrangian*:

$$\mathcal{L} = \frac{1}{4} \mathrm{Tr}[(\partial_\mu U)(\partial^\mu U^+)] + \mathcal{L}_{SB}, \qquad (1a)$$

where

$$\mathcal{L}_{SB} = -\mathrm{Tr}[M(U + U^+)]. \qquad (1b)$$

Here M is proportional to the 3x3 quark mass matrix and U(x) is the 3x3 field matrix transforming under chiral U(3)xU(3) transformations as $(3,\bar{3})$ representation. This matrix satisfies the constraint

$$U(x)\, U^+(x) = f^2 \qquad (2)$$

and can be parametrized as follows:

$$U(x) = f \exp(i \sum_{j=1}^{8} \frac{\lambda_j \phi_j(x)}{f}), \qquad (3)$$

where f is the pion decay constant (f = 93 MeV), $\phi_j(x)$'s (j = = 1,2,...8) represent the pseudoscalar Goldstone fields and λ's are the Gell-Mann λ matrices normalized to $\mathrm{Tr}(\lambda_i \lambda_j) = 2\,\delta_{ij}$. Because of the condition (2) the field matrix U(x) must not change under dilatations $x \to \rho x$ and thus its dimension (conformal weight) is zero[11]. Then after elementary calculations the trace of the improved energy-momentum tensor $\theta_{\mu\nu}$[12] of the Lagrangian (1) is found to have the following form**

$$(\theta_\mu^{\ \mu})_1 = -\frac{1}{2} \mathrm{Tr}[(\partial_\mu U)(\partial^\mu U^+)] - 4\mathcal{L}_{SB}, \qquad (4)$$

where index "1" labels correspondence to eq.(1). On the other hand, in QCD the exact (on the mass shell and at nonzero momentum) result for the trace of the energy-momentum tensor is given

* We neglect a pseudoscalar non-Goldstone boson singlet field (and correspondingly, a term in eq.(1a) which solves the U(1)-problem) since this neglect is not essential in our consideration provided the scalar glueball is light and cannot decay into the $\eta\eta'$ nor $\eta'\eta'$ systems. For discussions and more references concerning this Lagrangian see, e.g., ref.[10].

** An arbitrary constant can be added to eq.(4). With operators on the r.h.s. of eq.(4) in normal order, the constant is chosen by right normalization of $\langle\theta_\mu^{\ \mu}\rangle_0$. As usual, we shall calculate in tree approximation and states will be normalized covariantly: $\langle p | p' \rangle = (2\pi)^3\, 2\omega_p\, \delta^{(3)}(p - p')$.

as[8]

$$(\theta_\mu^{\ \mu})_{QCD} = \frac{\beta(g)}{2g} F_{\mu\nu}^{(a)} F^{(a)\mu\nu} - (1 + \gamma_m(g))\, \mathcal{L}_{SB}^{QCD}, \qquad (5)$$

where $F_{\mu\nu}^{(a)}$'s (a = 1,...,8) are gluon field strength tensors, $\beta(g)$ is the Callan-Symanzik function and $\gamma_m(g)$ is the mass anomalous dimension. The term $\mathcal{L}_{SB}^{QCD}(x) = \sum_i m_{q_i} \bar{q}_i(x)\, q_i(x)$ (m_{q_i} and $q_i(x)$ being the mass and field, respectively, of the quark of flavour i) represents the chiral symmetry breaking term in the QCD Lagrangian.

In the pseudoscalar Goldstone boson sector described by eq. (1) the relation (5) is effectively represented by eq.(4). However, chiral noninvariant pieces of eqs.(4) and (5) are formally different (naive comparison gives the unacceptable result $\gamma_m(g)$ = = 3) because of different dimensions (conformal weights) of the terms $\mathcal{L}_{SB}$ and $\mathcal{L}_{SB}^{QCD}$ in eqs.(4) and (5). Although such a difference is legal for effective Lagrangians, nevertheless being guided by eq.(5) we would like to enlarge eq.(1) in a way to include a scalar field to it. In fact to be closer to eq.(5) the improvement of the conformal weight of eq.(1b) is needed. This requires the existence of a dimensional, flavour independent scalar field $\sigma(x)$ (dimension $d_\sigma = 1$) which can be used to write down the following symmetry breaking term:

$$\mathcal{L}'_{SB}(x) = -[\sigma(x)]^{(3-\gamma_m)} \mathrm{Tr}[M(U(x) + U^+(x))] \qquad (6)$$

