объединенный
институт
ядерных
исследований
дубна

E2-84-256

G.A.Kozlov, S.P.Kuleshov, V.I.Savrin,
V.V.Sanadze, N.B.Skachkov

ON THE BEHAVIOUR OF FORM FACTORS OF DECAYS OF PSEUDOSCALAR SYSTEMS $P \to \gamma \ell \bar{\ell}$ IN THE REGION OF SMALL INVARIANT MASSES OF A LEPTON $\ell \bar{\ell}$-PAIR

Submitted to XXII International Conference
on High Energy Physics (DDR, 1984)

1984

1. INTRODUCTION

In ref.[1] it has first been suggested that the application of relativistic theory of bound states [2,3] leads to a possible theoretical prediction of a new effect in the behaviour of the form factor of decay $\pi^o \to \gamma e^+ e^-$. The effect consists in that at small invariant masses squared of an electron-positron pair, x, the slope parameter of the form factor may change sign. It is shown that in this approach for the form factor of decay $\pi^o \to \gamma e^+ e^-$ ($\tilde{F}_{\pi^o \to \gamma e^+ e^-}(x)$) at $x \leq 0.2$ there occurs a "dip"-effect. The form factor first decreases from the normalized value $\tilde{F}_{\pi^o \to \gamma e^+ e^-}(x=0)=1$ by 5-7%, then it starts to increase; in the interval $x = 0.2$-0.3 it crosses the line $\tilde{F}_{\pi^o \to \gamma e^+ e^-}(x) = 1$ and continue to grow by the law $(1-x)^{-1}$. Note that data on the slope parameter of form factors of decays of light π^o- and η-mesons into the Dalitz pair and γ-quantum are not reliable. For instance, in refs.[4,5] and [6] devoted to the measurement of decays $\pi^o \to \gamma e^+ e^-$ and $\eta \to \gamma e^+ e^-$, respectively, negative values have been obtained for the parameter a defining the slope of the form factor $\tilde{F}_{\pi^o, \eta \to \gamma e^+ e^-}(x)$ by the formula $\tilde{F}_{\pi^o, \eta \to \gamma e^+ e^-}(x) = 1 + ax$ at small invariant masses squared of e^+e^--pair x, whereas for $\tilde{F}_{\pi^o \to \gamma e^+ e^-}(x)$ in [7] and for $\tilde{F}_{\eta \to \gamma \mu^+ \mu^-}(x)$ in [8] positive values have been obtained for $a (a > 0)$.

Theoretical calculations by perturbative methods of quantum field theory give positive values for a (a review of results can be found in [9]).

We have also raised the point whether or not the effect we have discovered may exist in other, more suitable for a theoretical study processes, for instance, in the decay of the electrodynamic bound state $(\mu^+\mu^-)_{s=0} \to \gamma e^+ e^-$. In this case there is no problem with a confinement potential, and there are no free parameters of the problem: masses of particles, constituents of a bound state, interaction constants. It has been shown[10] that in this system the "dip"-effect is indeed admissible theoretically and amounts to about 0.5%. This small value as compared to the case of π^o-meson, is caused by the smallness of the coupling constant in quantum electrodynamics and by an almost nonrelativistic nature of the relative motion of constituent particles.

Here we apply the approach developed in [11,10] to describe the η- and η_c-mesons. It is shown to describe well the experimental data on the form factor of decay $\eta \to \gamma \mu^+ \mu^-$ [8] and to provide a prediction for the width $\Gamma(\eta_c \to \gamma\gamma)$ and branching ratios of the decays $\eta_c \to \gamma e^+ e^-$ and $\eta_c \to \gamma \mu^+ \mu^-$. We have established that for more heavy η- and η_c-mesons than π^o-meson the "dip"-effect is also admissible theoretically, and in the given case it amounts to about 0.4-0.7%.

