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QUANTUM SUPERTWISTORS AND FUNDAMENTAL SUPERSPACES

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Introduction

Supersymmetry [1] plays an important role in a recent field theory. Two main approaches exist to the construction of supersymmetric theories: a component and a superfield (superspace) one. In the first of them the supersymmetry is realized on fields while in the second directly on the superspaces.

The superspace approach arises rather naturally and perhaps has some conceptual advantages compared with the component approach, in particular, an explicitly covariant geometrical formulation of the supersymmetric theories. An important problem of this approach is to find such superspaces that allow to formulate the supersymmetric theory in terms of unconstrained superfields. This problem is solved only in the case of N=1 supersymmetry [2] when such a superspace turns out to be N=1 chiral superspace $C^{4,2}$. A crucial role in selecting this space comes from the requirement of existence of an irreducible chiral representation in a curved superspace [3].

Recently a chiral approach has been developted to supersymmetric theories. It is founded on the assumption of the simplest structure elements of superspace which are called "twistors" [4] for the usual space. The first papers on supertwistors [5,6] are devoted to solve classical equations. The structure approach is apparently more fundamental than the superspace one, and therefore an idea arises to use the structure approach for selecting extended superspaces which are most adequate to the geometrical (superspace) description of supersymmetric theories.

In this paper we would like to point out one of possible ways of realizing this idea. This way is based on quantization of the supertwistors [7], which are considered as fundamental ingradients of the superconformal group generators. This quantum supertwistors act in the superFock space by which one may construct unitary irreducible representations (UIR) of the group SU(2,2/N). We will show that such a construction selects some class of superspaces.

In part I we define a canonical symplectic structure on the supertwistor space. In part II we quantize supertwistors and construct superconformal algebra generators and oscillator-like UIR of N-extended conformal supergroup SU(2,2/N) in terms of the coherent states.

In part III we propose a consistent group interpretation of the extended superspaces as parameters of the super coherent states. We select some class of superspaces which are a straightforward generalization of the N=1 chiral superspace and find their group of translations.

I. Canonical Symplectic Structure on the Supertwistor Space

The supertwistors define a (4+N)-dimensional superspace

$$T_\alpha = \begin{pmatrix} \lambda^A \\ \bar\mu_{\dot A} \\ \xi_i \end{pmatrix}, \tag{1}$$

where λ^A and $\bar\mu_{\dot A}$ are standard complex Weyl spinors and $\xi_i \ (i=1,\dots,N)$ are anticommuting complex variables. The dual supertwistor is given by

$$\bar T^\alpha = (\mu_A, \bar\lambda^{\dot A}, -\xi_i). \tag{2}$$

These objects transform according to the fundamental representation of the group SU(2,2/N)*). We can define in the supertwistor space SU(2,2/N) an invariant bilinear form

$$F_{12} = \bar T_1^{\alpha} T_{2\alpha} = \mu_{1A}\lambda_2^A + \bar\lambda_1^{\dot A}\bar\mu_{2\dot A} - \bar\xi_1^i \xi_{2i}.$$

It is easy to check that the one form

$$\theta = \bar T^\alpha d T_\alpha = \mu_A d\lambda^A + \bar\lambda_1^{\dot A} d\bar\mu_{\dot A} - \bar\xi^i d\xi_i$$

is superconformally invariant, too. The symplectic two-form on the supertwistor space then is given by

$$\begin{aligned} &\omega_s = d\theta = \omega_{\alpha\beta}\, d\bar T^\alpha \wedge d T^\beta \ ; \ d\omega_s = 0 \\ &\omega_s = \delta_B{}^A d\mu_A \wedge d\lambda^B + \delta_{\dot A}{}^{\dot B} d\bar\lambda^{\dot A} \wedge d\bar\mu_{\dot B} - \delta_i{}^j d\bar\xi^i \wedge d\xi_j, \end{aligned} \tag{3}$$

where the external product $\wedge$ obeys the following relations

$$\begin{aligned} &d\mu_A \wedge d\lambda_B = -d\lambda_B \wedge d\mu_A \\ &d\bar\lambda^{\dot A} \wedge d\bar\mu_{\dot B} = -d\bar\mu_{\dot B} \wedge d\bar\lambda^{\dot A} \\ &d\bar\xi^i \wedge d\xi_j = d\xi_j \wedge d\bar\xi^i. \end{aligned} \tag{4}$$

*) Some notation and supertwistor transformation rules are given in Appendix A.