instead of eq.(1b). Here γ_m is a parameter to be specified later. We should remark that consistency with spontaneous chiral symmetry breaking (requiring VEV $\langle\sigma\rangle_0 = \sigma_0 \neq 0$) and correct behaviour of $\sigma(x)$ under dilatations ($x \to \rho x$, $\sigma(x) \to \rho^{-1}\sigma(x)$) need introduction of the actual field $\tilde{\sigma}(x)$ through the parametrization[13,14]

$$\sigma(x) = \sigma_0 \exp(\frac{\tilde{\sigma}(x)}{\sigma_0}), \qquad (7)$$

where under $x \to \rho x$, $\tilde{\sigma}(x) \to \tilde{\sigma}(x) - \sigma_0 \ln\rho$. It is worth noting here that there is no need to change the dimension of the first chiral invariant but dilatation noninvariant term in eq.(1), since just this term gives a chiral symmetrical contribution to eq.(4) in agreement with the QCD trace anomaly (eq.(5)). Moreover,this piece of eq.(4) effectively represents the low-energy theorems of refs.[6,7] in the chiral symmetry limit:

$$\langle P(p_1)\, \bar{P}(p_2)\, | \theta_\mu^{\ \mu} | 0 \rangle = q^2, \qquad (8)$$

where $q^2 = 2p_1 \cdot p_2 = (p_1 + p_2)^2$ is the invariant $(\mathrm{mass})^2$ of the $P\bar{P}$ system.

So, minimal enlargement of eq.(1) including σ-field is assumed to be as follows:

$$\mathcal{L}_{compl} = \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{4}\mathrm{Tr}[(\partial_\mu U)(\partial^\mu U^+)] - V(\sigma) + \mathcal{L}'_{SB}\ , \qquad (9)$$

where U, σ and $\mathcal{L}'_{SB}$ are given by eqs.(3), (7) and (6), respectively, and $V(\sigma)$ is a flavour independent potential. The Lagrangian (9) gives

$$(\theta^\mu_\mu)_9 = -\frac{1}{2}\mathrm{Tr}[(\partial_\mu U)(\partial^\mu U^+)] + 4V(\sigma) - \sigma\frac{dV}{d\sigma} - (1+\gamma_m)\mathcal{L}'_{SB} \qquad (10)$$

instead of eq.(4), and eq.(10) is already formally consistent with eq.(5). We expect even that the parameter γ_m in eqs.(6) and (10) is approximately given by perturbation theory at some low normalization point μ, i.e., $\gamma_m \doteq \gamma_m(g(\mu))$.

The potential $V(\sigma)$ will be found through the expansion in the field $\tilde\sigma$:

$$V(\sigma) = V(\sigma_0) + \langle\frac{dV}{d\tilde\sigma}\rangle_0\,\tilde\sigma + \frac{1}{2}\langle\frac{d^2V}{d\tilde\sigma^2}\rangle_0\,\tilde\sigma^2 + \ldots\ . \qquad (11)$$

Using parametrizations (3) and (7) in eq.(9) and eliminating term linear in $\tilde\sigma$ from eq.(9) by the use of vacuum stability condition

$$\langle\frac{dV}{d\tilde\sigma}\rangle_0 = \frac{1}{2}\,\frac{3-\gamma_m}{\sigma_0}\,f^2(2m_K^2 + m_\pi^2)\ , \qquad (12)$$

we obtain correct form of Lagrangian (9). From this Lagrangian one can easily find, e.g., the σ particle $(\mathrm{mass})^2$

$$m_\sigma^2 = \langle\frac{d^2V}{d\tilde\sigma^2}\rangle_0 - \frac{3-\gamma_m}{\sigma_0}\langle\frac{dV}{d\tilde\sigma}\rangle_0 \qquad (13)$$

and, the very important interaction term

$$\mathcal{L}_{\sigma P\bar P}(x) = -\frac{1}{2}\,\frac{3-\gamma_m}{\sigma_0}\,\tilde\sigma(x)\sum_{i=1}^{8} m_i^2\phi_i^2(x)\ , \qquad (14)$$

where m_i's ($i = 1,2,\ldots,8$) are the masses of the pseudoscalar meson octet members. It is worth noting here that just the constructed Lagrangian (9) gives (combining eqs.(10)-(14)) a generalized version (for nonzero quark masses) of eq.(8) in the following form:

$$\langle P(p_1)\bar P(p_2)|\theta_\mu^\mu|0\rangle = 2p_1\cdot p_2 + (3-\gamma_m)m_P^2\frac{m_\sigma^2}{m_\sigma^2 - q^2} + (1+\gamma_m)m_P^2\ , \qquad (15)$$

which for higher σ-particle mass ($m_\sigma^2 > q^2 \geq 4m_P^2$) behaves as

$$\langle P(p_1)\bar P(p_2)|\theta_\mu^\mu|0\rangle = q^2 + 2m_P^2 + O(q^4) + O(m_P^4) \qquad (16)$$

in full accordance with such a generalization of the low-energy theorem in ref. /7/.

