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RADIATIVE CORRECTIONS TO PHOTON STATES BETWEEN PLATES

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1. INTRODUCTION

Boundary conditions are a simple model for the interaction with an external field[1]. The corresponding examples are the interaction of the electromagnetic field with macroscopic bodies (conductors), the bag model in QCD, field theory at finite temperature (boundary conditions in imaginary time) and so on.

An interesting example here is the propagation of light in the presence of boundary conditions when radiative corrections are included. It is known that in heated QED the photon acquires a mass[2]. There are further mass effects coming from boundary conditions (see ref.[3] for example). They are due to radiative corrections. In this paper we investigate the radiative corrections to photon states between superconducting plates. This seems to be the simplest example for such corrections and has a direct and obvious physical interpretation. Furthermore though the order of magnitude of these corrections turns out to be very small they are much larger than the thermal photon mass for example.

To handle radiative corrections it is useful to work in covariant gauge. There were some confusions in literature on the canonical quantization of covariant gauge electrodynamics in the presence of boundary conditions. Namely, the authors of ref. [4] state that it would be necessary to introduce ghosts in this case, especially in order to obtain the right result for the Casimir energy. In ref.[5], however, it was shown that there is really no reason for ghosts and that the usual Gupta-Bleuler quantization method works well and is equivalent to the path-integral quantization method for the same problem derived in ref.[6]. The essential point in refs. [5,6] was the observation that the presence of boundary conditions does not influence the gauge freedom, so that the function $\phi(x)$ in the gauge transformation $A_\mu(x) \to A_\mu(x) + \partial_{x_\mu} \phi(x)$ is not restricted by the boundary conditions.

For our investigation of the photon states between superconducting plates we use the quantization procedure of ref.[5]. In doing this we show, that this procedure is consistent within the perturbation theory at least at the one-loop level. For the photon states it turns out that they remain massless, whereas the photon energy will be shifted. This shift can be interpreted as a renormalization of the distance between the plates, seen by the photon. We use a simple model, in which the boundary conditions are realized by two parallel, infinitely large and thin superconducting plates, oriented perpendicular to the third axis and intersecting it at $x_3 = 0$ and $x_3 = a$. Further we assume that the electrons do not feel the boundaries, so that we have boundary conditions to the electromagnetic field only.

2. THE RADIATIVE CORRECTIONS TO THE PHOTON STATES

We start with the QED Lagrangian in covariant gauge ($\alpha = 1$)

$$L(x) = \frac{1}{2} A_\mu(x)\,\partial_x^2 g^{\mu\nu} A_\nu(x) + \bar{\psi}(x)(i\hat{\partial} - m + e\hat{A})\psi(x) \qquad (1)$$

and the corresponding action

$$S(A, \bar{\psi}, \psi) = \int d^4x\, L(x). \qquad (2)$$

The boundary conditions for the electromagnetic field read

$$n^\mu F^*_{\mu\nu}(x) = 0 \qquad x_3 = 0, a \qquad (3)$$

with

$$n^\mu = (0, 0, 0, 1), \quad F^*_{\mu\nu}(x) = \epsilon_{\mu\nu\alpha\beta} F^{\alpha\beta}(x),$$

whereas there are boundary conditions for the spinor field.

Now, we are interested in the photon states only, so we can exclude the spinor field in order to get an effective action. Because we have no boundary conditions to the spinor field, this can be done in the usual manner and we get the effective action

$$S(A) = \frac{1}{2} \int d^4x\, d^4y\, A_\mu(x)\,[\partial_x^2 g^{\mu\nu}\,\delta(x-y) - \Pi^{\mu\nu}(x-y)]\,A_\nu(y) \qquad (4)$$

with the standard polarization operator

$$\Pi_{\mu\nu}(x-y) = -\, ie^2 \mathrm{Sp}[\gamma_\mu \hat{S}(x-y)\gamma_\nu \hat{S}(y-x)]. \qquad (5)$$

