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MODEL OF π N-INTERACTIONS. S-WAVE LENGTHS AND PROTON-NEUTRON MASS DIFFERENCE

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1. Introduction

In the early fifties, in virtue of the successful results of quantum electrodynamics, of more interest has become the weak coupling theory as applied to strong interactions. The calculations under the weak coupling assumption have been performed for a lot of various meson models (results of these calculations are the subject of the book of R.Marshak [1]).

The predictions of the above models, as a rule, were not consistent with experiment in many important aspects. Nevertheless, even on the basis of results on the low-energy πN -scattering it was possible to give preference to the model of πN -interactions with the pseudovector coupling (the PV-model). This model, at least, rather well described the energy dependence of cross sections up to about 200 MeV.

For higher energies the PV-model led to the linear growth of cross sections while the experimental values increase to a maximum and then decrease. This model also gave zeroth values for the s -wave scattering length which contradicts the expeimental data. From the theoretical point of view the PV-model was considered to be unacceptable due to its nonrenormalizability.

In previous paper [2] we have attempted to remove the difficulties on nonrenormalizabitily of this model. We raised there the problem whether it is possible, within the framework of the usual PV-model, to obtain the finite results for observable effects without introducing any additional (indefinite) parameters into this model. We have succeeded in solving this problem with the use of the summation procedure of Redmond-Bogolubov-Logunov-Shirkov [3] applied to a part of diagrams of the meson Green function.

The model we have suggested had some formal advantages (it is renormalizable and the correspondence rules are rather simple), however, it did not describe satisfactorily the πN -interactions.

Before to state a new problem, let us try to ascertain what requirements should be given to the quantum field model which pretends to describe the strong interactions. In our opinion, the basic requirements are as follows:

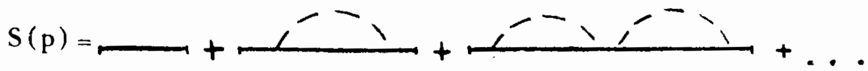
1) Since the meson-nucleon coupling constant is large a great amount of virtual mesons is tightly linked with nucleon, therefore it might be well that the model, even at the first stage, would take into account (at least, in part) the back action on nucleon of the meson field produced by the nucleon itself, i.e., it would take into account the self-energy effects.

2) Some indications (see, e.g., ref. /4/) exist in favour of that in the limit of large momenta the strong interactions should become extremely weak; this requirement can be satisfied if these interactions are described within the framework of super-renormalizable theory.

3) As we have no methods other than those of perturbation theory, it is desirable that in the lowest order the model does not give serious disagreements with experiment. In particular, if one bases on the PV-model, it is necessary to allow for a mechanism which, on the one hand, would lead to correct values of the s-waves and, on the other hand, would prevent the energy growth for the πN-scattering total cross sections (the latter requirement is connected, to some extent, with condition (2)).

The above listed conditions allow one to state the following problem: Does a method exist to make the PV-model super-renormalizable? These conditions also indicate partly a way to possible solution of the problem. Obviously (see(1÷3)) it is necessary to sum up preliminary some class of self-energy nucleon diagrams which will result in the nucleon Green function with the asymptotic behaviour appropriate for the super-renormalizability of the model.

Following paper /2/ we consider the chain of the diagrams of the nucleon Green functions:



$$= S_0 + S_0 \Sigma S_0 + S_0 \Sigma S_0 \Sigma S_0 + \dots \tag{1.1}$$

Here $S_0(p)$ is the nucleon propagator, $\Sigma(p)$ is the operator of self-energy of the second order. To a part of diagrams (1.1) the summation of Redmond-Bogolubov-Logunov-Shirkov /3/ is now applied.

The nucleon Green function

$$S_\lambda(p) = M S_\lambda^{(1)}(p^2) + \hat{p}\, S_\lambda^{(2)}(p^2) \tag{1.2}$$

found as a result of partial summation, will not have false poles on the first sheet, and with this we have

$$|S_\lambda^{(j)}(p^2)|_{|p^2| \gg M^2} = O\left(\frac{1}{|p^2|^2}\right) \quad (j = 1, 2) \tag{1.3}$$

Further we shall regroup the perturbation expansion series for the PV-model in such a way that each term include the function $S_\lambda(p)$, instead of the nucleon propagator $S_0(p)$. Conditions (1,3)) will ensure the super-renormalizability of such a model. Due to these conditions it is possible to overcome the difficulty of growing πN - scattering cross section at high energies. As far as the function $S_\lambda(p)$ has no additional singularities on the physical sheet, the every term of the series will keep the correct analytic properties.

