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SOME IMPLICATIONS OF THE CP-INVARIANCE FOR MIXING OF MAJORANA NEUTRINOS

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1. The problem of neutrino masses and neutrino mixing continues to be widely discussed [1]. In the present paper we shall consider some implications of CP-invariance in the leptonic sector for schemes with mixing of massive Majorana neutrinos [2] (i.e., schemes with Majorana [3] or Dirac and Majorana [4-8] mass terms). The mixing of neutrinos with Majorana masses in the case of CP-invariance has been discussed by several authors [9-12]. However, the CP-transformation properties of the Majorana fields have not always been correctly taken into account. This has led to some contradictory and ambiguous statements in the literature on the subject.

The leptonic part of the standard charged-current weak-interaction Lagrangian has the form*

$$\mathcal{L}_I = i\frac{g}{\sqrt{2}} \sum_{\ell=e,\mu,\tau\ldots} \bar{\ell}_L \gamma_\alpha \nu_{\ell L} W_\alpha + h.c. \quad (1)$$

We shall assume that

$$\nu_{\ell L} = \sum_k U_{\ell k} \chi_{kL}, \quad (2)$$

where U is a unitary mixing matrix, χ_k is the field of a neutrino with Majorana mass m_k and ℓ is the field of a charged lepton with mass m_ℓ. The index k in eq. (2) runs from 1 to n in the case of Majorana mass term and from 1 to $2n$ if the neutrino mass Lagrangian contains both Dirac and Majorana terms. The neutrino mass term has the canonical form

$$\mathcal{L}_M = -\frac{1}{2}\sum_k \bar{\chi}_k \chi_k m_k, \quad (3)$$

where the field χ_k satisfies the Majorana condition

$$C\bar{\chi}_k^T(x) = \xi_k \chi_k(x). \quad (4)$$

Here C is the charge conjugation matrix ($C^+C = 1$, $C^T = -C$ and $C\gamma_\alpha^T C^{-1} = -\gamma_\alpha$). We shall choose $\xi_k = \pm 1$ (see the discussion at the end of this section).

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*The general case of n lepton flavours will be considered.

It should be emphasized that the factors ξ_k have no physical meaning as the weak interaction Lagrangian is not invariant with respect to the charge conjugation. Condition (4) implies only that $\chi_k(x)$ is the field of a truely neutral particle with two helicity states.

It is obvious that the mass term (3) is CP-invariant. Let us impose now the requirement of CP-invariance on the Lagrangian (1). Under the CP-transformation we have

$$U_{CP}\chi_k(x)U_{CP}^{-1} = \eta_k^{CP}\gamma_4\chi_k(x'). \quad (5)$$

Here $x' = (-\vec{x}, ix_0)$ and η_k^{CP*} is the CP-parity of the Majorana neutrino with mass m_k. Is should be stressed that the CP-parities of Majorana particles can take values* $\pm i$. From eqs. (4) and (5) we get for the left-handed components of the Majorana fields

$$U_{CP}\chi_{kL}(x)U_{CP}^{-1} = \eta_k^{CP}\xi_k\gamma_4 C\bar{\chi}_{kL}^T(x'). \quad (6)$$

Because of the phase arbitrariness inherent to the Dirac fields, their CP-phase factors can always be set equal to unity

$$U_{CP}\ell_L(x)U_{CP}^{-1} = \gamma_4 C\bar{\ell}_L^T(x'). \quad (7)$$

One also has

$$U_{CP}W_\alpha(x)U_{CP}^{-1} = W_\alpha^+(x'). \quad (8)$$

The requirement of CP-invariance leads to the following constraint on the lepton mixing matrix

$$U_{\ell k}\eta_k^{CP}\xi_k = U_{\ell k}^*, \quad (9)$$

where we have made use of eqs. (1), (2) and (6)-(8). It is convenient to write the CP-parity of the Majorana neutrinos as

$$\eta_k^{CP} = i\eta_k, \quad (10)$$

where $\eta_k = \pm 1$. For the lepton mixing matrix we obtain then

$$U_{\ell k} = O_{\ell k}\, e^{-i\pi/4\,\xi_k\eta_k}, \quad (11)$$

where O is a real orthogonal matrix.