2. FORM FACTOR OF THE DECAY $\pi^o \to \gamma e^+ e^-$

Consider the π^o-meson with mass M as a bound state of a quark and an antiquark with mass m. The diagram of the decay is drown in Fig.1. The amplitude of the decay $\pi^o \to \gamma e^+ e^-$ involving composite particles in a single-time formalism is written as follows [1,11]:

$$M_{\pi^o \to \gamma e^+ e^-}(q_2|P) = \frac{1}{(2\pi)^3} \int \frac{d^3\vec{k}_1}{2k_1^o} T^{\sigma_1 \sigma_2}_{q\bar{q} \to \gamma e^+ e^-}(q_2; k_1|P)\, \Psi_{BK\,\sigma_1 \sigma_2}(\vec{k}_1), \quad (1)$$

where

$$M_{\pi^o \to \gamma e^+ e^-}(q_2|P) = F_{\pi^o \to \gamma e^+ e^-}(q^2, 0) \frac{1}{q_2^2} j^\mu(p_1, p_2)\, e_1^\nu q_2^\rho P^\sigma \epsilon_{\mu\nu\rho\sigma}, \quad (2)$$

$F_{\pi^o \to \gamma e^+ e^-}(q_2^2, 0)$ is the form factor of the decay $\pi^o \to \gamma e^+ e^-$, $\Psi_{BK\,\sigma_1 \sigma_2}(\vec{k}_1)$ is the relativistic single-time wave function for the relative motion of a quark and an antiquark in a hadron, $T^{\sigma_1 \sigma_2}_{q\bar{q} \to \gamma e^+ e^-}(q_2; k_1|P)$ is the amplitude of annihilation of a quark with polarization σ_1 and an antiquark with polarization σ_2 into a Dalitz pair and a γ-quantum [1]:

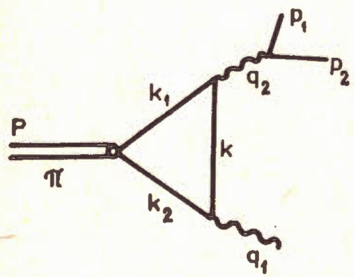

Fig.1

$$T^{\sigma_1 \sigma_2}_{q\bar{q} \to \gamma e^+ e^-}(q_2; k_1 | P) = \frac{(4\pi\alpha)^{3/2} S_q v_{\bar{q}}^{\sigma_2}(k_2) \hat{e}_1 (\hat{k}_1 - \hat{q}_2 + m) \hat{e}_2 u_q^{\sigma_1}(k_1)}{(k_1 - q_2)^2 - m^2} + (q_1 \leftrightarrow q_2), \quad (3)$$

where $S_q = \sqrt{n_c}(e_u^2 - e_d^2)$, n_c is the number of colours, e_u and e_d are charges of u- and d-quarks, repectively. For the other notation see Fig.1.

The single-time wave function $\Psi^{\sigma_1\sigma_2}_{BK}(\vec{k}_1)$ is expressed in terms of the Bethe-Salpeter wave function through a covariant equating of times [3]:

$$\Psi^{\sigma_1\sigma_2}_{BK}(\vec{k}_1) = \int d^4x\, e^{i(k_1-k_2)x}\, \delta(\lambda x)\, \bar{u}^{\sigma_1}_q(k_1) v^{\sigma_2}_{\bar{q}}(k_2) \times$$

$$\times \langle 0|T\{\psi_q(x)\psi_{\bar{q}}(0)\}|\vec{P}, M\rangle , \tag{4}$$

where $x = x_1 - x_2$ is a relative coordinate of a quark and an antiquark, $u^{\sigma_1}_q(k_1)$ and $v^{\sigma_2}_{\bar{q}}(k_2)$ are bispinors of a quark and an antiquark with polarizations σ_1 and σ_2, respectively ($k_1^2 = k_2^2 = m^2$, m being a quark mass), $P = k_1 + k_2$, $M = \sqrt{P^2}$ is the invariant mass of a system composed of two quarks; $\lambda^\mu = (M^{-1}P^\mu)$ is a four-vector in the c.m.s. with components $\lambda^\mu = (1,\vec{0})$, as a result, in this c.m.s. the δ-function in (4) provides the equating of times $(x_1)_0 = (x_2)_0$.

Making use of results of [1] we write the form factor of decay $\pi^0 \to \gamma e^+e^-$ in the following form (see also [18]):

$$F_{\pi^0\to\gamma e^+e^-}(q_2^2, 0) = \frac{8mS_q a}{(2\pi)M}\Big\{\int_0^\infty \frac{dk_1k_1^2}{2k_1^0}\frac{\bar{\phi}_{BK}(k_1)}{2q_1k_1}\int_{-1}^{+1}\frac{d\cos\theta_{k_1}}{q_1^0k_1^0/q_1k_1 - \cos\theta_{k_1}} + \tag{5}$$

