2/VII 84

объединенный
институт
ядерных
исследований
дубна

E2-84-260

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INTRINSIC GEOMETRY OF N=2 SUPERSYMMETRY AND SUPERGRAVITY

Submitted to XXII International Conference
on High Energy Physics (DDR, 1984)

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1984

1. Up to now there were serious difficulties in constructing the explicitly covariant supersymmetric extended Yang-Mills (SYM), supergravity (SG) and matter theories in terms of unconstrained superfields (SF). A partial success along this line was a nongeometric formulation of the N=2 Abelian Yang-Mills theory[/1/] and its extension to the non-Abelian case within some recurrence procedure[/2/]. At the same time nowadays almost everybody understands the necessity of such a construction. Probably the main reason is the drastic simplification of the study of ultraviolet cancellations.

2. Such a construction for N=2 SUSY is given in the present paper. It is based on the introduction of harmonic variables $u^{\pm}_{i}$ (connected with the sphere SU(2)/U(1)) in addition to the standard even coordinates. $u^{\pm}_{i}$ have indices of the two different types: i are SU(2) indices and $\pm$ are U(1) ones. Being "zweibeins", $u^{\pm}_{i}$ provide a bridge[x)] between these groups. Taking into account a normalization condition

$$u^{+i}u^{-}_{i}=1 \qquad (1)$$

one may transform standard spinor coordinates $\theta_{\alpha i}$ having SU(2) indices (i) into ones $\theta^{\pm}_{\alpha}$ having U(1) indices ($\pm$):

$$\theta^{\pm}_{\alpha}=\theta^{i}_{\alpha}u^{\pm}_{i}\ ,\quad \bar\theta^{\pm}_{\dot\alpha}=\bar\theta^{i}_{\dot\alpha}u^{\pm}_{i} \qquad (2)$$

and, conversely,

$$\theta^{i}_{\alpha}=u^{+i}\theta^{-}-u^{-i}\theta^{+},\ \bar\theta^{i}_{\dot\alpha}=u^{+i}\bar\theta^{-}_{\dot\alpha}-u^{-i}\bar\theta^{+}_{\dot\alpha}\ . \qquad (3)$$

It is of great importance that subspace parametrized by coordinates

$$\{\zeta_{A}=(x^{m}_{A},\theta^{+}_{\alpha},\bar\theta^{+}_{\dot\alpha}),u^{\pm}\},\ x^{m}_{A}=x^{m}-2i\,\theta^{(i}\sigma^{m}\bar\theta^{j)}u^{+}_{i}u^{-}_{j} \qquad (4)$$

($\theta^{-},\bar\theta^{-}$ are absent) is closed with respect to N=2 supersymmetry transformations:

$$\begin{aligned}&\delta x^{m}_{A}=-2i\,(\varepsilon^{k}\sigma^{m}\bar\theta^{+}+\theta^{+}\sigma^{m}\bar\varepsilon^{k})u^{-}_{k}\,,\\ &\delta\theta^{+}_{\alpha}=\varepsilon^{i}_{\alpha}u^{+}_{i}\,,\ \delta\bar\theta^{+}_{\dot\alpha}=\bar\varepsilon^{i}_{\dot\alpha}u^{+}_{i}\,,\ \delta u^{\pm}=0.\end{aligned} \qquad (5)$$

We shall refer to subspace (4) as the analytic one and to SF's $\Phi^{(q)}(\zeta_{A},u^{\pm})$ as analytic SF's (they do not depend on $\theta^{-},\bar\theta^{-}$). The decomposition of an analytic scalar superfield in powers of $\theta^{+},\bar\theta^{+}$ is written down as

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x) An analogy: the vierbeins e_{am} in the ordinary gravity have both the indices (m) of the general coordinate group and those (a) of the tangent Lorentz group, so making a bridge between these groups.