Indeed, we get from (5) $U_{CP}C\bar{\chi}_k^T(x)U_{CP}^{-1} = -\eta_k^{CP*}\gamma_4 C\bar{\chi}_k^T(x')$. By using eq. (4) one finds $U_{CP}\chi_k(x)U_{CP}^{-1} = -\eta_k^{CP*}\gamma_4\chi_k(x')$. Comparing this relation with (5) we obtain $\eta_k^{CP} = -\eta_k^{CP*}$. Thus, $\eta_k^{CP} = \pm i$

Thus, we see that if the CP-invariance holds in the leptonic sector, the phases of the elements of the mixing matrix can take values $\pm\pi/4$ (under the conventions used), the sign being determined by CP-parities of the Majorana fields and by the factors ξ_k in the Majorana condition (4). We are going to show now that the measurable quantities may depend only on the (relative) CP-parities of the massive Majorana neutrinos.

As is well known, the problem of neutrino masses and neutrino mixing is studied in the experiments searching for neutrino oscillations and neutrinoless double β-decay as well as in the experiments aimed at a direct determination of neutrino masses (measurements of the electron spectrum in the tritium β-decay, etc.).

We shall discuss first the oscillations of neutrinos. Let us assume that at some initial moment $t = 0$ a beam of neutrinos ν_ℓ (antineutrinos $\bar\nu_\ell$) with momentum $\vec p$ is formed. In the case of neutrino mixing (2), the probabilities to find in this beam neutrino $\nu_{\ell'}$ (antineutrino $\bar\nu_{\ell'}$) at moment t are given by the standard expressions /2/:

$$P_{\nu_{\ell'};\nu_\ell}(t) = |\sum_k U_{\ell' k}\, e^{-iE_k t}\, U^*_{\ell k}|^2$$
$$P_{\bar\nu_{\ell'};\bar\nu_\ell}(t) = |\sum_k U^*_{\ell' k}\, e^{-iE_k t}\cdot U_{\ell k}|^2 , \qquad (12)$$

where $E_k = \sqrt{\vec p^{\,2} + m_k^2}$.
It follows from (11) that in the case of CP-invariance we consider

$$P_{\nu_{\ell'};\nu_\ell}(t) = |\sum_k O_{\ell' k}\, e^{-iE_k t}\, O_{\ell k}|^2 = P_{\bar\nu_{\ell'};\bar\nu_\ell}(t) . \qquad (13)$$

Hence, the phases of the mixing matrix elements do not enter into the expressions for the probabilities of the transitions* $\nu_\ell \to \nu_{\ell'}$ and $\bar\nu_\ell \to \bar\nu_{\ell'}$.

We would like to note that the equality of the $\nu_\ell \to \nu_{\ell'}$ and $\bar\nu_\ell \to \bar\nu_{\ell'}$ transition probabilities, obtained for the case of Dirac /13/ and of Majorana /5,9,14/ mass terms, takes place also in the case of Dirac and Majorana mass term for any values of the CP-parities of the massive Majorana neutrinos.

Let us turn now to the neutrinoless double β-decay ($(\beta\beta)_{0\nu}$ -decay). The neutrino fields enter into the S-matrix element of this process in the following way (see, e.g., refs. /15,16/)

*Obviously, this conclusion is valid for the probability of transition of ν_ℓ into a sterile antineutrino $\bar\nu_{\ell' L}$ as well.

$$A(x_1, x_2) = \sum_k U^2_{ek}\, \chi_{kL}(x_1)\, \chi^T_{kL}(x_2) . \qquad (14)$$

Taking eq. (4) into account, we obtain

$$A(x_1, x_2) = -\sum_k U^2_{ek}\, \xi_k\, \frac{1+\gamma_5}{2}\, \chi_k(x_1)\, \bar\chi_k(x_2)\, \frac{1+\gamma_5}{2}\, C . \qquad (15)$$