In section 3, with the use of the obtained nucleon Green function $S_\lambda(p)$, we calculate the s -wave scattering lengths in the Born approximation and in section 4 we find the proton-neutron mass difference in the lowest order of perturbation expansion. Our results:

$$a_1 - a_3 = 0.2904$$
$$a_1 + 2a_3 = -0.0371$$
$$\delta M = M_p - M_n = -0.455 \text{ MeV}$$

are rather close to the available experimental data, therefore, as we think, the model is worthy of further study.

2. Partial Summation and the Nucleon Green Function

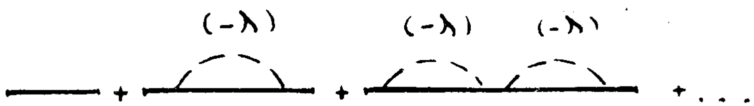
The Lagrangian for pion-nucleon interaction is taken in the following form*:

$$\mathcal{L}_{\pi N}(x) = \frac{f}{m} : \overline{\Psi}(x)\gamma_\mu \gamma_5 \tau_i \Psi(x) \frac{\partial \phi_i}{\partial x_\mu} : \qquad (2.1)$$

where m is the pion mass, f the coupling constant.

Consider the series of diagrams (1.1) for the nucleon Green function. We want to make correspond (following the summation methods for asymptotical series) to it or to its part such a function $S_\lambda(p)$ which, having correct analytic properties, would decrease for $|p^2| \to \infty$ not slower than $O(1/|p^2|^2)$.

It is just the behaviour that ensures super-renormalizability of model (2.1). Obviously, the formal summation of chain (1.1), as the summation under the spectral integral /3/, both do not give the desirable result. The nucleon Green function obtained in this way will have either false poles on the physical sheet or the same asymptotic behaviour as that for the free propagator $S_0(p)$. We shall proceed in the following way. First we pick out of the series (1.1) the part

$S_\lambda(p) =$ 

$$= S_0 + S_0(-\lambda\Sigma)S_0 + S_0(-\lambda\Sigma)S_0(-\lambda\Sigma)S_0 + \dots \qquad (2.2)$$

$$= MS_\lambda^{(1)}(p^2) + \hat{p}S_\lambda^{(2)}(p^2)$$

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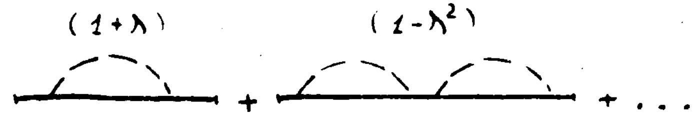
* The following notation is used:

$$\gamma_\alpha\gamma_\beta + \gamma_\beta\gamma_\alpha = 2g_{\alpha\beta}, \quad \gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$$

$$g_{\alpha\beta} = \mathrm{diag}(1,-1,-1,-1), \quad \gamma_5 = \gamma_5^+.$$

(Here λ an arbitrary parameter and M the nucleon mass). Regrouping formally the remaining terms of the series (1.1) allows us to express them in terms of the functions $S_\lambda(p)$ and $\Sigma(p)$ in the following manner:

$S'_\lambda(p) =$ 

$$= (1+\lambda)S_0\Sigma S_0 + (1-\lambda^2)S_0\Sigma S_0\Sigma S_0 + \dots \qquad (2.3)$$

$$= S_\lambda(1+\lambda)\Sigma S_\lambda + S_\lambda(1+\lambda)\Sigma S_\lambda(1+\lambda)\Sigma S_\lambda + \dots$$

If one will succeed in summing (2.2.) in such a way that the functions $S_\lambda^{(j)}(p^2)$ $(j=1,2)$ have correct analytic properties and an asymptotic behaviour similar to $O(1/|p^2|^2)$, then every term of the series (2.3) will also retain these properties. And what is more, any different diagram for the perturbation expansion for the interaction Lagrangian (2.1) calculated with the use of the nucleon Green function $S_\lambda(p)$ becomes finite because in the degree of divergence the theory (2.1) is equivalent to the model with Lagrangian $\mathcal{L} = g\phi^3$.