Hereby we assume that the renormalization in $\Pi_{\mu\nu}(x-y)$ is carried out in the usual way. Now the effective action $S(A)$ (4) has to be considered together with the boundary conditions (3). Following the quantization procedure of ref.[5] we introduce the polarization basis $E^s_\mu(-i\partial_x)$

$$E^1_\mu(-i\partial_x) = \begin{pmatrix} 0 \\ i\partial_{x_2} \\ -i\partial_{x_3} \\ 0 \end{pmatrix} \frac{1}{\sqrt{-\partial^2_{x_\parallel}}}, \quad E^2_\mu(-i\partial_x) = \begin{pmatrix} -\partial^2_{x_\parallel} \\ -\partial_{x_0}\partial_{x_1} \\ -\partial_{x_0}\partial_{x_2} \\ 0 \end{pmatrix} \frac{1}{\sqrt{-\partial^2_{x_\parallel}}\sqrt{-\partial^2_{x_0}+\partial^2_{x_\parallel}}}, \qquad (6)$$

$$E^3_\mu(-i\partial_x) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \qquad E^0_\mu(-i\partial_x) = \begin{pmatrix} -i\partial_{x_0} \\ -i\partial_{x_1} \\ -i\partial_{x_2} \\ 0 \end{pmatrix} \frac{1}{\sqrt{-\partial^2_{x_0} + \partial^2_{x_\parallel}}}.$$

Here $s = 0,1,2,3$ denote the polarizations. The subscript $\parallel$ means the 1,2-directions: $x^2_\parallel = x_1^2 + x_2^2$. The polarizations (6) satisfy the following orthogonality relation

$$E^s_\mu(-i\partial_x)\, g^{\mu\nu} E_\nu(-i\partial_x) = \tilde{g}^{st}$$

with $\tilde{g}^{st} = \mathrm{diag}(1,-1,-1,-1)$. The normalizations in (6) are chosen so that the root on-shell (i.e., for $\partial^2_{x_0} + \partial^2_x = -\partial^2_{x_3}$) be real and that the operators $E^s_\mu(-i\partial_x)$ be Hermitean. The decomposition of the potential $A_\mu(x)$ over this basis reads

$$A_\mu(x) = \sum_{s=0}^{3} E^s_\mu(-i\partial_x)\, a_s(x). \qquad (7)$$

The boundary conditions (3) imply for $a_s(x)$

$$a_s(x) = 0 \quad \text{for} \quad s = 1,2, \; x_3 = 0, a. \qquad (8)$$

Substituting the decomposition (7) into (4) we obtain the effective action in the form

$$S = \frac{1}{2} \int d^4x\, d^4y\, \overline{a_s(x)}\, [\partial_x^2 \tilde{g}^{st} \delta(x-y) - \Pi^{st}(x-y)]\, a_t(y). \qquad (9)$$

Here Π^{st} is the projected polarization operator

$$\Pi^{st}(x-y) = E^s_\mu(-i\partial_x) E^t_\nu(i\partial_y)\, \Pi^{\mu\nu}(x-y). \qquad (10)$$

Using [7]

$$\Pi_{\mu\nu}(x-y) = (g^{\mu\nu}\partial_x^2 - \partial_x^\mu \partial_x^\nu)\, \tilde{\Pi}(x-y) \qquad (11)$$

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Here Π^{st} is the projected polarization operator

$$\Pi^{st}(x - y) = E^s_\mu(-i\partial_x)\, E^t_\nu(i\partial_y)\, \Pi^{\mu\nu}(x - y)\,. \tag{10}$$

Using [7]

$$\Pi_{\mu\nu}(x - y) = (g^{\mu\nu}\partial_x^2 - \partial_x^\mu \partial_x^\nu)\, \tilde{\Pi}(x - y) \tag{11}$$

we rewrite (10) in the form

$$\Pi^{st}(x-y) = \left\{ \tilde{g}^{st}\,\partial_x^2 - \begin{pmatrix} i\sqrt{-\partial^2_{x_0}+\partial^2_{x_\parallel}} \\ 0 \\ 0 \\ \partial_{x_3} \end{pmatrix}^s \begin{pmatrix} i\sqrt{-\partial^2_{x_0}+\partial^2_{x_\parallel}} \\ 0 \\ 0 \\ \partial_{x_3} \end{pmatrix}^t \right\} \tilde{\Pi}(x-y). \qquad (12)$$

From equation (12) it is clear that $\Pi^{st}(x-y)$ is diagonal in the physical polarizations (for $s=1,2$ as well as for $s=3$, when it is independent of x_3).