$$+ \int_0^\infty \frac{dk_1k_1^2}{2k_1^0}\frac{\bar{\phi}_{BK}(k_1)}{2q_2k_1}\int_{-1}^{+1}\frac{d\cos\theta_{k_1}}{(2q_2^0k_1^0 - q_2^2)/2q_2k_1 - \cos\theta_{k_1}}\Big\},$$

where $k_1 = |\vec{k}_1|$, $q_1 = |\vec{q}_1|$, $q_2 = |\vec{q}_2|$, $\bar{\phi}_{BK}(k_1)$ is the scalar wave function for s-state connected with the spinor wave function (4) by

$$\Psi^{\sigma_1\sigma_2}_{BK}(\vec{k}_1) = \bar{u}^{\sigma_1}_q(k_1)\gamma^5 v^{\sigma_2}_{\bar{q}}(k_2)\frac{\bar{\phi}_{BK}(\vec{k}_1)}{2(Pk_1)/M} \tag{6}$$

and the normalization condition is as follows

$$(2\pi)^{-3}\int d^3\vec{k}_1 |\bar{\phi}_{BK}(\vec{k}_1)|^2 = M. \tag{7}$$

Upon calculating in (5) the integrals over $d\cos\theta_{k_1}$ we get

$$F_{\pi^0\to\gamma e^+e^-}(q_2^2, 0) = \frac{8mS_q a}{(2\pi)M}\int_0^\infty \frac{dk_1k_1^2}{2k_1^0}\bar{\phi}_{BK}(k_1) \times \tag{8}$$

$$\times\Big[\frac{1}{2q_1k_1}\ln\Big|\frac{k_1^0 + k_1}{k_1^0 - k_1}\Big| + \frac{1}{2q_2k_1}\ln\Big|\frac{q_2^2 - 2k_1^0q_2^0 - 2q_2k_1}{q_2^2 - 2k_1^0q_2^0 + 2q_2k_1}\Big|\Big].$$

In what follows it will be more convenient to work with another variable, the quark rapidity χ_k conjugate to the quark relative momentum: $\chi_k = \ln[(k_1^0 + k_1)/m]$. Then form factor (8) divided by $F_{\pi^0\to\gamma e^+e^-}(q_2^2, 0)$ at $q_2^2 = 0$ assumes the form:

$$\tilde{F}_{\pi^0\to\gamma e^+e^-}(x) \equiv F_{\pi^0\to\gamma e^+e^-}(q_2^2, 0)/F_{\pi^0\to\gamma\gamma}(0,0) = (1-x)^{-1}[1+(4I_N)^{-1}J(x)], \tag{9}$$

where

$$J(x) = \int_0^\infty d\chi_k \ln|X(x,\chi_k)|\phi(\chi_k), \tag{10}$$

$$X(x,\chi_k) = [1 - xe^{-\chi_k}(M/m - e^{-\chi_k})][1 - xe^{\chi_k}(M/m - e^{\chi_k})]^{-1}, \tag{11}$$

$$I_N = \int_0^\infty d\chi_k\, \phi(\chi_k)\chi_k\,;\quad 4\pi\phi(\chi_k) = k_1\bar{\phi}_{BK}(k_1);$$
$$k_1^0 = m\,\mathrm{ch}\chi_k\,;\quad k_1 = m\,\mathrm{sh}\chi_k\,;\quad x = (p_1 + p_2)^2/M^2. \tag{12}$$

From (9) and (10) two things may be observed: i) the form factor of decay $\pi^0\to\gamma e^+e^-$ depends essentially on the form of interaction between a quark and an antiquark, i.e., on the form of wave function $\phi(\chi_k)$, and ii) the second term in the brackets of (9) is negative in sign, when $J(x) < 0$. It can easily be verified that $J(x) < 0$ for $|X(x,\chi_k)| < 1$. The condition $|X(x,\chi_k)| < 1$ is equivalent to the inequality:

$$\mathrm{sh}\Big[\frac{M}{2m} - \mathrm{ch}\chi_k\Big] < 0, \tag{13}$$

which will be fulfilled for all χ_k, when the energy of a bound state is negative, i.e., $M < 2m$.