This implies that the $(\beta\beta)_{0\nu}$ -decay amplitude is proportional to $\sum_k U^2_{ek}\, \xi_k m_k$. Using eq. (9) we get:

$$\sum_k U^2_{ek}\, \xi_k m_k = -\sum_k O^2_{ek}\, \eta^{CP}_k m_k . \qquad (16)$$

Hence the $(\beta\beta)_{0\nu}$ -decay amplitude depends on the relative CP-partities of the massive Majorana neutrinos (and does not depend on the factors ξ_k in the Majorana condition (4)!). As a consequence, the contributions of the massive Majorana neutrinos with opposite CP-parities to the $(\beta\beta)_{0\nu}$ -decay probability tend to compensate each other. The possibility of such a compensation in the case of CP-invariance was discussed first by L.Wolfenstein /10/.

Let us note that a similar compensation can take place also if right-handed charged currents exist. To be more specific, we shall choose the following form /15/ of the leptonic part of the charged current weak interaction Lagrangian:

$$\mathcal{L} = \frac{i}{\sqrt 2} \sum_{\ell, k} (g_L \bar\ell_L \gamma_\alpha U^L_{\ell k}\, \chi_{kL}\, W^L_\alpha + g_R \bar\ell_R \gamma_\alpha U^R_{\ell k}\, \chi_{kR}\, W^R_\alpha) + \text{h.c.} \qquad (17)$$

Here $U^{L,R}$ and $g_{L,R}$ are the appropriate mixing matrices and coupling constants. The CP-invariance implies in this case

$$U^{L,R}_{\ell k}\, \xi_k\, \eta^{CP}_k = U^{L,R*}_{\ell k} . \qquad (18)$$

It is easy to see that the contribution to the $(\beta\beta)_{0\nu}$ -decay amplitude from the interference of the left- and right-handed currents is proportional to the factor

$$\sum_k |U^L_{ek}|\, |U^R_{ek}|\, \eta^{CP}_k , \qquad (19)$$

and therefore the Majorana neutrinos with opposite CP-partities give again mutually compensating contributions.

As for the experiments aimed at a direct determination of the neutrino masses, the quantities measured in these experiments are independent of the phases in the lepton mixing matrix

U (only $|U_{ek}|^2$ enter the expression for the electron spectrum in the tritium β-decay, etc.).

We would like to conclude this section with some remarks concerning the Majorana condition. It has been assumed earlier that the factors ξ_k in eq. (4) take values ± 1. However, the fields χ_k may obey a more general Majorana condition in which ξ_k are arbitrary phase factors. It is not difficult to see that in the case of CP-invariance under consideration these phase factors are physically irrelevant. Indeed, instead of eq. (9) we now have

$$U_{\ell k}\eta_k^{CP}\xi_k^* = U_{\ell k}^* \qquad (20)$$

and, for example, the $(\beta\beta)_{0\nu}$ -decay amplitude is proportional to $\sum_k U_{ek}^2 \xi_k^* m_k = \sum_k |U_{ek}|^2 \eta_k^{CP*} m_k$.

It should also be added that if CP is not conserved, the phase factors ξ_k can include some of the phase parameters associated with the CP-violation and can be relevant physically. Indeed, in the case of CP-nonconservation the mixing matrix can be written as

$$U = U_D S. \qquad (21)$$

Here the matrix U_D has the form of a mixing matrix for massive Dirac neutrinos containing $(n-1)(n-2)/2$ CP-violating phases and $S_{mk} = \delta_{mk} e^{i\beta_k}$, where β_k are the specific CP-violating parameters associated with massive Majorana neutrinos /5,7-9/. Instead of χ_k $(C\bar\chi_k^T = \chi_k)$ one can introduce the fields

$$\phi_k = e^{i\beta_k}\chi_k. \qquad (22)$$

They satisfy the Majorana condition

$$C\bar\phi_k^T = \xi_k\phi_k, \qquad (23)$$

where

$$\xi_k = e^{-2i\beta_k}. \qquad (24)$$

Written in terms of the fields ϕ_k the charged lepton current has the form (1) with U_D playing the role of lepton mixing matrix, and the CP-violating phases typical for Majorana neutrinos can appear in the amplitudes of the physical processes only through the Majorana condition (23). For the case of two flavours this possibility was considered in detail in ref /17/.

