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DYNAMIC ORIGIN
OF WESS-ZUMINO MASSLESS MODEL

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I. Introduction

Radiative corrections to an initial Green function lead to a divergent expression for a total Green function in quantum field theory. After a subtraction procedure there remains an essential uncertainty for finite parts in a regularized expression. This uncertainty might be reduced by taking into account higher symmetries of the theory like gauge and supersymmetry. A concrete form of divergent expressions is not important.

However, a more detailed analysis of divergences in quantum field theory can shed light on hidden symmetries of the models under consideration. For example, the appearance of a local gauge symmetry in the model with a four-fermion "current x current" interaction in common space-time is discussed in [1]. A study of ultraviolet singular solutions of renormalization group equations allows one to discover Lagrangians with an internal symmetry[2].

In this paper it is suggested to use no finite <u>initial</u> kinetic terms at all for any fields from an <u>initial</u> lagrangian of an interaction. These terms will arise as solutions of self-consistent equations[3] after summing of leading logarithmic-divergent terms and a subsequent renormalization of fields. Then, all uncertainties in the definition of divergent integrals will be contained in the only renormalized coupling constant, which is the solution of self-consistent equations for vertices. Under such a "soft" introduction of kinetic terms and of additionally induced interactions the theory itself selects dynamically favourable relationships between renormalization constants and coupling constants.

II. An Interaction between the Weyl Spinor and a (Pseudo) Scalar Field

Let the Lorentz-invariant Hermitial combination of (pseudo) scalar and Weyl spinor fields be chosen as an initial interaction

$$\mathcal{L}_{int} = \frac{1}{2} g_0 \Psi \sigma_2 \Psi \cdot B + \frac{1}{2} g_0 \Psi^+ \sigma_2 \Psi^+ B^* , \qquad (2.1)$$

here σ_2 stands for a charge-conjugate matrix. This corresponds to two Feynman's diagrams (fig. 1).

Fig. 1.

Since the coupling constant g_0 in initial interactions is assumed to be a small value, finite parts of radiative corrections to Green functions are of an order of g_0^2 and higher. Thus, the kinetic terms of free fields (which are of an order of $O(1)$) cannot be generated by these corrections. However, the kinetic terms might be obtained, due to the presence of uncertainties in the form of divergent integrals. Then, these uncertainties are removed by renormalization procedure into the coupling constant.

Ultraviolet asymptotics of propagators are necessary to analyse divergent expressions. Let the fermion propagator $S(p)$ go as $(p^2)^{-\alpha}$ and the boson propagator $D(p)$ as $(p^2)^{-\beta}$ in this limit. Then, their Lorentz-covariant expressions are

$$S(p) = Z_F \frac{ip_\mu \sigma^\mu}{(p^2)^{\alpha+\frac{1}{2}}}, \qquad (2.2a)$$

$$D(p) = Z_B \frac{i}{(p^2)^\beta}, \qquad (2.2b)$$

where Z_F and Z_B are the renormalization constants of spinor $\Psi = \sqrt{Z_F}\,\Psi_R$ and (pseudo) scalar $B = \sqrt{Z_B}\,B_R$ fields. The fermion propagator is denoted by a solid line (fig. 2a); and the boson propagator, by a dashed line (fig. 2b) in the diagrams.

Fig. 2

Consider the primitive Feynman diagrams for Green functions. Necessary conditions for the existence of solutions of self-consistent equations may come out from dimensional analysis. For example, self-energy contributions of fermion and (pseudo) scalar fields are determined out of the diagrams a) and b) in fig. 3, respectively,

Fig. 3

which ought to lead to initial Green functions (2.2) in the lowest approximation [4]. This gives the first equation for unknown parameters α and β :

$$4\alpha + 2\beta = 4. \qquad (2.3)$$

The primitive vertices for the self-interaction of the (pseudo) scalar field (fig. 4) effectively give rise to an interaction shown in fig. 5.