It is natural that the condition $J(x) < 0$ being fulfilled does not yet mean an essential deviation of the behaviour of form factor (9) from the trivial dependence $(1-x)^{-1}$ coming from the integration in (5) of the quark propagator over angles. It is just this deviation that is most interesting as it is due to the contribution of the bound-state wave function (6). So, for instance, if we shall not take account of the relativistic relative motion of quarks in a hadron and consider only the static limit, we immediately obtained from (9) [12]:

$$(1-x)\tilde{F}^{stat}_{\pi^0\to\gamma e^+e^-}(x) = 1 + \frac{(M/2m - 1)x}{1 - (M/m - 1)x}. \tag{14}$$

Hence we find that for small $x \ll 1$ the slope parameter of form factor (14) $a \equiv (M/2m) > 0$. In the case of ultrarelativistic motion of quarks, i.e., when to the integrals (9) a main contribution comes from large χ_k, we obtain the following formal expression for the slope of form factor (9):

$$(1-x)\frac{1}{\tilde{F}_{\pi^o\to\gamma e^+e^-}(x)}\cdot\frac{d\tilde{F}_{\pi^o\to\gamma e^+e^-}(x)}{dx}=1-\frac{1}{x\tilde{F}_{\pi^o\to\gamma e^+e^-}(x)}\frac{\int_0^\infty d\chi_k\phi(\chi_k)}{\int_0^\infty d\chi_k\chi_k\phi(\chi_k)} \quad (15)$$

From the latter formula it can be easily seen that for small x it may happen that the slope parameter $a \equiv d\tilde{F}(x)/dx$ will be negative, and the form factor will decrease from the normalized value $\tilde{F}_{\pi^o\to\gamma e^+e^-}(x=0)=1$. The absolute value of this effect can be obtained only by calculating integrals in (9) with wave function $\phi(\chi_k)$.

3. APPLICATION OF MODEL WAVE FUNCTIONS

To calculate the form factor (9) numerically, we make use of a specific form of the wave functions (WF) only for extreme cases of interaction: the chromodynamic "coulomb" interaction [13]:

$$\phi^{QCD}(\chi_k)=\frac{C_{QCD}\,\chi_k}{(\mathrm{ch}\chi_k-M/2m)[\chi_k^2+(\arccos M/2m)^2]}; \quad (16)$$

the confinement "oscillatory-like" interaction [14]:

$$\phi^{OSC}(\chi_k)=C_{OSC}\exp(-m\,\mathrm{ch}\chi_k/2\omega), \quad (17)$$

where C_{QCD} and C_{OSC} are normalization constants defined from(7). The numerical results for form factor (9) are presented in Fig.2 for the values of parameters m = 315 MeV for WF (16) and m = = 138 MeV, ω = 268 MeV for WF (17). From that figure it is seen that the form factor $\tilde{F}_{\pi^o\to\gamma e^+e^-}(x)$ in the region $x_0 \le x \le 0.35$ ($x_0 = 4m_e^2/M^2$, m_e is the electron mass) is characterized by a "dip"-effect. The magnitude of this effect is explicitly dependent on the form of interaction between quarks in a hadron and on the WF parameters.

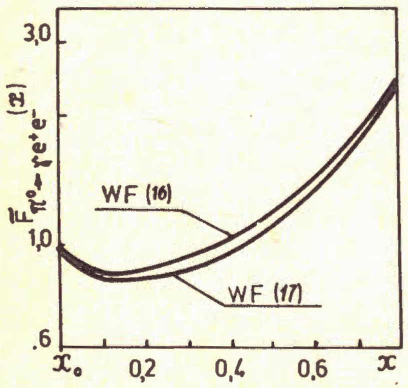

Fig.2

In ref.[10] the electromagnetic form factor of decay $(\mu^+\mu^-)_{s=0}\to\gamma e^+e^-$ has been calculated with the use of the wave function [15]:

$$\phi^{QED}(\chi_k)=\frac{C_{QED}\,\mathrm{sh}\chi_k}{(\mathrm{ch}\chi_k-M/2m)^2}, \quad (18)$$

where M is the mass of the bound state $(\mu^+\mu^-)_{s=0}$, m is a muon mass, C_{QED} is the normalization constant. As has been noted above, in the decay of a weakly bound system $(\mu^+\mu^-)_{s=0}$ into a Dalitz pair and a γ-quantum the "dip"-effect is theoretically admissible and amounts to about 0.5%.