The Poisson-brackets algebra is determined /8/ on the symplectic manyfold as

$$[f, g] = \Big(f \frac{\overleftarrow{\partial}}{\partial \bar T^\alpha}\Big)(\omega^{-1})^{\alpha\beta}\Big(\frac{\overrightarrow{\partial}}{\partial T_\beta} g\Big),$$

where the arrows over derivatives mean right and left differentiations. Using eqs. (3) and (4) we obtain

$$\begin{aligned} [f, g] = {} & \frac{\partial}{\partial \mu_A} f \frac{\partial}{\partial \lambda^A} g - \frac{\partial}{\partial \lambda^A} f \frac{\partial}{\partial \mu_A} g + \frac{\partial}{\partial \bar\lambda^{\dot A}} f \frac{\partial}{\partial \bar\mu_{\dot A}} g - \\ & - \frac{\partial}{\partial \bar\mu_{\dot A}} f \frac{\partial}{\partial \bar\lambda^{\dot A}} g + f \frac{\overleftarrow{\partial}}{\partial \bar\xi^i} \frac{\overrightarrow{\partial}}{\partial \xi_i} g + \Big(\frac{\overrightarrow{\partial}}{\partial \xi_i} f\Big)\Big(g \frac{\overleftarrow{\partial}}{\partial \bar\xi^i}\Big). \end{aligned} \tag{5}$$

These relations define a superalgebra /8/.

The Poisson brackets (5) lead to the following canonical relations for the supertwistor components

$$\begin{aligned} &[\mu_A, \lambda^B] = -[\lambda^B, \mu_A] = \delta_A{}^B \\ &[\bar\lambda^{\dot B}, \bar\mu_{\dot A}] = -[\bar\mu_{\dot A}, \bar\lambda^{\dot B}] = \delta_{\dot A}{}^{\dot B} \\ &[\xi_i, \bar\xi^j] = [\bar\xi^j, \xi_i] = \delta_i{}^j. \end{aligned} \tag{6}$$

They can be written in a more compact form

$$[\bar T^\alpha, T_\beta] = \underline{\delta}^\alpha{}_\beta, \tag{7}$$

where

$$\underline{\delta}^\alpha{}_\beta = \begin{pmatrix} \delta_A{}^B & 0 & 0 \\ 0 & \delta_{\dot A}{}^{\dot B} & 0 \\ 0 & 0 & -\delta_i{}^j \end{pmatrix}.$$

All the other Poisson brackets are equal to zero.

II. Quantization of the Supertwistors and Unitary Irreducible Representations of the Group SU(2,2/N)

To quantize a classical dynamical system means to construct a Hilbert-space representation of a given subsuperalgebra of the infinite-dimensional Lie superalgebra of Poisson brackets (5), which contains basic observables *). We shall require that the selected subsuperalgebra includes the canonical variables T and $\bar T$ as well as the enveloping algebra of the Lie superalgebra SU(2,2/N) /9/.

*) By definition observables are functions of supertwistors invariant under the action of the SU(2,2/N) group.

The supertwistor operators T and $\tilde{T}$ obey in the Hilbert space canonical commutation relations

$$[\tilde{T}^{\alpha}, T_{\beta}\} = \underline{\delta}^{\alpha}{}_{\beta}, \qquad (8)$$

where $[\ ,\ \}$ is to be understood as an anticommutator between any two fermionic components and as a commutator otherwise.

At first let us consider the N=1 case. We postulate that the Hilbert space contains vacuum state, i.e.,a minimal eigenvalues of the Hamiltonian. The Hamiltonian is defined by

$$H = \bar{\lambda}\lambda + \mu\bar{\mu} + \bar{\xi}\xi = \tilde{T}^{\alpha} B^{\beta}{}_{\alpha} T_{\beta},$$

where the matrix B has the following form

$$B = \begin{pmatrix} A & 0 \\ 0 & -1 \end{pmatrix} \quad ; \quad A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

and $\tilde{T} = T^{+}B$.

