объединенный
институт
ядерных
исследований
дубна

E2-84-230

V.A.Nesterenko, A.V.Radyushkin

QCD SUM RULE ANALYSIS OF THE PION FORM FACTOR BEHAVIOUR IN SMALL-Q^2 REGION

Submitted to "Письма в ЖЭТФ"

1984

QCD sum rules [1] are now practically the only quantitative approach in QCD that works in situations when the major role is played by nonperturbative effects. This method was applied first to various two-point functions $\pi(p^2)$ (correlators) and enabled one to calculate masses and leptonic widths of the mesons [1] as well as similar characteristics of the baryons [2]. In refs. [3,4] the method was extended onto three-point functions. In particular, analysing the amplitude

$$T(Q^2, p_1^2, p_2^2) = \frac{n_\alpha n_\beta n_\mu}{2(nP)^3} i^2 \int d^4x \, d^4y \, \langle 0 | T\{ j_\alpha^+(-\frac{y}{2}) j_\beta(\frac{y}{2}) J_\mu(x)\} | 0 \rangle \cdot \exp(i \frac{p_1 + p_2}{2} y - iqx), \quad (1)$$

where $j_\alpha = \bar{u}\gamma_5\gamma_\alpha d$, $J_\mu = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d$, n is a light-like vector orthogonal to $q = p_2 - p_1$: $n^2 = 0$, $(nq) = 0$, $(np_1) = (np_2) \equiv (nP)$ in the symmetric kinematics $|p_1^2| \sim |p_2^2| \sim Q^2 \gtrsim m_\rho^2$ (where $Q^2 = -q^2$), the authors of refs. [3,4] calculated the behaviour of the pion electromagnetic form factor for intermediate momentum transfers: $m_\rho^2 \lesssim Q^2 \lesssim$ 3-4 GeV^2. However, the approach [3,4] is not directly applicable in the small Q^2 region because there are contributions in $T(Q^2, p_1^2, p_2^2)$ that essentially depend on the dynamics on the distances $\sim 1/Q$. If $Q^2 \lesssim m_\rho^2$ = 0.6 GeV^2, then there is no justification to use the asymptotic freedom (i.e., perturbation theory) for estimates of the magnitude of such contributions.

To improve the estimates (i.e., to take into account the nonperturbative effects), one should perform an additional factorization of the contributions due to short and long distances. The problems of such a type were considered earlier [5-7] when the QCD sum rules were applied to the nucleon magnetic moments [5,6] and the g_A constant [7]. Our approach to the factorization is based on the analysis of the asymptotic behaviour of the Feynman diagrams developed in ref. [8]. Using this technique, it is easy to establish that in the kinematics $|p_1^2| \sim |p_2^2| \sim Q^2$ the power behaviour of $T(Q^2, p_1^2, p_2^2)$ with respect to $1/p_1^2$, $1/p_2^2$ may result not only from the region where all the intervals $(y^2, (x + y/2)^2, (x - y/2)^2)$ are small, but also from the region where y^2 is small, while $(x + y/2)^2$ and $(x - y/2)^2$ are large.

The latter possibility gives additional contributions of the structure $C(p^2)[\pi(q^2) - \pi^{pert}(q^2)]$, where $\pi(q^2)$ is an "exact" correlator of some composite operator with the electromagnetic current J while $\pi^{pert}(q^2)$ is its pertirbative counterpart (cf. ref. [6]). The addition of such a term corresponds, evidently, to the substitution of a perturbative estimate for the contribution $C(p^2)\pi(q^2)$ (that has an essential dependence on the 1/Q-dynamics) by an expression, in which the nonperturbative effects are explicitly taken into account. Thechnically this is achieved with the help of the dispersion relation

$$\pi(q^2) - \pi^{pert}(q^2) = \frac{1}{\pi}\int_0^\infty \frac{ds}{s - q^2}(\rho(s) - \rho^{pert}(s)), \quad (2)$$

where $\rho(s)$ is the "exact" spectral density and $\rho^{pert}(s)$ is the perturbative one. The absence of subtractions in eq. (2) is due to the fact that $\pi(q^2) - \pi^{pert}(q^2) \to 0$ as $|q^2| \to \infty$, owing to the asymptotic freedom (see ref. [9], Appendix C). In actual calculations we used the model representation for $\rho(s)$:

$$\rho(s) = f\,\delta(s - m_\rho^2) + \rho^{pert}(s) \cdot \theta(s > s_0)$$

with the parameters f, s_0 extracted in a standard way [1] from the QCD sum rule for the relevant correlator $\pi(q^2)$. In the case $\rho^{pert}(s) = 0$ such an approximation is equivalent to the vector dominance model (cf. ref. [6]). However, for the $\langle\bar{\psi}\psi\rangle^2$-contributions (that are significant numerically) it makes sense to improve this approximation parametrizing the higher states by an effective ρ'-resonance, i.e., to take $\rho(s) f_1\,\delta(s - m_\rho^2) + f_2\,\delta(s - m_{\rho'}^2)$ [10].

As a result, we obtained the following sum rule (SR) for $B(p^2, M^2) T(Q^2, p^2, p^2)$ (where $B(p^2, M^2)$ is the SVZ-Borel transformation [1]):

$$\frac{f_\pi^2 F_\pi(Q^2)}{M^4} + \frac{c_\pi(Q^2)}{M^2} + e^{-m_{A_1}^2/M^2}\left\{\frac{f_{A_1}^2 F_{A_1}(Q^2)}{M^4} + \frac{c_{A_1}(Q^2)}{M^2}\right\} + \ldots =$$

$$= \frac{3}{2\pi^2 M^2}\int_0^1 dx \int_0^1 d\xi\, x(1 - x)\exp\left(-\frac{x}{1 - x}\xi(1 - \xi)\frac{Q^2}{M^2}\right) +$$

$$+ \frac{1}{40\pi^2 M^6}\left[Q^4 \ln\left(\frac{Q^2}{Q^2 + s_0}\right) + Q^2 s_0 - \frac{s_0^2}{2}\frac{Q^2}{Q^2 + m_\rho^2}\right] + \frac{\alpha_s\langle G_{\mu\nu}^a G_{\mu\nu}^a\rangle}{24\pi M^6}\left(1 + \left[\frac{m_\rho^2}{Q^2 + m_\rho^2}\right]\right) + \quad (3)$$

$$+ \frac{16}{81}\frac{\pi\alpha_s\langle\bar{\psi}\psi\rangle^2}{M^8}\left(5 + 6\left[\frac{1{,}6 m_\rho^2}{Q^2 + m_\rho^2} - \frac{0{,}6 m_{\rho'}^2}{Q^2 + m_{\rho'}^2}\right]\right),$$

where s_0 = 1.5 GeV^2, $m_{\rho'}^2$ = 2.0 GeV^2, $m_{A_1}^2$ = 1.6 GeV^2.