4. FORM FACTOR AND WIDTHS OF DECAY $\eta\to\gamma\gamma$, $\eta\to\gamma\ell\bar{\ell}$, $\eta_c\to\gamma\gamma$, $\eta_c\to\gamma\ell\bar{\ell}$

If a spin-unitary part of the η-meson wave function is taken into account, the decay width $\Gamma(\eta\to\gamma\gamma)$ is given by the formula

$$\Gamma(\eta\to\gamma\gamma)=\frac{16\alpha^2}{3\pi M_\eta}|(|u\bar{u}>+|d\bar{d}>)(\sqrt{\tfrac{1}{2}}\cos\theta+\sin\theta)m_u\int_0^\infty d\chi_k\chi_k\phi^{\eta u\bar{u}}(\chi_k)+$$
$$+|s\bar{s}>(\sin\theta-\sqrt{2}\cos\theta)m_s\int_0^\infty d\chi_k\chi_k\phi^{\eta s\bar{s}}(\chi_k)|^2, \quad (19)$$

where m_u and m_s are masses of u- and s-quarks, respectively, M_η is the η-meson mass, $\phi^{\eta u\bar{u}}(\chi_k)$ and $\phi^{\eta s\bar{s}}(\chi_k)$ are wave functions of $u\bar{u}$- and $s\bar{s}$-quarkonia satisfying the normalization condition (7), θ is the angle of η-η'-mixing.

The form factor of decay $\eta\to\gamma\ell\bar{\ell}$ is calculated by the formula

$$\tilde{F}_{\eta\to\gamma\ell\bar{\ell}}(x)=\frac{(1-x)^{-1}}{4(I_u-I_s)}\{m_uA_u\int_0^\infty d\chi_k(4\chi_k+\ln|X(x,\chi_k,m_u)|)\phi^{\eta u\bar{u}}(\chi_k)-$$
$$-m_sA_s\int_0^\infty d\chi_k(4\chi_k+\ln|X(x,\chi_k,m_s)|)\phi^{\eta s\bar{s}}(\chi_k)\}, \quad (20)$$

where

$$A_u=(|u\bar{u}>+|d\bar{d}>)(\sqrt{\tfrac{1}{2}}\cos\theta+\sin\theta),\quad A_s=|s\bar{s}>(\sin\theta-\sqrt{2}\cos\theta);$$
$$X(x,\chi_k,m_q)=[1-xe^{-\chi_k}(M_\eta/m_q-e^{-\chi_k})][1-xe^{\chi_k}(M_\eta/m_q-e^{\chi_k})]^{-1} \quad (21)$$
$$(m_q: m_u,\ \text{or}\ m_s);$$
$$I_u=m_uA_u\int_0^\infty d\chi_k\chi_k\phi^{\eta u\bar{u}}(\chi_k),\quad I_s=m_sA_s\int_0^\infty d\chi_k\chi_k\phi^{\eta s\bar{s}}(\chi_k).$$

Table 1

	$\Gamma(\eta \to \gamma\gamma)$, eV	$\rho(\eta \to \gamma e^+e^-)$	$\rho(\eta \to \gamma\mu^+\mu^-)$
Theory	556,8 (m_u = 380 MeV, m_s = 520 MeV)	$1,35 \cdot 10^{-2}$	$7,5 \cdot 10^{-4}$
Experiment	(560 ± 160) [16]	$(1.32 \pm 0.32) \cdot 10^{-2}$ [6]	$(7,1 \pm 2,1) \cdot 10^{-4}$ [8]