We can define the vacuum state explicitly with the help of the matrix B as follows:

$$\tilde{T}\tfrac{1}{2}(1 + B)|0\rangle = 0\ ;\ \tfrac{1}{2}(1 - B)T|0\rangle = 0. \qquad (9)$$

In a basis, in which A is diagonal

$$A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

this amounts to setting

$$T = \begin{pmatrix} a^{a+} \\ b^{m} \\ \psi \end{pmatrix}, \quad \tilde{T} = (a_a, -b_m^{+}, -\xi^{+}), \qquad (10)$$

where $a = 1,2$, $m = 1,2$ and

$$a|0\rangle = 0 \quad ; \quad \begin{pmatrix} b \\ \psi \end{pmatrix}|0\rangle = 0,$$

i.e., the operators a_a and $\psi_N = (b^m, \psi)$ are annihilation operators. The space F is then defined as the Hilbert-space closure of a set of vectors $\mathcal{P}(a^{+}, \psi^{+})|0\rangle$, where $\mathcal{P}$ is an arbitrary polynomial. Using (8) it is easy to check that the operators a, b and ψ satisfy standard canonical commutation relations for Bose and Fermi annihilation and creation operators

$$[a_a, a^{b+}] = \delta_a{}^{b}$$
$$[b^{m}, b_n^{+}] = \delta_n{}^{m} \qquad (11)$$
$$\{\psi, \psi^{+}\} = 1 .$$

The transformation between two bases is realized with the help of the matrix P

$$B' = PBP^{-1} \qquad T' = PT$$
$$P = \begin{pmatrix} S & 0 \\ 0 & 1 \end{pmatrix} \qquad S = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}. \qquad (12)$$

From (10) and (12) we find the relation between the creation and annihilation operators a, b and ψ and the components of the supertwistors in the realization (1), (2)

$$a^{+} = \frac{1}{\sqrt{2}}(\lambda + \bar{\mu}) \qquad a = \frac{1}{\sqrt{2}}(\bar{\lambda} + \mu)$$
$$b = \frac{1}{\sqrt{2}}(\bar{\mu} - \lambda) \qquad b^{+} = \frac{1}{\sqrt{2}}(\mu - \bar{\lambda}) \qquad (13)$$
$$\psi = \psi \qquad \psi^{+} = \bar{\psi} .$$

In the general case for any N the quantum supertwistors are defined by

$$T_{\alpha} = \begin{pmatrix} \xi^{A+} \\ \psi^{N} \end{pmatrix}, \quad \tilde{T}^{\alpha} = (\xi_A, -\psi_N^{+}), \qquad (14)$$

where

$$\xi^{A+} = \begin{pmatrix} a^{a+} \\ \xi^{i+} \end{pmatrix} \qquad \xi_A = (a_a, \xi_i)$$
$$\psi^{N} = \begin{pmatrix} b^{n} \\ \psi^{k} \end{pmatrix} \qquad \psi_N^{+} = (b_n^{+}, \psi_k^{+}) \qquad (15)$$

and indices i, j, k, l take values $i,j = 1,\dots,n$; $k,l = 1,\dots,q$; $n + q = N$.

The commutation relations are given by

$$[\tilde{T}^{\alpha}, T_{\beta}\} = \underline{\delta}^{\alpha}{}_{\beta},$$

where

$$\underline{\delta}^{\alpha}{}_{\beta} = \begin{pmatrix} \delta_a{}^{b} & 0 & 0 & 0 \\ 0 & \delta_i{}^{j} & 0 & 0 \\ 0 & 0 & \delta_m{}^{n} & 0 \\ 0 & 0 & 0 & -\delta_k{}^{l} \end{pmatrix}. \qquad (16)$$

The matrix B which connects the supertwistor T and its dual $\tilde{T}$ now is given by

$$B_\alpha{}^\beta = \begin{pmatrix} \delta_a{}^b & 0 & 0 & 0 \\ 0 & \delta_i{}^j & 0 & 0 \\ 0 & 0 & -\delta_m{}^n & 0 \\ 0 & 0 & 0 & -\delta_k{}^l \end{pmatrix} \; ; \tilde{T} = T^+ B . \qquad (17)$$