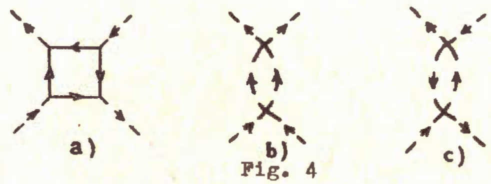


Fig. 4

From the self-consistent equation we obtain the condition

$$8\alpha = 4\beta = 4 \qquad (2.4)$$

which is valid in any order of the perturbation theory. This leads to the canonical form of propagators:

$$S(p) = Z_F \frac{ip_\mu \sigma^\mu}{p^2}, \qquad (2.5)$$

$$D(p) = Z_B \frac{i}{p^2} . \qquad (2.6)$$

III. Calculation of Primitive Feynman Diagrams

In order to sum the leading logarithmic-divergent integrals in any order *), it is necessary to calculate one-loop diagrams in figs. 3 and 4. The self-energy of a fermion with the help of Feynman's rules is

$$\Sigma(p) = \frac{i g_0^2 Z_F Z_B}{(4\pi)^2} \frac{i}{\pi^2} \int \frac{\sigma_\mu q^\mu d^4q}{q^2 (p+q)^2} = \frac{1}{2} g^2 C \frac{ip_\mu \sigma^\mu}{Z_F} + \frac{1}{Z_F} g^2 (\text{finite part}). \qquad (3.1)$$

*) They are of the same order of the coupling constant.

Here $g^2 = \frac{g_0^2 Z_F^2 Z_B}{(4\pi)^2}$ is renormalized coupling constant, and $C = -\frac{i}{\pi^2} \underset{\varepsilon}{Reg} \int \frac{d^4 q}{q^4}$ is a regularized value of a logarithmic-divergent integral.

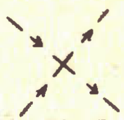
There is a difference of principle in extracting the divergences in an ordinary theory and in an approach of the dynamic generation of Green functions. In a standard theory the form of divergent parts is not essential. These uncertainties remain in finite expressions after a subtraction procedure and have no dependence on subtracted infinite terms due to a quasilocal arbitrariness/5/. In this paper we shall extract divergent integrals like $-\frac{i}{\pi^2} \int \frac{d^4 q}{q^4}$ in any calculations and use them for solving self-consistent equations as if these integrals are regularized. As we shall show later by solving the self-consistent equation, an uncertainty in extracting an infinite part of Green function as well as of a finite part of it turns out to be connected with an uncertainty in the coupling constant g^2.

Now calculate a contribution to the self-energy of (pseudo) scalar field (fig. 3b). Taking into account a combinatorial multiplier $1/2$, we have

$$\Pi(p) = \frac{i g_0^2 Z_F^2}{(4\pi)^2} \frac{i}{\pi^2} \int d^4 q \frac{q^2 + (qp)}{q^2 (p+q)^2} = \frac{i}{Z_B} M^2 + \frac{1}{2} g^2 C \frac{i p^2}{Z_B} + \frac{1}{Z_B} g^2 (\text{finite part}), \quad (3.2)$$

where $M^2 = \underset{\varepsilon}{Reg} \frac{i g^2}{\pi^2} \int \frac{d^4 q}{q^2}$ *).

There exist no one-loop radiative corrections to the interaction between the spinor and (pseudo) scalar fields. So, this vertex cannot appear in a dynamic way and it has been introduced as an initial one. The self-interaction of the (pseudo) scalar field $-\frac{\lambda_0}{4} (B^* B)^2$ (fig. 5)



$-i\lambda_0$

Fig.5

originates from the diagrams in fig. 4a. This vertex in its turn leads to diagrams (fig. 4b,c) which contribute to the Green function for the vertex in fig. 5 in the one-loop approximation. When external momenta are zero in the diagrams in fig. 4, we have

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*) In this paper an origin of mass terms is not discussed.

$$\Box = -i \frac{\lambda}{Z_B^2} \left[\frac{2g^4}{\lambda} - \frac{5}{2} \lambda \right] C + \frac{\lambda}{Z_B^2} \cdot (\text{finite part}), \quad (3.3)$$

where $\lambda = \lambda_0 Z_B^2$ is the renormalized coupling constant.

It is easy to see there are no more one-loop divergent diagrams at all.

IV. Summation of the Leading Logarithmic Divergences with the Help of the Renormalization Group

In order to write down renormalization group equation, it is necessary to calculate anomalous dimensions of the Green functions. They are determined as functions of charges:

$$\gamma(g_1, \dots, g_n) = g_i \frac{\partial}{\partial g_i} \left[\frac{\partial Z(C)}{\partial C} \Big|_{C=0} \right], \quad (4.1)$$

where renormalization constants Z can be obtained out of radiative corrections (3.1)-(3.3)*).

Thus, for the propagators of fermion and boson fields, we have

$$\gamma_F(g^2, \lambda) = -\frac{1}{2} g^2, \quad (4.2)$$

$$\gamma_D(g^2, \lambda) = -\frac{1}{2} g^2, \quad (4.3)$$

and for the interaction between the (pseudo) scalar and spinor fields and for the self-interaction of the (pseudo) scalar field -

$$\gamma_\Gamma(g^2, \lambda) = 0, \quad (4.4)$$

$$\gamma_\Box(g^2, \lambda) = \frac{5}{2} \lambda - \frac{2g^4}{\lambda}, \quad (4.5)$$

respectively.