It is seen from eqs.(5), (10), and (11) that the chiral invariant piece of the trace anomaly is effectively given as

$$H(x) \equiv -\frac{\beta(g)}{2g}F^2(x) = \frac{1}{2}\mathrm{Tr}[(\partial_\mu U)(\partial^\mu U^+)] + H_0 + H_1\tilde\sigma(x) + H_2\tilde\sigma^2(x) + O(\tilde\sigma^3), \qquad (17)$$

where

$$H_0 = -\langle\frac{\beta(g)}{2g}F^2\rangle_0 = \sigma_0\langle\frac{dV}{d\tilde\sigma}\rangle_0 - 4V(\sigma_0)\ ,$$

$$H_1 = \sigma_0\langle\frac{d^2V}{d\tilde\sigma^2}\rangle_0 - 4\langle\frac{dV}{d\tilde\sigma}\rangle_0\ , \qquad (18)$$

$$H_2 = \frac{1}{2}[\sigma_0\langle\frac{d^3V}{d\tilde\sigma^3}\rangle_0 - 4\langle\frac{d^2V}{d\tilde\sigma^2}\rangle_0]\ ,$$

$\ldots$.

Using the low-energy theorem of ref. /9/, i.e.,

$$i\int dx\,\langle 0|T(H(x)H(0))|0\rangle = 4H_0[1 + O(m_q)]\ , \qquad (19)$$

then from eq.(17) one finds

$$H_1^2 = 4m_\sigma^2 H_0[1 + O(m_q)]\ . \qquad (20)$$

Analogously, generalizations of eq.(19), for example,

$$i^2\int dx\int dy\,\langle 0|T(H(x)H(y)H(0))|0\rangle = 16H_0,\ \text{etc.},$$

can help us to calculate (unfortunately, in chiral limit only) all the coefficients H_i in terms of, e.g., m_σ and H_0, and thus to reconstruct the potential (11) /14/. Combining eqs.(12), (13), (18) and (20) we get

$$m_\sigma^2\sigma_0^2 = 4H_0[1 + O(m_q)] \qquad (21)$$

and then eq.(14) can be rewritten in the following form (neglecting $O(m_q^2)$ contributions):

$$\mathcal{L}_{\sigma P\bar P}(x) = -\frac{1}{4}(3-\gamma_m)\frac{m_\sigma}{\sqrt{H_0}}\tilde\sigma(x)\sum_{i=1}^{8} m_i^2\phi_i^2(x)\ , \qquad (22)$$

where for the $SU(3)_c$ and three light flavours

$$H_0 = -\langle \frac{\beta(g)}{2g} F^2 \rangle_0 \doteq \frac{9}{8}\langle \frac{\alpha_s}{\pi} F^2 \rangle_0 + O(\alpha_s^2),$$

$$\gamma_m \doteq \gamma_m(g(\mu)) \doteq \frac{2\alpha_s(\mu)}{\pi} + O(\alpha_s^2) \doteq 0.5 + O(\alpha_s^2). \qquad (23)$$

Here for definiteness we have taken $\alpha_s(\mu) \doteq 0.7$ at $\mu = 0.2$ GeV [15]. Eq.(22) allows us to write down explicitly formulae for partial decay widths of σ into pseudoscalar pairs:

$$\Gamma_{\sigma\to\pi^+\pi^-} = 2\Gamma_{\sigma\to\pi^0\pi^0} = m_\pi^4 A(m_\pi^2),$$

$$\Gamma_{\sigma\to K^+K^-} = \Gamma_{\sigma\to K^0\bar{K}^0} = m_K^4 A(m_K^2), \qquad (24)$$

$$\Gamma_{\sigma\to\eta\eta} = \frac{1}{2} m_\eta^4 A(m_\eta^2),$$

where

$$A(m^2) = \frac{(1-\frac{\gamma_m}{3})^2 m_\sigma}{8\pi\langle \frac{\alpha_s}{\pi} F^2\rangle_0}\left[1 - \frac{4m^2}{m_\sigma^2}\right]^{1/2}.$$