2. As we have seen, the requirement of CP-invariance of the interaction Lagrangian (1) leads to constraint (9) for the

elements of the lepton mixing matrix. It is easy to see that in this case the current neutrino fields $\nu_{\ell L}$ have definite CP-transformation properties:

$$U_{CP}\nu_{\ell L}(x)U_{CP}^{-1} = \gamma_4 C\bar\nu_{\ell L}^T(x'). \qquad (25)$$

Usually, however, one starts with eq. (25) (the weak interaction Lagrangian (1) is then automatically CP-invariant) and imposes the requirement of CP-invariance on the neutrino mass Lagrangian, written in terms of the fields $\nu_{\ell L}$. We shall compare next these two approaches.

As an example consider the Majorana mass term

$$\mathcal{L}_M = -\frac{1}{2}\overline{\nu_L^C} M \nu_L + h.c. \qquad (26)$$

Here $\nu_L = \begin{pmatrix}\nu_{\ell L}\\ \nu_{\mu L}\\ \vdots\end{pmatrix}$, M is a complex symmetric matrix and $\nu_L^C(x) = C\bar\nu_L^T(x)$. Our considerations can be easily extended to the case of Dirac and Majorana mass term.

The condition of CP-invariance of the mass Lagrangian (26) implies*

$$M^+ = -M. \qquad (27)$$

Taking into account that $M^T = M$ we have

$$M = iOm'O^T, \qquad (28)$$

where O is a real orthogonal matrix and m' is a real diagonal matrix, $m'_{ik} = \delta_{ik} m'_k$. The mass eigenvalues m'_k can be either positive or negative. One has

$$m'_k = m_k\rho_k, \qquad (29)$$

where $m_k = |m'_k|$, while $\rho_k = \pm 1$. Taking into account (28) and (29), we can rewrite the mass matrix M in the form

$$M = (U^+)^T m U^+, \qquad (30)$$

where

$$U_{\ell k} = O_{\ell k} e^{-i\pi/4\rho_k}. \qquad (31)$$

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* The matrix M will be hermitian if the CP-phases of the fields $\nu_{\ell L}$ are chosen to be equal to i.

From (26) and (30) we get

$$\mathcal{L}_M = -\frac{1}{2}\sum_k \overline{\chi_k}\,\chi_k\, m_k \,. \qquad (32)$$

Here

$$\chi_k = (U^+\nu_L)_k + (U^+\nu_L)_k^C = C\,\bar{\chi}_k^T \qquad (33)$$

is a Majorana field with mass m_k.

It is not difficult to convince oneself that the CP-parity of the field χ_k is $i\rho_k$. Indeed, from (33) and (25) we obtain

$$U_{CP}\,\chi_{kL}(x)\,U_{CP}^{-1} = \sum_\ell U_{\ell k}^*\,\gamma_4\, C\,\bar{\nu}_{\ell L}^T(x'). \qquad (34)$$

Further, using eq. (31) one finds that

$$U_{CP}\,\chi_{kL}(x)\,U_{CP}^{-1} = i\rho_k\,\gamma_4\, C\,\bar{\chi}_{kL}^T(x'). \qquad (35)$$

By construction of the fields $\chi_k(x)$, $\xi_k = 1$. Comparing (35) and (5) we have

$$\eta_k^{CP} = i\rho_k \,. \qquad (36)$$

Thus, the signs of eigenvalues of the neutrino mass matrix determine the CP-parities of the corresponding massive Majorana neutrinos*.