Now let us apply the summation method of Redmond-Bogolubov-Logunov-Shirkov /3/ to the set of diagrams (2.2). In this case, allowing for notation (2.2), we get

$$S_\lambda^{(j)}(p) = \frac{1}{M^2 - p^2} + \int_{(M+m)^2}^{\infty} \frac{dz\,\rho_\lambda^{(j)}(z)}{z - p^2 - i\epsilon}, \quad (j=1,2). \qquad (2.4)$$

Here

$$\pi\rho_\lambda^{(1,2)}(z) = \mathrm{Im}\left\{ \frac{1 \mp \Sigma_{1,2}(z)}{M^2[1-\lambda\Sigma_1(z)]^2 - z[1+\lambda\Sigma_2(z)]^2} \right\}. \qquad (2.5)$$

The functions Σ_1 and Σ_2 are defined by the expansion

$$\Sigma(p) = M\,\Sigma_1(p^2) + \hat{p}\,\Sigma_2(p^2) \tag{2.6}$$

and have the form

$$\Sigma_1(x) = \frac{3f^2}{16\pi^2 Q}\{-2W + \ell n Q\,[\frac{Q(1-Q)}{2x} - \frac{7}{2}Q + 3Q^2] -$$

$$-5Q - 2 - \frac{1}{2}x - \frac{Q(6Q-20)}{\sqrt{4/Q-1}}\,\mathrm{arctg}\,(\frac{4}{Q}-1)^{1/2} - \tag{2.7}$$

$$-\frac{Q\sqrt{\lambda(x)}}{2x}[\ell n\frac{x-1-Q+\sqrt{\lambda(x)}}{x-1-Q-\sqrt{\lambda(x)}} - 2i\pi\theta(x-(1+\sqrt{Q})^2)]\},$$

$$\Sigma_2(x) = \frac{3f^2}{16\pi^2 Q}\{W(3-x) + 2(1+2Q) +$$

$$+\ell n Q\,[\frac{Q(1-Q-x)}{2x} - \frac{(1-x)\{(1-Q)^2 - 2x + x^2\}}{4x^2} - 2Q^2 + 3Q] -$$

$$-\frac{(1-x)(2-2Q+x)}{4x} + \frac{4Q(Q-3)}{\sqrt{4/Q-1}}\,\mathrm{arctg}(4/Q-1)^{1/2} -$$

$$-\frac{\sqrt{\lambda(x)}}{2x}[Q - \frac{(1-x)(1-x-Q)}{2x}][\ell n\frac{x-1-Q+\sqrt{\lambda(x)}}{x-1-Q-\sqrt{\lambda(x)}} - \tag{2.8}$$

$$-2i\pi\theta(x-(1+\sqrt{Q})^2)]\}.$$

Here we have introduced the following notation: $x = p^2/M^2$, $Q = m^2/M^2$, $\lambda(x) = (1-x-Q)^2 - 4xQ$, W is an arbutrary subtraction constant.

As has been mentioned, the model (2.1) is super-renormalizable if the condition (1.3) holds. For this it is sufficient to require that

$$\mathcal{F}^{(j)}(\lambda, W) = 1 + \int_{(M+m)^2}^{\infty} dz\,\rho_\lambda^{(j)}(z) = 0, \quad (j=1,2). \tag{2.9}$$

The numerical analysis shows that in terms of the variables λ and W the system of equations (2.9) has a single solution (A-solution) for $\lambda f^2 \approx 0.8$ and $W \approx -1$ and many solutions (B-solutions) for $0<\lambda f^2 \lesssim 0.04$. To fix uniquely these parameters, in the next section, using the function $S_\lambda(p)$, we calculate the πN s-wave scattering lengths in the Born approximation. It will turn out that only the A-solution will result in the correct values of the wave lenghts (the B-solutions give the combinations $a_1 - a_3$ and $a_1 + 2a_3$ of the same sign that contradicts the experimental data).

3. The πN s-Wave Scattering Lengths

In the lowest order in the coupling constant f the πN-interaction is described by two diagrams

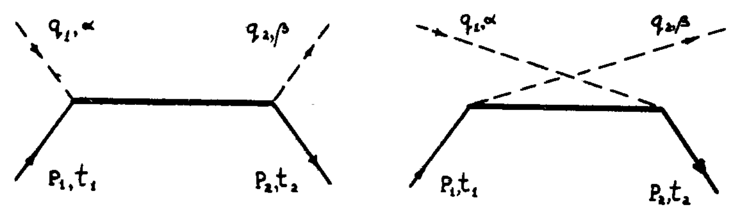


Fig. 1

The solid line stands for the nucleon Green function S_λ, $p_{1,2}$ - the nucleon momenta, $q_{1,2}$ the meson momenta, $t_{1,2}$, α, β the isotopic indices of nucleons and mesons.