$$\Phi^{(q)}(\zeta_A, u^\pm) = F^{(q)}(x_A, u^\pm) + \theta^+ \psi^{(q-1)}(x_A, u^\pm) + \bar\theta^+ \bar\varphi^{(q-1)}(x_A, u^\pm) + $$
$$+ \theta^+\theta^+ M^{(q-2)}(x_A, u^\pm) + \bar\theta^+\bar\theta^+ N^{(q-2)}(x_A, u^\pm) + \theta^+ \sigma^a \bar\theta^+ A_a^{(q-2)}(x_A, u^\pm) +$$
$$+ \bar\theta^+\bar\theta^+ \theta^+ \xi^{(q-3)}(x_A, u^\pm) + \theta^+\theta^+ \bar\theta^+ \chi^{(q-3)}(x_A, u^\pm) + \theta^+\theta^+\bar\theta^+\bar\theta^+ D^{(q-4)}(x_A, u^\pm). \quad (6)$$

As a whole $\Phi^{(q)}$ is a representation of the U(1) group with the charge q. Its components have charges varying from q to q-4 depending on the number of $\theta^+, \bar\theta^+$ in a given term of decomposition (6). Every component itself is a function of new coordinates $u^\pm_i$, e.g.,

$$F^{(q)}(x_A, u^\pm) = \sum_{n=0}^{\infty} f^{(i_1 \ldots i_{n+q} j_1 \ldots j_n)}(x_A)\, u^+_{i_1} \ldots u^+_{i_{n+q}} u^-_{j_1} \ldots u^-_{j_n} \, . \quad (q \geq 0) \quad (7)$$

It is remarkable that in (6),(7) U(1)-charges are carried only by harmonics $u^\pm_i$ and, correspondingly, spinor coordinates $\theta^+_\alpha, \bar\theta^+_{\dot\alpha}$ while component fields $f^{(i_1 \ldots j_n)}(x_A)$ are U(1)-chargeless,beeing at the same time nontrivial representations of the SU(2) group. To explain this fact note that (6), (7) is nothing but the harmonic expansion on the homogeneous space SU(2)/U(1) (see,e.g.,[3]) (after fixing the U(1)-gauge the number of independent parameters in $u^\pm_i$ is reduced to two spherical coordinates).

The harmonic expansion in symmetrized powers of $u^\pm_i$ (the spheric harmonics) (6),(7) does not terminate.Supermultiplets are characterized by two quantum numbers, just by superspin and superisospin. An analysis shows that for the scalar analytic SF's with U(1) charge q the superspin is zero while the superisospin I has an infinite spectrum

$$\Phi^{(q)}(\zeta_A, u^\pm) : I = \left|\frac{q}{2} - 1\right| + n \, , \; n = 0, 1, \ldots \quad (8)$$

For analytic SF's there exists a covariant harmonic derivative D^{++}. Acting on an analytic SF D^{++} preserves its analyticity. It also respects the normalization (1)

$$D^{++}\Phi^{(q)} = \left(u^{+i}\frac{\partial}{\partial u^{-i}} - 2i\,\theta^+ \sigma^m \bar\theta^+ \frac{\partial}{\partial x^m_A}\right)\Phi^{(q)} . \quad (9)$$

So we have succeeded in such a generalization of the Grassmann analyticity[4] that the SU(2)-symmetry is maintained! Analytic variables[3] correspond to a particular choice $u^{+1} = -\frac{1}{\sqrt{2}}, u^{+2} = -\frac{1}{\sqrt{2}}$ in (2). Note also that a coordinate of the type x^m_A (4) appeared previously in[5]. One more note. There exists a conjugation preserving the analytic subspace (4). It is given by a product of the complex conjugation (converting,e.g., θ^+ into $\bar\theta^-$) and the new involution $*$

$$* : \quad (u^\pm{}_i)^* = \pm u^{\mp i} \quad (10)$$

(converting $\bar\theta^-$ into $-\bar\theta^+$). Using the combined operation $\overset{*}{-}$ one can define real analytic SF's.