As was shown in ref. /6/, the physical state space is spanned by the polarizations E^s_μ with amplitudes $a_s(x)$ ($s=1,2,3$) with the restriction $a_3(x)$ to be independent of x_3. So this diagonality shows, that the physical state space is invariant with respect to the interaction, represented by the polarization operator (5). It seems to be nearly obvious that a higher loop correction does not change anything.

Now, once the physical polarizations including radiative corrections are clear, the problem is reduced to a scalar one for a_1 and a_2. For the amplitude $a_3(x) = \mathrm{const}(x_3)$ it reduces to the corresponding problem in (2+1) dimensions without boundary conditions. In this case it is well known that the polarization operator can be absorbed into the wave function normalization, so that this contribution is not interesting from the point of view of boundary conditions.

It remains to consider the scalar problem for $a_1(x)$ and $a_2(x)$ (hereafter denoted by $a(x)$) only. The action reads

$$S(a) = \frac{1}{2}\int d^4x\, d^4y\, a(x)\,[-\partial_x^2\delta(x-y) + \partial_x^2\tilde{\Pi}(x-y)]\,a(y) \qquad (13)$$

with boundary conditions

$$a(x)\,|_{x_3=0,a} = 0. \qquad (14)$$

Now the variational principle yields the equation of motion

$$-\partial_x^3 a(x) + \int d^4y\, \partial_x^2\tilde{\Pi}(x-y)\, a(y) = 0 \qquad (15)$$

for $x_3 \neq 0, a$ (remember $\delta a(x) = 0$ for $x_3 = 0, a$ due to the boundary conditions). To satisfy the conditions (14), we introduce the corresponding Fourier representation

$$a(x) = \int \frac{d^3p_\alpha}{(2\pi)^3} e^{ip_\alpha x^\alpha} \{\theta(-x_3)\int_0^\infty \frac{dp_3}{\pi/2}\sin p_3x_3 \quad a_{-1}(p_\alpha,p_3) +$$

$$+\,\theta(x_3)\theta(a-x_3)\frac{2}{a}\sum_{n>0}\sin\omega_n x_3 \quad a_0(p_\alpha,\omega_n) + \qquad (16)$$

$$+\,\theta(x_3-a)\int_0^\infty \frac{dp_3}{\pi/2}\sin p_3(x_3-a) \quad a_1(p_\alpha,p_3)\} \equiv \qquad (\alpha = 0,1,2)$$

$$\equiv \sum_{i=-1}^{1}\int d^4(p)\,\psi_i(x,(p))\,a_i((p)).$$

Substituting (16) into (15) we get the equations of motion in the form

$$(\Gamma^2 - (p_3)_i^2)\,a_i(p_\alpha,(p_3)) -$$

$$-\sum_{j=-1}^{1}\int d(k_3)_j\,\Pi_{ij}(\Gamma^2,(p_3),(k_3))\,a_j(p_\alpha,(k_3)) = 0 \qquad (17)$$

with $i=-1,0,+1$ and $\Gamma^2 = p_0^2 - p_1^2 - p_2^2$. Here is $(p_3)_i = p_3$ for $i=\pm1$, $(p_3) = \omega_n \equiv \pi n/a$ for $i=0$, and $\int d(k_3)_{\pm1} = \int_0^\infty dk_3$ $i=\pm1$, $\int d(k_3)_0 = \sum_{n>0}$ for $i=0$. The polarization operator Π_{ij} is given by

$$\delta^3(p_\alpha - k_\alpha)\,\Pi_{ij}(\Gamma^2,(p_3),(k_3)) = \int d^4x\,d^4y\,\overline{\psi_i(x,(p))}\,\partial_x^2\Pi(x-y)\,\psi_j(y,(k))$$

and it is symmetric under the change $i \to j$. With the standard momentum representation

$$\tilde{\Pi}(x-y) = \int \frac{d^4p}{(2\pi)^4} e^{ip(x-y)}\,\tilde{\Pi}(p^2) \qquad (18)$$

we get

$$\Pi_{ij}(\Gamma^2,(p_3),(k_3)) = \qquad (19)$$

$$= \int_{-\infty}^{\infty} dq_3 \frac{p_3}{p_3^2-q_3^2}\,\frac{k_3}{k_3^2-q_3^2}\,M_{ij}(\Gamma^2-q_3^2)\,\tilde{\Pi}(\Gamma^2-q_3^2)$$

with

$$M_{ij} = \begin{cases} \frac{1}{\pi^2} & \text{for } i=j=\pm 1 \\ \frac{1}{a\pi}(1-(-1)^n e^{iq_3 a}) & \text{for } i=-1,\ j=0\ * \\ \frac{1}{\pi^2} e^{iq_3 a} & \text{for } i=-1,\ j=+1 \\ \frac{1}{a\pi}(1-(-1)^n e^{iq_3 a})(1-(-1)^{n'} e^{-iq_3 a}) & \text{for } i=j=0\ **. \end{cases}$$