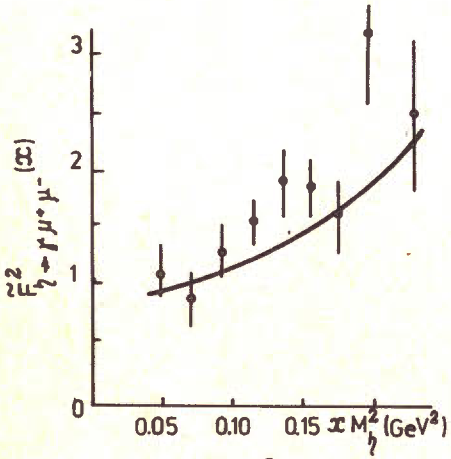

Fig.3

The numerical results for $\Gamma(\eta \to \gamma\gamma)$ and $\rho = \Gamma(\eta \to \gamma \ell\bar{\ell})/\Gamma(\eta \to \gamma\gamma)$ obtained with the use of the model WF (16) represented in Table 1 are in a satisfactory agreement with experiment. In Fig.3 the behaviour of the form factor squared of decay $\eta \to \gamma\mu^+\mu^-$ is shown, which is calculated by formula (20) with WF (16). As parameters we have used m_u = 380 MeV and m_s = = 520 MeV which give a satisfactory value for the width $\Gamma(\eta \to \gamma\gamma)$ (see Table 1). From Fig.3 a satisfactory agreement of our curve is observed with experimental data [8]. Note that the "dip-effect" of the form factor $\tilde{F}_{\eta \to \gamma\mu^+\mu^-}(x)$ is small, about 0.3% near a minimal kinematically admissible value $x_0 =$ = 0.15. In the case of decay $\eta \to \gamma e^+e^-$ the "dip"-effect amounts to 0.7% in the interval $x_0 \le x \le 0.15$.

For practical calculations of decays of the η_c-meson we assume that the wave function of the lowest state of two bound heavy c and $\bar{c}$ quarks may be approximated by the Coulomb wave function (16). As the experimental value of the width $\Gamma(\eta_c \to \gamma\gamma)$ is not known, we show in Fig.4 the dependence of the width $\Gamma(\eta_c \to \gamma\gamma)$ on the c-quark mass.

Note that for m_c = 1.57 GeV the value of $\Gamma(\eta_c \to \gamma\gamma)$ is in good agreement with the one calculated within quantum chromodynamics sum rules [17,18] and nonrelativistic quark model [19]. From the calculation by formula (9) it follows that the "dip"-effect in the form factors of decays $\eta_c \to \gamma e^+e^-$ and $\eta_c \to \gamma\mu^+\mu^-$ is theoretically permissible. The "dip"-effect in the form factor $\tilde{F}_{\eta_c \to \gamma\ell\bar{\ell}}(x)$ is predicted in the interval $x_0 \le x \le 0.025$ and amounts to about 0.4%. In Table 2 we present predictions for the width of decay $\eta_c \to \gamma\gamma$ and branching ratios $\rho = \Gamma(\eta_c \to \gamma\ell\bar{\ell})/\Gamma(\eta_c \to \gamma\gamma)$ at m_c=1.57 GeV. The predictions for ρ are in agreement with the calculations performed in ref. [12].

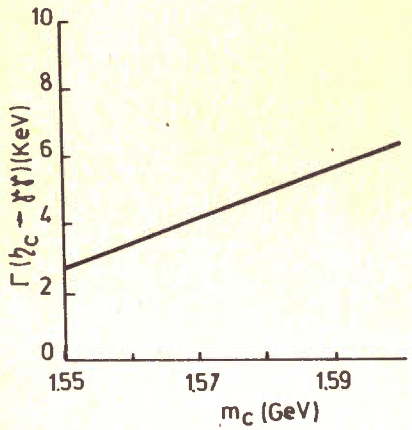

Fig.4

Table 2

Decay	$\eta_c \to \gamma\gamma$	$\eta_c \to \gamma e^+e^-$	$\eta_c \to \gamma\mu^+\mu^-$
	$\Gamma(\eta_c \to \gamma\gamma)$ = 4,24 keV	$\rho = 1,95 \cdot 10^{-2}$	$\rho = 5.52 \cdot 10^{-3}$

5. CONCLUSION

In this work we have attempted to show that the theory of bound states [2,3] as applied to $P \to \gamma\ell\bar{\ell}$ decay in the case of η and η_c mesons leads (like in [1,10]) to a theoretical possibility of the "dip"-effect in the behavior of form factors $\tilde{F}_{P \to \gamma\ell\bar{\ell}}(x)$ in the region of small x ($x_0 \le x \le 0.15$). The use of the model wave function (16) allows us to estimate this effect and to make predictions for $\Gamma(\eta_c \to \gamma\gamma)$ and $\rho = \Gamma(\eta_c \to \gamma\ell\bar{\ell})/\Gamma(\eta_c \to \gamma\gamma)$. The "dip"-effect in the behaviour of form factors of particle decays with heavy quarks $\eta \to \gamma\ell\bar{\ell}$, $\eta_c \to \gamma\ell\bar{\ell}$ is significantly smaller than for $\pi^0 \to \gamma e^+e^-$ decay. The results obtained for the widths $\Gamma(\eta \to \gamma\gamma)$, $\Gamma(\eta \to \gamma\ell\bar{\ell})$, and form factor $\tilde{F}_{\eta \to \gamma\mu^+\mu^-}(x)$ are in good agreement with available experimental data [16,6,8].