When constructing action we shall need the notion of an integral over analytic superspace. This integral involves integration over harmonic variables. It can be determined by the rules

$$\int du \cdot 1 = 1 \, , \quad \int du\, u^{+(i_1} \ldots u^{+i_n} u^{-j_1} \ldots u^{-j_m)} = 0 \, , \; n+m > 0. \quad (11)$$

The volume element of analytic superspace is

$$d\zeta_A^{(-4)} du = d^4 x_A\, d^2\theta^+ d^2\bar\theta^+ du ,$$

where $d\zeta_A^{(-4)}$ has negative U(1)-charge (- 4)because the Grassmann integration is equivalent to differentiation.

3. Now we proceed to the construction of the N=2 supersymmetric theories. We shall begin with the Fayet-Sohnius hypermultiplet[6,7]. It has superspin 0 and superisospin 1/2 and contains two isospin 1/2 - spin 0 fields and two isospin 0 - spin 1/2 fields. According to (8) the simplest SF with this content is an analytic SF q^+. The action has the form

$$S = \int d\zeta_A^{(-4)} du \left(\overset{*}{q}{}^+ D^{++} q^+ + \lambda (\overset{*}{q}{}^+)^2 (q^+)^2 \right) + c.c. \quad (12)$$

It leads to the equations of motion

$$D^{++} q^+ = -2\lambda\, \overset{*}{q}{}^+ (q^+)^2 . \quad (13)$$

By examining q^+ one discovers new property. As follows from (8) q^+ contains an infinite tower of auxiliary fields assigned to multiplets with superspin 0 and infinitely growing superisospins $\frac{1}{2}, \frac{3}{2}, \ldots$ Eq. (13) tells us that all the fields with superisospin exceeding 1/2 are equal to zero in the free case and are expressed in terms of fields with superisospin 1/2 if $\lambda \neq 0$.

An analogous situation takes place also for the other type of hypermultiplet, that of Howe, Stelle and Townsend[8]. It has superspin 0 and superisospin 1 (containing isosinglet and isotriplet scalar fields and an isodoublet of spin 1/2 fields). Correspondingly, it is described by the analytic SF $\omega(\zeta_A, u^\pm)$ having superisospins 1,2... The free action is

$$S_0 = \int d\zeta_A^{(-4)} du \; D^{++}\omega D^{++}\omega . \quad (14)$$

An analysis of the corresponding equations of motion $(D^{++})^2\omega = 0$ shows that the infinite number of auxiliary supermultiplets with superisospins ≥ 2 vanish on-shell. It is easy to build self-interaction of ω , e.g.,of the form

$$S_0 + S_{int} = \int d\zeta_A^{(-4)} du \; g^{ab}(x\omega) D^{++}\omega_a D^{++}\omega_b \, . \quad (15)$$

where g^{ab} is a "metric" and $\varkappa$ is a coupling constant.

The origin of difficulty with introduction of a self-interaction in[8] is traced to the fact that there the constraint (in our notation) $(D^{++})^3\omega=0$ has been used. It is compatible only with the free equations of motion.

4. N=2 SYM-theory has been known to date in three forms: in the component formulation[9,6], as a constrained SF theory[10] x) and in the unconstrained non-geometric SF description with the prepotentials of high dimension[1,2]. N=2SYM multiplet contains a vector field $A_a(x)$, scalar fields $M(x)$, $N(x)$, $D_{ij}(x)$ and Majorana isodoublet $\psi^i_\alpha(x)$, $\bar\psi_{\dot\alpha i}(x)$. It has zero superspin and superisospin and can be described by a SF $V^{++}(\zeta_A, u^\pm)$ (superspin 0, superisospins 0,1,...). Redundant superisospins are purely gauge owing to the invariance under transformations

$$(V^{++})' = \frac{1}{ig} e^{i\lambda(\zeta_A,u)} (D^{++} + igV^{++}) e^{-i\lambda(\zeta_A,u)} \; ; \; \lambda=\lambda_i T_i , \quad (16)$$

T_i being generators of the internal symmetry group. The transformations (16) literally mimics those of the N=0 YM theory in three aspects: derivative $\partial/\partial x^m$ is changed by D^{++}, the connection V_m by V^{++}, the gauge function $\lambda(x)$ - by a gauge analytic SF $\lambda(\zeta_A, u^\pm)$. The remarkable new phenomenon emerges: instead of one gauge degree of freedom in $\lambda(x)$ we have now an infinite number of gauge degrees of freedom in $\lambda(\zeta_A, u^\pm)$! We postpone discussion of the action to the more detailed paper. Notice only that coupling to the matter is introduced straightforwardly: one should lengthen harmonic derivative, $D^{++} \to D^{++} + igV^{++}$ in eqs. (12), (14), (15).