As we have seen earlier, the phases of the mixing matrix elements depend not only on the CP-parities of the massive Majorana neutrinos but also on the factors ξ_k, associated with the freedom in the choice of the Majorana fields. That freedom can be used to make the lepton mixing matrix real/10/. Indeed, according to eq. (11), this is realized by the choice**

$$\xi_k = \rho_k \,. \qquad (37)$$

In order to have eq. (37) fullfilled, we introduce the fields

$$\chi_k' = e^{i\pi/4(1-\rho_k)}\,\chi_k \,. \qquad (38)$$

* This statement was made first in ref./10/. However, it was based on some not quite accurate definitions.

** Such a choice was implicitly made in refs./10,18/. In this case the CP-parities of the Majorana neutrinos appear, e.g., in the $(\beta\beta)_{0\nu}$-decay and neutrino radiative decay amplitudes only through the Màjorana condition.

One has

$$C\,\bar{\chi}_k'^T = \rho_k\,\chi_k' \,. \qquad (39)$$

As can be easily seen from (5), the CP-parities of the fields χ_k and χ_k' coincide. Further, it follows from eqs. (33) and (38) that

$$\chi_k' = (U'^+\nu_L)_k + \rho_k\,(U'^+\nu_L)_k^C \,, \qquad (40)$$

where

$$U_{k\ell}'^+ = e^{i\pi/4}\, O_{\ell k} \,. \qquad (41)$$

From (40) we get

$$\nu_{\ell L} = \sum_k U_{\ell k}'\,\chi_{kL}' \,. \qquad (42)$$

Thus, the elements of the mixing matrix differ from those of an orthogonal matrix by an insignificant common phase factor* $e^{-i\pi/4}$.

3. To summarize, the mixing of neutrinos with Majorana masses (Majorana or Dirac and Majorana mass terms) has been considered under the assumption of CP-invariance in the leptonic sector. It was shown that

i) the phases of elements of the lepton mixing matrix are determined by the CP-parities of the massive majorana fields as well as by arbitrary sign factors in the relevant Majorana conditions. The observable quantities do not depend on these latter factors.

ii) The CP-parities of the Majorana neutrinos have no effect on the neutrino oscillations. The $(\beta\beta)_{0\nu}$ -decay rates depend on the relative CP-parities of massive Majorana neutrinos. This statement remains valid in the schemes with right-handed currents as well.

iii) The CP -parities of the massive Majorana neutrinos are specified by the signs of the neutrino mass matrix eigenvalues, and are independent of the choice of the sign factors in Majorana conditions.

In conclusion we would like to thank B.M.Pontecorvo, J.Hošek and F.Niedermayer for useful discussions.

* The phase factor $e^{-i\pi/4}$ ensures, however, correct CP-transformation properties for the Majorana fields χ_k'. It vanishes if the CP-phases of the current fields $\nu_{\ell L}(x)$ are chosen to be equal to i.

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СР-инвариантность в лептонном секторе и смешивание нейтрино с майорановскими массами

Рассматривается смешивание нейтрино с майорановскими массами в предположении, что имеет место СР-инвариантность в лептонном секторе. Показано, что фазы элементов матрицы смешивания определяются как СР-четностями нейтрино Майораны, так и произвольными знаковыми множителями ξ_k,входящими в условия, которым удовлетворяют поля Майорана. Показано также, что вероятности безнейтринного двойного β-распада зависят только от относительных СР-четностей нейтрино с майорановскими массами /и не зависят от ξ_k / и что СР-четности нейтрино Майораны не входят в выражения для осцилляций нейтрино.

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Some Implications of the CP-Invariance for Mixing of Majorana Neutrinos

Some implications of CP-invariance for the mixing of neutrinos with Majorana masses are discussed. It is shown that the phases of the mixing matrix elements are determined both by the CP-partities of the Majorana neutrinos and by arbitrary sign factors ξ_k in the relevant Majorana conditions. It is also shown that the neutrinoless double β-decay rates depend on the relative CP-parities of the massive Majorana neutrinos but do not depend on ξ_k. The CP-parities have no effect on the neutrino oscillations.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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