The commutation relation between annihilation and creation operators ξ and η can be find from eq. (16)

$$[\xi_A , \xi^{B+}] = \delta_A{}^B$$
$$[\eta^N , \eta^+_M] = \delta_M{}^N . \qquad (18)$$

The generators of the group $SU(2,2/n+q)$ can be constructed with the help of the supertwistor operators (14) as follows

$$K^\alpha{}_\beta = \tilde{T}^\gamma B_\gamma{}^\alpha T_\beta - \frac{1}{4(2-n)} (1+B)^\alpha{}_\beta \tilde{T}(1+B)T +$$

$$+ \frac{1}{4(2-q)} (1-B)^\alpha{}_\beta \tilde{T}(1-B)T \qquad (19)$$

$$N = \frac{1}{2(2-n)} \tilde{T}(1+B)T - \frac{1}{2(2-q)} \tilde{T}(1-B)T .$$

The group $SU(2,2/n+q)$ is noncompact. An even subgroup of $SU(2,2/n+q)$ is $S(U(2,2) \times U(n+q))$, where $SU(2,2)$ is noncompact with the maximal compact subgroup $S(U(2)xU(2))$. The maximal compact sub-supergroup of $SU(2,2/n+q)$ is $S(U(2/n)xU(2/q))$.

From eqs. (19) we see that there are singularities in the case $n,q = 2$. It is because the group $SU(2/2)$ has not a fundamental representation /10/. That contradicts our starting proposition to construct generators from fundamental representations. Therefore we shall not consider these cases. Using methods developed by Bars and Gunaydin /11/ we can construct the unitary irreducible representations of the group $SU(2,2/n+q)$ in the superFock space F.

Let the set of states $|\Phi^{A\ldots}_{M\ldots}\rangle$ in F that transforms according to some irreducible representation of the maximal compact subsupergroup $S(U(2/n)xU(2/q))$ be annihilated by all operators $K_A{}^M$. Then an infinite set of states obtained by a repeated application of the $K^A{}_M$ operators to the $|\Phi^{A\ldots}_{M\ldots}\rangle$

$$|\Phi^{A\ldots}_{M\ldots}\rangle , \; K^B{}_N |\Phi^{A\ldots}_{M\ldots}\rangle , \; K^C{}_P K^B{}_N |\Phi^{A\ldots}_{M\ldots}\rangle , \ldots$$

forms the basis of unitary irreducible representations of supergroup $SU(2,2/n+q)$.

The unitary action of the group $SU(2,2/n+q)$ in the superFock space F is given by

$$\hat{U} = \exp\{ i \tilde{T} M T \}$$

The matrix M can be represented in the following form /7/

$$M = \left(\begin{array}{c|c} \varkappa & -i\nu \\ \hline -i\nu^+ & \varepsilon \end{array} \right) .$$

where the blocks $\varkappa^A{}_B$, $\nu^A{}_N$ and $\varepsilon^M{}_N$ are functions of the super-group parameters.

The fundamental representation can, in general, be decomposed as

$$U = t . h ,$$

where h is an element of the maximal compact subsupergroup $S(U(2/n)xU(2/q))$; and t, of the coset space $SU(2,2/n+q)/S(U(2/n)xU(2/q))$. We can represent h as follows

$$h = \exp\left\{ i \begin{pmatrix} \varkappa & 0 \\ 0 & \varepsilon \end{pmatrix} \right\} = \begin{pmatrix} u & 0 \\ 0 & v \end{pmatrix} ,$$

and t as

$$t = \exp\left\{ \begin{pmatrix} 0 & \nu \\ \nu^+ & 0 \end{pmatrix} \right\} = \begin{pmatrix} \frac{1}{\sqrt{1-ZZ^+}} & Z \frac{1}{\sqrt{1-Z^+Z}} \\ \frac{1}{\sqrt{1-Z^+Z}} Z^+ & \frac{1}{\sqrt{1-Z^+Z}} \end{pmatrix} , \qquad (20)$$

where $$Z^A{}_N = \left(\frac{\tanh \sqrt{\nu\nu^+}}{\sqrt{\nu\nu^+}} \nu \right)^A{}_N .$$