Since the σ-particle dominates scalar gluonic current in eq.(17) it must be identified with a scalar glueball. Taking eqs.(15) and (16) for all eight pseudoscalar mesons we easily see that present model suggests bound $m_\sigma > 2m_\eta \doteq 1.1$ GeV. The announced scalar glueball candidate $g_s(1240)$ [2] satisfies this bound. Moreover, because it is so light its dominant hadronic decays should be given by eqs.(24) and then total width $\Gamma \doteq \Gamma_{g_s\to\pi\pi} + \Gamma_{g_s\to K\bar{K}} + \Gamma_{g_s\to\eta\eta}$. Labelling $x_\pi = \Gamma_{g_s\to\pi\pi}/\Gamma$, $x_K = \Gamma_{g_s\to K\bar{K}}/\Gamma$ and using $m_\sigma = m_{g_s} = 1.24$ GeV [2] we obtain $(x_\pi x_K)^{1/2} = 0.06$ from eqs.(24) and for $\gamma_m \doteq 0.5$, $\langle\frac{\alpha_s}{\pi}F^2\rangle_{0\,SVZ} = 0.012$ GeV^4 [15] we find $\Gamma = 270$ MeV while for $\langle\frac{\alpha_s}{\pi}F^2\rangle_0 \doteq 2\langle\frac{\alpha_s}{\pi}F^2\rangle_{0\,SVZ}$ one gets $\Gamma = 135$ MeV. The agreement with the experimental values $(x_\pi x_K)^{1/2} = 0.04$ (without errors indicated) and $\Gamma = (140 \pm 10)$ MeV is remarkable [2]. Because of the lack of knowledge of precise values of the phenomenological parameters H_0 and γ_m (see eqs.(23)) it is hard to say whether consistency with experiment requires definitely higher value of $\langle\frac{\alpha_s}{\pi}F^2\rangle_0$. Due to uncertainties in H_0 we see that the free quark model value $\gamma_m = 0$ is acceptable phenomenologically too. It should be noted also that the decay pattern of another announced scalar glueball candidate G(1590) [3] is not consistent with eqs.(24)

In conclusion we want to stress that Lagrangian (9) illustrates explicitly (eqs.(14), (22) or (24)) the SU(2)xSU(2) pattern for coupling between a scalar glueball and pseudoscalar Goldstone mesons. At the same time just this Lagrangian (9) leads to eq.(17) having besides a dominant glueball contribution also a pseudoscalar meson (i.e., quark) contribution on the r.h.s. and thus representing explicitly mixing of gluon and quark degrees of freedom . This mixing [9] gives low-energy theorems of refs.[6,7] and can be simultaneously responsible for the SU(2)xSU(2) rather than SU(3) [4] coupling rule. If $g_s(1240)$ together with its decay pattern predicted by eqs.(24) is confirmed then we obtain support for both the presented decay scheme based on the SU(2)xSU(2) rule and the glueball assignment of the scalar state $g_s(1240)$ [2]. At the end we note that tensor glueball candidate $\theta(1640)$ also seems to obey the SU(2)xSU(2) rule for its coupling to pseudoscalar mesons [16].

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Ланик Й. E2-84-266

Возможная связь скалярного глюболя с псевдоскалярными мезонами Голдстоуна

Предлагается эффективный лагранжиан, описывающий взаимодействия между скалярным глюболем и псевдоскалярными мезонами Голдстоуна. Эта модель правильно отражает аномалию следа тензора энергии-импульса КХД и некоторые низкоэнергетические теоремы и явно показывает $SU(2) \times SU(2)$ характер механизма связи между скалярным глюболем и псевдоскалярными мезонами. Модель находится в разумном согласии с отождествлением состояния g_s (1240) как глюбольного.

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A Possible Coupling of a Scalar Glueball to Pseudoscalar Goldstone Mesons

An effective Lagrangian describing interactions between a scalar glueball and pseudoscalar Goldstone mesons is suggested. This Lagrangian model reproduces correctly the trace anomaly of the energy-momentum tensor of QCD and some low-energy theorems. It illustrates explicitly the SU(2) x SU(2) coupling mechanism of a scalar glueball to pseudoscalar mesons. The model is in a reasonable agreement with glueball assignment for g_s (1240) state.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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