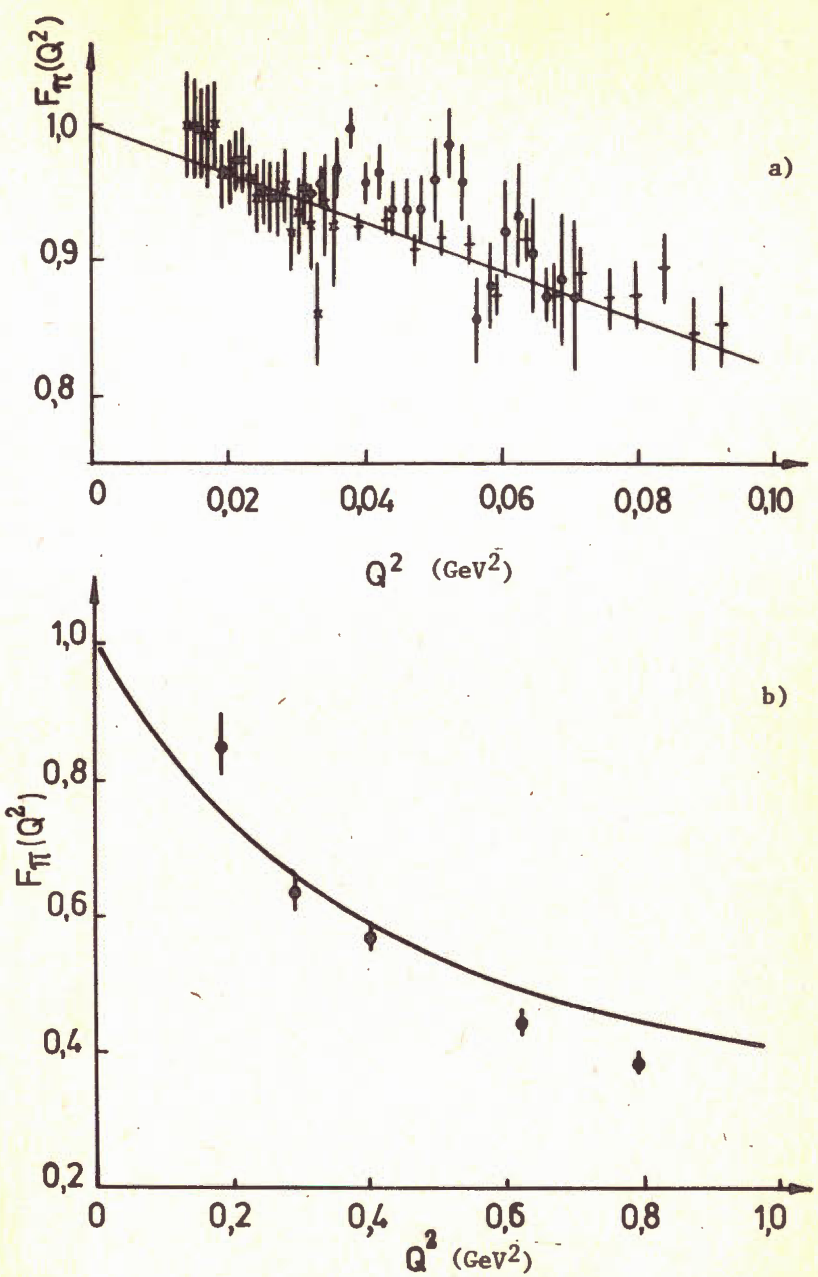
Let us concentrate on two points.

a) For $Q^2 = 0$ the r.h.s. of eq.(3) (multiplied by M^2) coincides with the expansion for the correlator $\pi \sim n^\alpha n^\beta \langle j_\alpha^+ j_\beta \rangle / (nP)^2$ of two axial currents, and hence $F_\pi(0) = 1$, $F_{A_1}(0) = 1$ and $c_\pi(0) = c_i(0) = 0$. This fact is a direct consequence of the Ward identity for T (cf.ref./7/). Subtracting from eq.(3) the SR for $Q^2 = 0$ we obtain the SR that allows one to calculate $(F_\pi(Q^2) - 1)/Q^2$. Using the standard procedure of handling the SR of such a type/5-7/ we extracted the value of the pion electromagnetic radius $\langle r_\pi^2 \rangle^{1/2} = 0.66 \pm 0.03$ fm that is in good agreement with the value $\langle r_\pi^2 \rangle^{1/2}_{exp} = 0.636 \pm 0.037$ fm (see figure a).

b) The above-mentioned additional contributions (in eq.(3) they are those in square brackets) must "die away" as Q^2 grows. It is easy to observe that this requirement is fulfilled in eq.(3). Note, however. that one cannot justify the extrapolation of the specific $1/Q^2$-law of the decrease beyond the region $Q^2 \lesssim m_\rho^2$ of formal applicability of SR (3), because the operator expansion for $T(Q^2, p_1^2, p_2^2)$ in the symmetric kinematics $Q^2 \sim |p^2|$ does not contain the $O(1/Q^2)$ contributions (as well as higher $(1/Q^2)^N$ ones) in $\langle GG \rangle$ and $\langle \bar\psi\psi \rangle^2$-terms. In other words, the additional contributions should vanish for $Q^2 \geq m_\rho^2$ faster than any power of $1/Q^2$. Eq.(3) overestimates these terms and, as a result, the extracted value of $F_\pi(Q^2 = 0.6\ GeV^2)$ is 10% as high as the experimental one (see figure b). On the other hand, if all the additional terms are set to zero (this corresponds to the use of the SR derived in the symmetric kinematics $Q^2 \sim |p^2| \geq m_\rho^2$), then $F_\pi(Q^2 = 0.6\ GeV^2)$ is 10% as small as the experimental value. This means that the SR corresponding to the two kinematic situations ($Q^2 \ll m_\rho^2$ and $Q^2 \gtrsim m_\rho^2$) within the necessary accuracy of 10-20% agree for $Q^2 = m_\rho^2$ both with each other and with the experimental curve.