The system (17) is non-diagonal in i. This is due to our assumption that electrons do not feel the plates, and consequently, photons penetrate them via an electron-positron pair creation. However, having in mind a realistic situation, where the plates are not infinitely thin, the nondiagonal terms are small like $\exp(-dm_e)$, where d is the thickness of the plates and m_e is the electron mass. Thus we neglect these terms.

After this simplification the system (17) gets diagonal. Consider the case $i=0$, i.e., the region between the plates. Here we have (writing a instead of a_0)

$$(\Gamma^2-\omega_n^2)\, a(\Gamma^2,\omega_n) - \sum_{n'>0} \omega_n\omega_{n'} f_{n,n'}(\Gamma^2)\, a(\Gamma^2,\omega_{n'}) = 0, \qquad (n=1,2,\dots) \tag{20}$$

with

$$f_{n,n'}(\Gamma^2) = \frac{1}{a\pi}\int_{-\infty}^{\infty} dq_3 \frac{(1-(-1)^n e^{iq_3 a})(1-(-1)^{n'} e^{-iq_3 a})}{(\omega_n^2-q_3^2)(\omega_{n'}^2-q_3^2)} \times (\Gamma^2-q_3^2)\,\bar{\Pi}(\Gamma^2-q_3^2)\,. \tag{21}$$

Equation (20) can be solved in perturbation theory. In a standard way*** we find that solutions are possible for

$$\Gamma^2 = \omega_n^2(1-f_{n,n}(\omega_n^2)), \quad \text{i.e., for} \quad p_0^2 = p^2 + \omega_n^2(1-f_{n,n}(\omega^2)) \tag{22}$$

*Here is $(k_3) = \omega_n$.

** Here is $(p_3) = \omega_n$, $(k_3) = \omega_{n'}$.

*** See the perturbation theory for a quantum mechanical system in ref. /8/ for example.

only. They look like

$$a(\Gamma^2,\omega_n) = \sum_{n>0}\left[\delta_{n,n'} - \frac{\omega_n\omega_{n'}}{\omega_{n'}^2-\omega_n^2} f_{n,n'}(\omega_{n'}^2)\right] \times \delta(\Gamma^2-\omega_{n'}^2(1-f_{n',n'}(\omega_{n'}^2)))\,\tilde{a}(p^2,\omega_{n'}), \tag{23}$$

where $\tilde{a}(p^2,\omega_{n'})$ are arbitrary parameters. Transforming (23) back into x-space we obtain the solutions in the form

$$a(x) = \theta(x_3)\,\theta(a-x_3)\int\frac{d^3p_\alpha}{(2\pi)^3}\frac{2}{a}\sum_{n>0}\sum_{n'>0} e^{ip_\alpha x^\alpha}\sin\omega_n x_3 \times \delta(p_0^2-p^2-\omega_{n'}^2(1-f_{n',n'}(\omega_{n'}^2)))\left[\delta_{n,n'} - \frac{\omega_n\omega_{n'}}{\omega_{n'}^2-\omega_n^2} f_{n,n'}(\omega_{n'}^2)\right]\tilde{a}(p^2,\omega_{n'}). \tag{24}$$