Using the formulation of the N=2 SYM theory in terms of the pre-prepotential V^{++} we can easily introduce a mass term

$$S_m = \frac{m^2}{8} \int d\zeta_A^{(-4)} du \, V^{++} V^{++} .$$

It can be shown that the full equations of motion imply the following irreducibility condition for V^{++}

$$m^2 D^{++} V^{++} = 0$$

which singles out just one real superspin 0, superisospin 0 massive multiplet.

5. N=2 Einstein SG. The basic gauge group is chosen from the requirement of preserving analytic representations, just as in the N=1 case it has been chosen to preserve chirality[13]. One must add, to the analytic coordinates, a central charge corrdinate x_A^5 (in order

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x) Rosly[11] have already used variables of the type u_i for analysis of the N=2 SYM constraints in the spirit of the Ward's paper[12].

to describe the graviphoton). The gauge group is primarily realized as a group of general coordinate transformations in $\{\zeta_A^M, x_A^5, u^\pm\}$ leaving invariant the subspace $\{\zeta_A^M, u^\pm\}$

$$\delta\zeta_A^M = \lambda^M(\zeta_A, u), \quad \delta x_A^5 = \lambda^5(\zeta_A, u). \quad (17)$$

Harmonic variables $u_i^\pm$ as before, undergo merely rigid SU(2) and U(1)-rotations. The basic geometric quantities of the theory are analytic vielbeins

$$V^{++M}(\zeta_A, u), \quad V^{++5}(\zeta_A, u). \quad (18)$$

They transform as follows

$$\delta V^{++M,5} = V^{++M,5\,\prime}(\zeta_A', u) - V^{++M,5}(\zeta_A, u) = \mathcal{D}_A^{++} \lambda^{M,5}(\zeta_A, u), \quad (19)$$

where

$$\mathcal{D}_A^{++} = u^{+i}\frac{\partial}{\partial u^{-i}} + V^{++M}(\zeta_A, u)\frac{\partial}{\partial \zeta_A^M} + V^{++5}(\zeta_A, u)\frac{\partial}{\partial x_A^5}$$

is a covariantization of the derivative (9).

It is a simple task to check that the component content of $V^{++M,5}$ in the W-Z gauge precisely coincides with the multiplet of N=2 SG (in its first version[14]). The formalism of differential geometry can be constructed by analogy with the case of N=2 SYM. All the conventional constraints are again solved in terms of the analytic pre-prepotentials $V^{++M,5}(\zeta_A, u)$.

6. Maybe, the main goal of our approach is to simplify an analysis of quantum structure of N=2 theories. We postpone it to future work. Here we shall give only the expression for the propagator of SYM pre-prepotential V^{++} (in the gauge $D^{++}V^{++}=0$):

$$< V^{++}(z^M, u) V^{++}(z^{M\prime}, u') > = \frac{1}{8\Box} (D^+)^4 \delta^4(x-x')\delta^8(\theta-\theta')\Delta(u,u'). \quad (20)$$

It is written in full N=2 harmonic superspace $\{z^M = (x^m = x_A^m + 2i\theta^{(+}\sigma^m\bar\theta^{-)}, \theta_\alpha^\pm, \bar\theta_{\dot\alpha}^\pm), u^\pm\}$, D^+ - are spinor derivatives D_α^+ and $\bar D_{\dot\alpha}^+$ and finally

$$\Delta(u,u') = \sum_{\substack{n=1 \\ m=0}}^{\infty} (-1)^m \frac{(m+n+1)!}{m!\,n!} u^{(+i_1} \dots u^{+i_n} u^{-j_1} \dots u^{-j_m)} \times u'^-_{(i_1} \dots u'^-_{i_n} u'^+_{j_1} \dots u'^+_{j_m)} \quad (21)$$

is the harmonic δ-function:

$$\int du \, f(u) \Delta(u,u') = f(u').$$

7. Thus, all the extendend N=2 SUSY theories admit an explicitly covariant unconstrained SF formulation in the harmonic superspace.