The variables Z parametrizing the supercoset space $SU(2,2/n+q)/$ $/S(U(2/n)xU(2/q))$ transforms nonlinearly under the action of the group $SU(2,2/n+q)$

$$Z' = (\alpha Z + \beta)(\gamma Z + \delta)^{-1} ,$$

where an element of the fundamental representation has the following form

$$g = \exp(iM) = \left(\begin{array}{c|c} \alpha & \beta \\ \hline \gamma & \delta \end{array}\right).$$

Let us consider the set of states $|\Phi^{A\dots}_{M\dots}\rangle$ which transform under the irreducible representation of the supergroup S(U(2/n)x xU(2/q) and are annihilated by the operators $K_A{}^M$. We define the supercoherent state as follows

$$\exp\{\psi^+_M Z^{+M}{}_A a^{A+}\}|\Phi^{B\dots}_{N\dots}\rangle = |\Phi^{B\dots}_{N\dots}, Z^+\rangle .$$

The supercoherent states form a complete basis of the unitary irreducible representations determined by the lowest state $|\Phi^{B\dots}_{N\dots}\rangle$. The unitary irreducible representations of the N -extended superconformal group can be realized with the help of this basis on the superHilbert space of holomorphic functions of a complex variable Z (these representations belong to the holomorphic discrete series).

III. Fundamental Superspaces

Unitary irredicible representations (analogs of UIR of the superconformal group constructed above) in the usual conformal theory [9] can be reduced to the Poincare UIR. Therefore the parameters of the coherent states play the role of coordinates of the space-time. It is naturally to connect the parameters of the supercoherent states with coordinates of the extended superspace. Then the scheme of constructing the UIR selects some class of superspaces which we will call "fundamental". They are a straightforward generalization of the N=1 chiral superspace. The matrix structure and group of translation of these superspaces are determined by the structure of the coset space SU(2,2/n+q)/S(U(2/n)xU(2/q)).

In the case N=1, for example, this coset space coincides with the chiral superspace

$$Z^A{}_N = (z^a{}_m, \theta^a).$$

When $N=2$, we have only one superspace (as a consequence of the restriction $n\neq 2, q\neq 2$)

$$Z^A{}_N = \begin{pmatrix} z^a{}_m & \theta^a \\ \chi_m & \lambda \end{pmatrix} \tag{21}$$

which is parametrized with an additional complex scalar variable.

In the general case ($N \geq 2$) one has a set of the fundamental superspaces

$$Z^A{}_N = \begin{pmatrix} z^a{}_m & \theta^a{}_i \\ \chi^e{}_m & \lambda^e{}_i \end{pmatrix}, \tag{22}$$

where $i=1,\dots,n \; ; \; e=1,\dots,q$

$q+n=N \; ; \; q\neq 2 \, , \, n\neq 2 .$

The superspace-translation generators are determined by the matrix of noncompact generators

$$\begin{pmatrix} a^{a+}b^+_m & a^{a+}y^+_e \\ \xi^{i+}b^+_m & \xi^{i+}y^+_e \end{pmatrix} .$$

In order to construct them, we go back to our starting basis[9] (1) and (2)

$$\begin{aligned} a^+ &\longrightarrow \mu = \frac{1}{\sqrt{2}}(a^+ + b) \\ b^+ &\longrightarrow \bar\mu = \frac{1}{\sqrt{2}}(b^+ + a) . \end{aligned} \tag{23}$$

Then for the translation generators we get

$$\begin{aligned} &P_{A\dot A} = \mu_A \bar\mu_{\dot A} \; , \; \Pi_{Ae} = \mu_A y^+_e \\ &\bar Q_{\dot A}{}^i = \xi^i \bar\mu_{\dot A} \; , \; C^i{}_e = \xi^{+i} y^+_e \\ &\bar\Pi^K_{\dot A} = \bar\mu_{\dot A} y^K \; , \; Q_{Ai} = \xi_i \mu_A . \end{aligned} \tag{24}$$