The authors are thankful to G.V.Efimov and A.V.Sidorov for useful discussions.

REFERENCES

1. Kozlov G.A. et al. Z.Phys., 1983, 21C, p.63.
2. Logunov A.A., Tavkhelidze A.N. Nuovo Cim., 1963, 29, p.380; Kadyshevsky V.G. Soviet Journ. ZhETh, 1964, 46, p.654,872; Logunov A.A. et al. Sov.Journ.Teor.i Mat.Fiz., 1971,6,p.157.
3. Faustov R.N. Ann.of Phys., 1973, 78, p.176.
4. Samios N.P. et al. Phys.Rev., 1961, 121, p.275.
5. Kobrak H. et al. Nuovo Cim., 1961,20,p.1115;Devons S. et al. Phys.Rev., 1969, 184, p.1356.
6. Jane M.R. et al. Phys.Lett., 1975, 59B, p.103.
7. Fischer J. et al. Phys.Lett., 1978, 73B, p.359.
8. Djhelyadin R.I. et al. Phys.Lett., 1980, 94B, p.548.
9. Efimov G.V. Ivanov M.A., Muradov R.H. JINR, P2-13007, Dubna, 1980.
10. Kozlov G.A. et al. JINR, E2-83-630, Dubna, 1983.
11. Savrin V.I., Skachkov N.B. Lett.Nuovo Cim., 1980, 29,p.363.
12. Bergström L., Snellman H. Z.Phys., 1981, 8C, p.363.
13. Kapshay V.N., Skachkov N.B. JINR, E2-81-618, Dubna, 1981.
14. Kuleshov S.P., Sidorov A.V., Skachkov N.B. JINR, P2-82-7, Dubna, 1982; Kapshay V.N., Kuleshov S.P., Skachkov N.B. JINR, P2-82-271, Dubna, 1982.
15. Kapshay V.N. et al. Sov.Journ.Teor.i Mat.Fiz., 1982, 53, p.388.
16. Weinstein A. et al. Phys.Rev., 1983, 28D, p.2896.
17. Shifman M.A., Vainstein A.I., Zakharov V.I. Nucl.Phys., 1979, B147, p.385,448; Shifman M.A. Z.Phys., 1980,C4,p.345; Reinders L.J., Rubinstein H.R., Yazaki S. Phys.Lett., 1982, 113B, p.411.
18. Kirschner R., Schiller A. Z.Phys., 1982, 16C, p.141.
19. Paschalis J.E., Gounaris G.J. Nucl.Phys., 1983, B222,p.473.

nt

Козлов Г.А. и др. E2-84-256

К вопросу о поведении формфакторов распадов псевдоскалярных систем $P \to \gamma \ell \bar{\ell}$ в области малых инвариантных масс лептонной $\ell \bar{\ell}$ -пары

В рамках релятивистской теории связанных состояний исследуется поведение формфакторов распадов $\eta \to \gamma \ell \bar{\ell}$, $\eta_c \to \gamma \ell \bar{\ell}$. Результаты вычислений ширин распадов $\eta \to \gamma\gamma$ и $\eta \to \gamma \ell \bar{\ell}$, а также формфактора распада $\eta \to \gamma \mu^+ \mu^-$, находятся в хорошем согласии с экспериментальными данными. Дается теоретическое предсказание ширин распадов $\eta_c \to \gamma\gamma$ и $\eta_c \to \gamma \ell \bar{\ell}$.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1984

Kozlov G.A. et al. E2-84-256

On the Behaviour of Form Factors of Decays of Pseudoscalar Systems $P \to \gamma \ell \bar{\ell}$ in the Region of Small Invariant Masses of a Lepton $\ell \bar{\ell}$ -Pair

The relativistic theory of bound states is applied to studying the behavior of form factors of decays $\eta \to \gamma \ell \bar{\ell}$, $\eta_c \to \gamma \ell \bar{\ell}$. The calculated widths of decays $\eta \to \gamma\gamma$ and $\eta \to \gamma \ell \bar{\ell}$ and form factor of decay $\eta \to \gamma \mu^+ \mu^-$ are in good agreement with experiment. A theoretical prediction is given for the width of decays $\eta_c \to \gamma\gamma$ and $\eta_c \to \gamma \ell \bar{\ell}$.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1984