It remains to get a more explicit representation for the function $f_{n,n'}(\omega_{n'}^2)$ than (21). Using the well-known representation (see ref. /7/ for example)

$$\bar{\Pi}(p^2) = \frac{e^2}{2\pi^2}\int_0^1 d\alpha\,\alpha(1-\alpha)\ln\left(1-\alpha(1-\alpha)\frac{p^2}{m_e^2}\right)$$

for the polarization operator we get after a deformation of the integration contour in (21)

$$f_{n,n'}(\omega_{n'}^2) = \frac{\alpha}{am_e}\,\frac{4}{3\pi}\int_{\sqrt{4-(\frac{\pi n'}{am_e})^2}}^{\infty} dk\,\frac{1+(-1)^{n+n'}-((-1)^n+(-1)^{n'})e^{-kam_e}}{2\left((\frac{\pi n}{am_e})^2+k^2\right)}\,\rho\!\left(k,\frac{\pi n'}{am_e}\right) \tag{25}$$

with

$$\rho(k,\omega) = \frac{\sqrt{k^2+\omega^2-4}\,(k^2+\omega^2+2)}{(k^2+\omega^2)^{3/2}}\,,$$

where α is the fine-structure constant *. Special values are

$$f_{0,0}(0) = \frac{\alpha}{am_e}\,\frac{3}{16}, \qquad f_{n,n}(\omega_n^2)\big|_{\omega_n=2m_e} = \frac{\alpha}{am_e}\,\frac{28\sqrt{2}}{9\pi} \tag{26}$$

* This is, of course, nothing else than the usual spectral representation.

(up to contributions small like $\exp(-2am_e)$). For $0 \leq \omega_n \leq 2m_e$ the function $f_{n,n}(\omega_n^2)$ is monotonic and analytic in n, at $\omega_n = 2m_e$ it has a cut, corresponding to the creation of real electron-positron pairs. Furthermore, the function $f_{n,n'}(\omega_{n'}^2)$ has for $\omega_{n'} < 2m_e$ the asymptotic form

$$f_{n,n'}(\omega_{n'}^2) \sim \frac{1}{n} \frac{2\alpha}{3} \frac{1+(-1)^{n+n'}}{2} \tag{27}$$

for $n \to \infty$.

What physics does this mean? First, for the stationary photon states between the plates one observes an energy shift due to the radiative corrections given by (22). One may imagine the photon to create near a plate an electron-positron pair that does not feel the boundary and cannot be reflected in order to contribute to the stationary wave. This can be seen also from equation (15). We write it in the form $\partial_x^2 a(x) = j(x)$ with $j(x) = \int d^4y\, \partial_x^2 \tilde{\Pi}(x-y)\, a(y)$ and consider the region $x_3, y_3 \in [0, a]$. One observes that integrating ∂_x^2 by parts is not possible because $\tilde{\Pi}(x-y)$ does not satisfy boundary conditions. So, we have $j(x) \neq 0$ for $\partial_x^2 a(x) = 0$. Consequently, the radiative corrections can be considered as a source generated by the photon when it is reflected from the boundary. For the same reason the x_3-dependence of the wave function for a given frequency ω_n (resp. for a given p_0) is no longer simply $\sin\omega_n x_3$, but

$$\phi_{n'}(x_3) = \sum_{n>0} \sin\omega_4 x_3 \left[\delta_{n,n'} - \frac{\omega_n \omega_{n'}}{\omega_{n'}^2 - \omega_n^2} f_{n,n'}(\omega^2)\right]. \tag{28}$$

Due to (27) the sum over n converges and $\phi_{n'}(x_3) = 0$ for $x_3 = 0, a$, i.e., $\phi_{n'}(x_3)$ is continuous at $x_3 = \epsilon$ and $x_3 = a - \epsilon$, $\epsilon > 0$, $\epsilon \to 0$.

What about the order of magnitude of these corrections? For real macroscopic plates the distance a between the plates is much larger than $1/m_e$ so that $1/am_e$ is very small. Furthermore, the energy shift (22) can be interpreted as a renormalization of the distance between the plates

$$a \to a_{ren} = a\left(1 + \frac{1}{2} f_{n,n}(\omega_n^2)\right). \tag{29}$$

So, due to radiative corrections a photon with energy $p_0 = \sqrt{p_\parallel^2 + \left(\frac{\pi n}{am_e}\right)^2}$ sees a larger distance $a_{ren} > a$. This distance renormalization, however, is much smaller than the uncertainty of the distance coming from the real surface structure of the plates. This comes, of course, from the mentioned above fact that

the virtual electron-positron pairs contribute essentially in the region $\sim 1/m_e$ only. The deviation of the x_3-dependence of the wave function from $\sin\omega_n x_3$ is of the same order. This deviation, however, is present in the whole x_3-region from $x_3 = 0$ to $x_3 = a$ and not near the surface only. Moreover, it is present also in the case of one plate only. This can be shown easily by taking the limit $a \to \infty$ in (22) and (28). For this aim one has to substitute

$$\omega_n \to p_3, \quad \frac{1}{a}\sum_{n>0} \to \frac{1}{\pi}\int_0^\infty dp_3 .$$