It is easy to check that in the case of conformal group ($N=0$) we find only one hermitian generator-operator of the momentum [9] $P_{A\dot A} = \mu_A \bar\mu_{\dot A}$ and in the case $N=1$ -usual translation generators of the supersymmetry

$$\bar Q_{\dot A} = \xi \bar\mu_{\dot A} , Q_A = \xi^+ \mu_A ; \{Q_A, \bar Q_{\dot A}\} = P_{A\dot A} . \tag{25}$$

The commutation relations for supertwistors (16) lead to the following algebra of the translation generators

$$\begin{aligned} &\{Q_{Ai}, \bar Q_{\dot A}{}^j\} = \delta_i{}^j P_{A\dot A} \, , \, \{\Pi_{Ae}, \bar\Pi_{\dot A}{}^K\} = \delta_e{}^K P_{A\dot A} \\ &[C^i{}_e, \bar\Pi_{\dot A}{}^K] = \delta_e{}^K \bar Q_{\dot A}{}^i , \, [C^i{}_e, Q_{Aj}] = \delta^i{}_j \Pi_{Ae} \\ &[C^i{}_e, \Pi_{Ae}] = [C^i{}_e, \bar Q_{\dot A}{}^j] = 0 . \end{aligned} \tag{26}$$

The rest of the commutators and anticommutators equal zero. Now it is easy to relate the fundamental superspace parameters (22) with the standard parametrization of translation group (26) element

$$K(x,\theta,\bar\theta,\varkappa,\bar\varkappa,\lambda)= \exp i\{ P_{A\dot A}x^{A\dot A}+ Q_i^A\,\theta_A{}^i+\bar\Pi_{\dot A}{}^{\kappa}\bar\varkappa^{\dot A}{}_{\kappa}+$$
$$+\Pi^A{}_e\,\varkappa_A{}^e+C^i{}_e\lambda^e{}_i+\bar\theta_i{}^{\dot A}\bar Q_{\dot A}{}^i\}. \qquad (27)$$

For this reason we must solve the equation

$$K(x,\theta,\bar\theta,\varkappa,\bar\varkappa,\lambda)= \exp i\{ P_{A\dot A}x_f^{A\dot A}+ Q^A{}_i(\theta_f)_A{}^i+\bar\Pi_{\dot A}{}^{\kappa}(\bar\varkappa_f)^{\dot A}{}_{\kappa}+$$
$$+C^i{}_e(\lambda_f)^e{}_i\}\exp i\{\bar\theta^{\dot A}{}_i\bar Q^i{}_{\dot A}+\Pi^A{}_e\varkappa_A{}^e\}$$

using the Hausdorff formula. As a result, we get the following relation between coordinates of the fundamental and full (27) superspaces.

$$(x_f)_{A\dot A}= x_{A\dot A}+\frac{i}{2}(\theta^i\bar\theta_i+\varkappa_e\bar\varkappa^e-\lambda_i{}^{\kappa}\varkappa_\kappa\bar\theta^i)$$
$$(\theta_f)_{Ai}=\theta_{Ai}-\frac{1}{2}\lambda_i{}^e\varkappa_{Ae}$$
$$(\bar\varkappa_f)_{\dot A}{}^e=\bar\varkappa_{\dot A}{}^e-\frac{1}{2}\lambda_i{}^e\,\theta_{\dot A}{}^i$$
$$(\lambda_f)_i{}^e=\lambda_i{}^e.$$

From the supertwistor structure of the SU(2,2/$n+q$) algebra generators it follows that the conjugate scalar variable $\bar\lambda$ should not be added to the fundamental superspaces (i.e.,the superfields on the fundamental superspace are analytical $\partial/\partial\bar\lambda\;\phi(Z)=0$) because the generator $\bar C=\xi\psi$ does not commute with generators (24). In order to have a closed algebra (including $\bar C$) we must add, except $\bar\lambda$, the parameters which are related with the compact group generators.

Conclusion

The classical supertwisotrs are usually considered as simplest structure elements of the superspace. The whole class of superspaces can be found imposing different constraints on the classical supertwistors.