While the N=1 SUSY has demanded a "complexification", in the N=2 case a new step, "harmonization", is required. Radically novel feature revealed in this case is the infinite array of gauge and (or) auxiliary degrees of freedom. It seems to us that this phenomenon is of general significance and may explain the origin of the well-known theorems about non-existence of the finite number of auxiliary fields in the N=4 theories. The latter are still to be analyzed. We sincerely thank B.Zupnik for useful discussions.

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Received by Publishing Department
on April 18, 1984.

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Гальперин А. и др. E2-84-260

Внутренняя геометрия N=2 суперсимметрии и супергравитации

Предлагается новый подход к N=2 суперсимметрии, основанный на концепции гармонического суперпространства. В его рамках дается геометрическое описание N=2 теории Янга-Миллса, теории супергравитации и гипермультиплетов материи. Гармоническое N=2 суперпространство в качестве независимых координат имеет, в дополнение к обычным, изоспинорные гармоники $u_i^{\pm}$ на сфере SU(2)/U(1). Роль $u_i^{\pm}$ состоит в установлении моста между группой SU(2), реализованной на компонентных полях, и группой U(1), действующей на подходящие суперполя. Введение гармонических координат делает возможным совмещение SU(2) с грассмановой аналитичностью. Решающим для нашей конструкции является существование аналитического подпространства общего гармонического N=2 суперпространства. Суперполя гипермультиплета и истинные препотенциалы /пре-препотенциалы/ N=2 теорий Янга-Миллса и супергравитации являются суперфункциями, свободными от ограничений, в этом аналитическом подпространстве. Пре-препотенциалы имеют прозрачную геометрическую интерпретацию как калибровочные связности по отношению к SU(2)/U(1) -направлениям. Возникает радикально новое явление: число калибровочных и вспомогательных степеней свободы становится бесконечным, тогда как число физических степеней свободы остается конечным. Другие новые результаты - это массивная N=2 теория Янга-Миллса и различные самодействия гипермультиплетов вне массовой оболочки. Приведен пропагатор для суперполя Янга-Миллса.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1984

Galperin A. et al. E2-84-260

Intrinsic Geometry of N=2 Supersymmetry and Supergravity

A new approach to N=2 supersymmetry based on the concept of harmonic superspace is proposed and is used to give an unconstrained superfield geometric description of N=2 super Yang-Mills and supergravity theories as well as of matter N=2 hypermultiplets. The harmonic N=2 superspace has as independent coordinates, in addition to the usual ones, the isospinor harmomics $u_i^{\pm}$ on the sphere SU(2)/U(1). The role of $u_i^{\pm}$ is to relate the SU(2) group realized on the component fields to a U(1) group acting on the relevant superfilds. Thier introduction makes it possible to SU(2)-covariantize the notion of Grassmann analyticity. Crucial for our construction is the existence of an analytic subspace of the general harmonic N=2 superspace. The hypermultiplet superfields and the true prepotentials (pre-prepotentials) of N=2 super Yang-Mills and supergravity are unconstrained superfunctions over this analytic subspace. The pre-prepotentials have a clear geometric interpretation as gauge connections with respect to the internal SU(2)/U(1) - directions. A radically new feature arises: the number of gauge and auxiliary degrees of freedom becomes infinite while the number of physical degrees of freedom remains finite. Other new results are the massive N=2 Yang-Mills theory and various off-shell self-interactions of hypermultiplets. The propagator for Yang-Mills superfield is given.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1984