Under such a definition of anomalous dimensions β-functions for the corresponding vertices are given as/6/:

$$\beta_\Gamma(g^2, \lambda) = g^2 \left[2\gamma_\Gamma(g^2, \lambda) - 2\gamma_F(g^2, \lambda) - \gamma_D(g^2, \lambda) \right] = \frac{3}{2} g^4, \quad (4.6)$$

$$\beta_\Box(g^2, \lambda) = \lambda \left[\gamma_\Box(g^2, \lambda) - 2\gamma_D(g^2, \lambda) \right] = \frac{5}{2} \lambda^2 + g^2 \lambda - 2g^4. \quad (4.7)$$

It is easy now to write down the Gell-Mann-Low equations for bare coupling constants g_B^2 and λ_B:

$$-\frac{\partial g_B^2}{\partial C} = \beta_\Gamma(g_B^2, \lambda_B) = \frac{3}{2} g_B^4, \quad (4.8)$$

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*) Renormalization constants Z in a MS scheme depend on a parameter $1/\varepsilon$. In our case a similar parameter is C.

$$-\frac{\partial \lambda_s}{\partial x} = \beta_\square(g_s^2,\lambda_s) = \frac{5}{2}\lambda_s^2 + g_s^2\lambda_s - 2g_s^4 \,. \tag{4.9}$$

One should pay attention to an opposite sign in the left-hand sides of equations (4.8) and (4.9) in contrast with their usual form for the effective charges $\bar{g}^2$ and $\bar{\lambda}$.

The solutions of these equations

$$g_s^2 = g_s^2(C, g^2, \lambda), \tag{4.10a}$$

$$\lambda_s = \lambda_s(C, g^2, \lambda) \tag{4.10b}$$

give us a possibility to sum up any leading logarithmic-divergent integrals corresponding to Feynman's diagrams without counter-terms*).

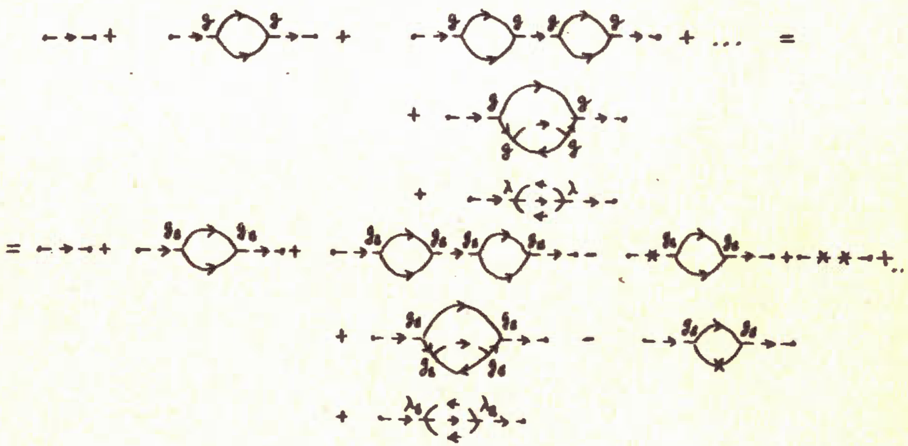

Fig. 6

The solution of equation (4.8) is

$$g_s^2 = \frac{g^2}{1+\frac{3}{2}g^2C} \,. \tag{4.11}$$

Substituting it into the second equation (4.9) we get the Riccati equation. Its solution is

$$\lambda_s = \frac{g^2}{1+\frac{3}{2}g^2C}\;\frac{\left(4+5\frac{\lambda}{g^2}\right)\left(1+\frac{3}{2}g^2C\right)^3 - 4\left(1-\frac{\lambda}{g^2}\right)}{\left(4+5\frac{\lambda}{g^2}\right)\left(1+\frac{3}{2}g^2C\right)^3 + 5\left(1-\frac{\lambda}{g^2}\right)} \,. \tag{4.12}$$

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*) The Green function re-expansion for the (pseudo) scalar field in coupling constants g_s^2 and λ_s of the second order is given in fig. 6., where —✱— and —✱— lines stand for the corresponding counter-terms for (pseudo) scalar and spinor fields.