In this paper the quantum supertwistors are considered as structure elements of supersymmetry generators. The supertwistortor operators act in the Hilbert space on which the UIR of the superconformal group are constructed. We show that the quantization of the supertwistors and construction of the UIR allow to select some class of superspaces which are a straightforward generalization of the N=1 chiral superspace and to find their group of translations.

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Appendix A

Spinors λ^A and $\bar\mu_{\dot B}$ $(A,B=1,\dots,2)$ transform as usually under the group $SL(2,C)$. Spinor indices are raised and lowered according to the rules:

$$\lambda^A=\varepsilon^{AB}\lambda_B\quad,\quad\lambda_A=\varepsilon_{AB}\lambda^B$$
$$\bar\lambda^{\dot A}=\varepsilon^{\dot A\dot B}\bar\lambda_{\dot B}\quad,\quad\bar\lambda_{\dot A}=\varepsilon_{\dot A\dot B}\bar\lambda^{\dot B},$$

where $\varepsilon_{12}=-\varepsilon^{12}=\varepsilon_{\dot1\dot2}=-\varepsilon^{\dot1\dot2}=1$.

The components of the supertwistors transform under the group SU(2,2/N) as follows

$$\delta\lambda^A=L^A{}_B\,\lambda^B+\frac{1}{2}(-D+iG)\lambda^A-iK^{A\dot B}\bar\mu_{\dot B}+\psi^{Ai}\xi_i$$
$$\delta\bar\mu_{\dot A}=-iP_{\dot AB}\lambda^B-\bar L^{\dot B}{}_{\dot A}\bar\mu_{\dot B}+\frac{1}{2}(D+iG)\bar\mu_{\dot A}+\bar\phi_{\dot A}{}^i\xi_i$$
$$\delta\xi_i=\phi_{Bi}\lambda^B+\bar\psi^{\dot B}{}_i\bar\mu_{\dot B}+\frac{2i}{N}G\xi_i+S_i{}^j\xi_j\,.$$

where D and G are real, $P_{A\dot B}$ and $K^{A\dot B}$ are hermitian; $L^A{}_B$ and S^{ij} are elements of the algebras $SL(2,C)$ and SU(N), respectively, ϕ_{Ai} and ψ^{Bi} are anticommuting spinors.

The supertwistors T and $\tilde T$ can be represented as

$$T_\alpha=\begin{pmatrix}\Pi\\ \psi\end{pmatrix}\qquad\tilde T^\alpha=(\tilde\Pi,-\bar\psi)\,,$$

where

$$\Pi=\begin{pmatrix}\lambda^A\\ \bar\mu_{\dot A}\end{pmatrix}\qquad\tilde\Pi=(\mu_A,\bar\lambda^{\dot A}).$$

Then there exists a real matrix A with two positive and two negative eigenvalues /9/ such that

$$\tilde\Pi=\Pi^+A\,.$$

In the basis (1), (2) the matrix A has the following form

$$A=\begin{pmatrix}0&1\\1&0\end{pmatrix}.$$

It is suitable to introduce the matrix B

$$B=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$$

which gives the relation between $\tilde T$ and T^+

$$\tilde T=T^+B\,.$$

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Квантовые супертвисторы и фундаментальные суперпространства

Квантовые супертвисторы рассматриваются как исходные элементы для построения генераторов суперalгебры. Предлагается интерпретация суперпространства как параметров когерентных состояний, которые задают базис для унитарных неприводимых представлений супергруппы. Показывается, что в случае расширенной суперсимметрии такой подход ведет к выделению определенного класса суперпространств и групп их движения. Эти суперпространства являются прямым обобщением кирального суперпространства ($N = 1$) и параметризованы дополнительными $n \times q$ ($N = n + q$; $n \neq 2$; $q \neq 2$) скалярными переменными.

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Quantum Supertwistors and Fundamental Superspaces

Quantum supertwistors are considered as initial elements for constructing superalgebra generators. An interpretation is proposed of superspace as parameters of coherent states. It is shown that in the case of extended supersymmetry such an approach leads to the separation of a class of superspaces and its group of motion. They are a straightforward generalization of the $N = 1$ chiral superspace and are parametrized with an additional $n \times q$ ($N = n + q$) bosonic variables.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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