Thus, the QCD sum rule approach allows one to get a good description of the experimental data for the pion electromagnetic form factor practically for the whole experimantally investigated region of momentum transfers $0 < Q^2 \lesssim 3\text{-}4\ GeV^2$.

In the course of our calculations we have been informed about the result obtained in ref./13/. The authors of ref./13/ attempted to calculate $\langle r_\pi^2 \rangle$ using a method similar to ours. However, they ignored the $\langle GG \rangle$ and $\langle \bar\psi\psi \rangle^2$-contributions, the calculation of which is in fact the most complicated part of our work. The inclusion of these terms increases the value of $\langle r_\pi^2 \rangle$ obtained in ref./13/ by a factor of 1,5, but the most important point is that the neglect of these corrections contradicts the basic principles of the QCD sum rule approach, since it is just these terms that accumulate information on the dynamics of the system investigated.



Comparison between experimental data for $F_\pi(Q^2)$ and theoretical results based on the SR(3). a) region $Q^2 \leq 0.1\ GeV^2$, data from ref./11/ ; b) region $Q^2 \leq 1$ GeV data from ref./12/.

We are most grateful to K.G.Chetyrkin, S.G.Gorishny, and S.A.Larin for stimulating discussions and to A.V.Efremov for interest in our work.

REFERENCES

1. Shifman M.A., Vainshtein A.I., Zakharov V.I. Nucl.Phys.B, 1979, 147, p.p.385,448.
2. Ioffe B.L. Nucl.Phys.B, 1981, 188, p.317.
3. Nesterenko V.A., Radyushkin A.V. JETP Lett., 1982,35,p.486; Phys.Lett.B, 1982, 115, p.410.
4. Ioffe B.L., Smilga A.V. Phys.Lett.B, 1982, 114, p.353.
5. Ioffe B.L., Smilga A.V. Pis'ma JETF, 1983, 37, p.250.
6. Balitsky I.I., Yung A.V. Phys.Lett.B, 1983, 129,p.388.
7. Belyaev V.M., Kogan Ya.I. Pis'ma JETF, 1983, 37, p.611.
8. Efremov A.V., Radyushkin A.V. Teor.Mat.Fiz., 1980,44,p.p. 17, 157, 327; 42, p.147; Radyushkin A.V. Fiz.Elem.Chast. i. At.Yadra., 1983, 14, p.58.
9. Vainshtein A.I. Fiz.Elem.Chastits i At.Yadra, 1982, 13, p.542; Novikov V.A. et al. Nucl.Phys.B, 1981, 191, p.301.
10. Belyaev V.M., Kogan Ya.I. Preprint ITEP-12, M., 1984.
11. Vodopianov A.S., Tsyganov E.N. Fiz.Elem.Chast.i At.Yadra. 1984, 15, p.5.
12. Bebek C. et al. Phys.Rev.D., 1978, 17, p.1693.
13. Chetyrkin K.G. et al. Preprint INR, P-337, M., 1984.

Received by Publishing Department
on April 10,1984.

Нестеренко В.А., Радюшкин А.В. E2-84-230

Анализ поведения формфактора пиона при малых Q^2 методом КХД правил сумм

Получено КХД правило сумм для электромагнитного формфактора пиона в области малых пространственно-подобных передач импульса $Q^2 < m_\rho^2$. Найденное значение электромагнитного радиуса пиона $<r_\pi^2>^{1/2} = 0,66 \pm 0,03$ фм хорошо согласуется с экспериментальными данными.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1984

Nesterenko V.A., Radyushkin A.V. E2-84-230

QCD Sum Rule Analysis of the Pion Form Factor Behavior in Small-Q^2 Region

The QCD sum rule is derived for the pion electromagnetic form factor in the region of small space-like momentum transfers $Q^2 \lesssim m_\rho^2$. The obtained value of the pion electromagnetic radius $<r_\pi^2>^{1/2} = 0.66 \pm 0.03$ fm is in good agreement with experimental data.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1984