Now for the total two-point Green functions of (pseudo) scalar and spinor fields in the leading logarithmic approximation, we have

$$\Delta(C,g^2,\lambda) = \exp\left\{\int_0^C dx\, \gamma_{F,D}\left[g_s^2(x,g^2,\lambda),\lambda_s(x,g^2,\lambda)\right]\right\} = \left(1+\frac{3}{2}g^2C\right)^{-\frac{1}{3}}, \tag{4.13}$$

and for the vertex of the self-interaction of the (pseudo) scalar field:

$$\square(C,g^2,\lambda) = \exp\left\{-\int_0^C dx\, \gamma_\square\left[g_s^2(x,g^2,\lambda),\lambda_s(x,g^2,\lambda)\right]\right\} = \tag{4.14}$$

$$= \frac{1}{\frac{\lambda}{g^2}\left(1+\frac{3}{2}g^2C\right)^{\frac{1}{3}}}\;\frac{\left(4+5\frac{\lambda}{g^2}\right)\left(1+\frac{3}{2}g^2C\right)^3 - 4\left(1-\frac{\lambda}{g^2}\right)}{\left(4+5\frac{\lambda}{g^2}\right)\left(1+\frac{3}{2}g^2C\right)^3 + 5\left(1-\frac{\lambda}{g^2}\right)} \,.$$

V. Solution of the Self-Consistent Equations for the Green Functions

In accordance with the hypothesis of the dynamic origin of Green functions, the sum of regularized divergent integrals without an initial propagator from Feynman's diagrams in fig. 6 has to generate the propagator (2.2b) /4/. The latter has been used in its turn to calculate radiative corrections in the perturbation theory. Thus, we have the equation of coupling

$$\left(1+\frac{3}{2}g^2C\right)^{-\frac{1}{3}} - 1 = 1 \,. \tag{5.1}$$

Its solution

$$g^2C = -\frac{7}{12} \tag{5.2}$$

shows that the uncertainty in the definition of coupling constant g^2 is completely caused by that of a divergent integral $-\frac{i}{\pi^2}\int\frac{d^4q}{q^4}$.

The self-consistent equation for the vertex of the self-interaction of the (pseudo) scalar field is

$$\square(C,g^2,\lambda) - 1 = 1 \,. \tag{5.3}$$

Using (5.1) we then have

$$\left(1-\frac{\lambda}{g^2}\right)\left(4+5\frac{\lambda}{g^2}\right) = 0 \,. \tag{5.4}$$

Both solutions of equation (5.4)

$$\lambda = g^2, \tag{5.5}$$

$$\lambda = -\frac{4}{5}g^2 \tag{5.6}$$

play the role of a restriction to coupling constants of interactions which arises due to their dynamic origin. Note, the condition

(5.5) leads to the supersymmetric massless Wess-Zumino model with an effective lagrangian:

$$\mathcal{L}_{eff} = i\psi_R^+ \sigma_\mu \frac{\partial}{\partial x_\mu}\psi_R + \frac{\partial B_R^*}{\partial x_\mu}\frac{\partial B_R}{\partial x^\mu} + \frac{1}{2}g(\psi_R\sigma_2\psi_R B_R + \psi_R^+\sigma_2\psi_R^+ B_R^*) - \frac{g^2}{4}(B_R^* B_R)^2, \quad (5.7)$$

and is valid in any order of the perturbation theory /7/. The condition (5.6), as is shown in /2/, is already broken down in the two-loop approximation, and it is correct only in the leading logarithmic approximation.

Thus, the approach under consideration shows once more an importance of the divergent expressions themselves in the field-theoretical calculations. Moreover, the idea of the dynamic origin of Green functions enables one to construct an interaction between any fields on the same principles, revealing the dynamic origin of particular geometric symmetries.

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Динамическое происхождение безмассовой модели Весса-Зумино

В качестве затравочного взаимодействия в уравнениях самосогласованности для кинетических членов и вершинных частей использована локальная лоренц-инвариантная эрмитова комбинация /псевдо/ скалярного и вейлевского спинорного полей. Решение этих уравнений в приближении главных логарифмически расходящихся членов приводит к суперсимметричной безмассовой модели Весса-Зумино.

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Dynamic Origin of Wess-Zumino Massless Model

A local Lorentz-invariant Hermitian combination of (pseudo) scalar and Weyl spinor fields is used as an initial interaction in self-consistent equations of kinetic terms. The solution of these equations in an approximation of the leading logarithmic-divergent diagrams provides the supersymmetric Wess-Zumino massless model effectively.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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