Then the energy shift, of course, vanishes. The function $\phi_n(x_3)$ gets simplified in this case, and we have after straightforward calculations

$$\phi_n(x_3) \xrightarrow[a\to\infty]{} \phi_p(x_3) = \sin p_3 x_3 + \alpha \frac{p_3}{m_e} \frac{2}{3\pi} f\left(\frac{p_3}{m_e}\right)(1 - \cos p_3 x_3) \tag{30}$$

with

$$f(p) = \int_{\sqrt{4-p^2}}^{\infty} dk\, \rho(k,p). \tag{31}$$

The function $f(p)$ (31) is essentially the same as (25), and we have $f(0) = 3/8$, $f(2) = \frac{56\sqrt{2}}{9\pi}$, $f(p)$ analytic and monotonic for $0 \leq p < 2$ and has a cut from $p = 2$. Now the small parameter is p_3/m_e. From (30) we see that the effect (i.e., the deviation from $\sin p_3 x_3$) does not decrease if one is far from the plate.

3. CONCLUSIONS

In the previous section we have considered the photon states between parallel superconducting plates when radiative corrections are included. We have used the canonical quantization procedure of covariant gauge electrodynamics with boundary conditions developed in ref. /5/. Thereby it was shown, that the physical polarizations of the photon in the presence of boundary conditions introduced in ref. /5/ are not mixed when radiative corrections are included. This shows at once that this quantization procedure is convenient for calculations in perturbation theory (at last at the one-loop level) and is therefore the appropriate extension of the Gupta-Bleuler method to the case with boundary conditions.

Concerning the radiative corrections to the photon states we get the result, that the polarizations in the first order of the perturbation theory are the same as in zeroth order. They are given by E_μ^s $(s = 1, 2)$ (6) as well as by E_μ^3 (6) with an amplitude independent of x_3. The last polarization turns out to have no radiative corrections specific for the presence of boundary conditions. For the first two polarizations the problem is reduced to a scalar one. It is shown that the radiative corrections to the corresponding scalar amplitude result in a shift of the photon energy, which can be interpreted as a renormalization of the distance between the plates seen by the photon. Furthermore, the x_3-dependence of the photon wave function differs slightly from the usual $\sin\omega_n x_3$. All corrections are small by an order of α/am_e, where α is the finestructure constant, a the distance between the plates, and m_e is the electron mass. Namely, the renormalization of the distance a is much smaller than the uncertainty of the distance due to the real surface structure and therefore not measurable. The corrections to the x_3-dependence of the photon wave function are also small by an order of α/am_e between the plates resp. by $\alpha p_3/m_e$ if there is one plate only (p_3 is the perpendicular momentum of the photon). However they are present in the whole x_3-region, and not only near the plate. This leads to the hope that they can be observed experimentally.

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Бордаг М. E2-84-126

Радиационные поправки к состояниям фотонов между зеркалами

Радиационные поправки к состояниям фотонов рассматриваются при наличии граничных условий, задаваемых двумя сверхпроводящими параллельными пластинами. Использована развитая ранее процедура канонического квантования электродинамики в ковариантной калибровке с учетом граничных условий, которая очень удобна в данном случае. Для фотонов, находящихся между пластинами, наблюдается смещение энергии, обусловленное радиационными поправками. Это смещение может быть интерпретировано как результат ренормировки расстояния между пластинами, которое увидит фотон. Кроме того наблюдается изменение волновых функций фотонов в импульсном пространстве, которое также обусловлено радиационными поправками.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Bordag M. E2-84-126

Radiative Corrections to Photon States between Plates

Radiative corrections to the photon states are considered in the presence of superconductor boundary conditions given by two parallel plates. The developed earlier canonical quantization method for covariant gauge electrodynamics in the presence of boundary conditions is used and it turns out to be convenient for handling this problem. For the photon states between the plates one observes a shift of the photon energy due to radiative corrections which can be interpreted as a renormalization of the distance between the plates seen by the photon. The photon wave functions are found to be spread in the momentum space, also due to radiative corrections.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1984