The invariant amplitudes $A^{(\pm)}$ and $B^{(\pm)}$ (we follow the notation of book[5]) can be represented in the form:

$$A^{(\pm)}(s,u) = -f^2 M\{(s-M^2)[S_\lambda^{(1)}(s)+S_\lambda^{(2)}(s)] \pm (u-M^2)\times \times[S_\lambda^{(1)}(u)+S_\lambda^{(2)}(u)]\}, \tag{3.1}$$

$$B^{(\pm)}(s,u) = f^2\{2M^2[S_\lambda^{(1)}(s) \mp S_\lambda^{(1)}(u)] + (s+M^2)S_\lambda^{(2)}(s) \mp (u+M^2)S_\lambda^{(2)}(u)\}. \tag{3.2}$$

Hence for the s-wave lengths we have the following relations

$$\frac{1}{3}(a_1-a_3) = \frac{f^2}{4\pi}\frac{M^2}{(M+m)}\{-M(S_1^+-S_1^-)+(M+m)(S_2^+-S_2^-)\}, \tag{3.3}$$

$$\frac{1}{3}(a_1+2a_3) = \frac{f^2}{4\pi}\frac{M^2}{(M+m)}\{-M(S_1^++S_1^-)+(M+m)S_2^+ +(M-m)S_2^-\}, \tag{3.4}$$

where $S_j^+ = S_\lambda^{(j)}((M+m)^2)$, $S_j^- = S_\lambda^{(j)}((M-m)^2)$.

The calculation results for (3.3) and (3.4) depending on the choice of solution are given in the Table. We see that only one solution

$$W = -1.195 \qquad \lambda f^2 = 0.782 \tag{3.5}$$

results in the correct values for (3.3) and (3.4), viz:

$$a_1 - a_3 = 0.2904 \qquad a_1 + 2a_3 = -0.0371 \tag{3.6}$$

(we have chosen $f^2/4\pi = 0.072$) that is in good agreement with the experimental data (we cite the value recommended in ref.[6]):

$$a_1 - a_3 = 0.29^{+0.01}_{-0.02} \qquad a_1 + 2a_3 = -0.045\pm0.045. \tag{3.7}$$

Table

λf^2	W	a_1-a_3	a_1+2a_3
0.782	-1.195	0.2904	-0.0371
0.0478	20.1	-0.0244	-0.0064
0.0474	20.4	-0.0243	-0.0064
0.0470	20.8	-0.0248	-0.0063

Here $f^2/4\pi = 0.072$. We list only few B-solutions; for others the results change slightly.

Thus, in comparing the calculation results with the experimental data on measurement of the s-waves it appears to be possible to fix uniquely all known parameters of the model.

In the next section, within the framework of the proposed model, using the obtained parameters (3.5) we shall calculate the contribution to the neutron-proton electromagnetic mass difference in the lowest perturbation expansion order.

4. Proton-Neutron Electromagnetic Mass Difference

In the lowest order of perturbation expansion the following diagram

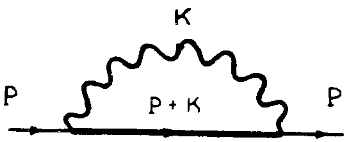


Fig. 2

contributes to the proton-neutron electromagnetic mass difference. The corresponding matrix element can be represented in the form

$$S_2(x,y) = -i : \bar{\Psi}(x)\, \Sigma(x-y)\, \Psi(y): \tag{4.1}$$

where

$$\Sigma(x-y) = \frac{1}{(2\pi)^4} \int d^4x\, e^{-ip(x-y)}\, \Sigma(p) \tag{4.2}$$

and

$$\Sigma(p) = \frac{ie^2}{(2\pi)^4} \int \frac{d^4k}{k^2}\, \gamma_\mu S_\lambda(p+k)\gamma^\mu . \tag{4.3}$$

Inserting here the spectral representations (2.4) for the functions $S_\lambda^{(1)}$ and $S_\lambda^{(2)}$ and integrating over k we get

$$\delta M = M_p - M_n = \Sigma(M)$$

where

$$\Sigma(M) = \frac{\alpha M}{2\pi} \left\{ \frac{1}{2} + \int_{(M+m)^2}^{\infty} dz \left[2\rho_\lambda^{(1)}(z) - \rho_\lambda^{(2)}(z) \right] \times \right. \\ \left. + \left[\frac{z}{M^2} \ln \frac{z-M^2}{z} - \ln \frac{z-M^2}{M^2} \right] \right\}. \tag{4.4}$$

In calculating the integral (4.3) we have used conditions (2.9) due to which this integral converges.

The subsequent integration in (4.4) was made numerically. For solution (3.5) one obtains

$$\delta M = -0.455 \text{ MeV} \tag{4.5}$$

that coincides both in the sign and in the order of magnitude with the experimental value

$$\delta M^{exp} = -1.293 \text{ MeV}. \tag{4.6}$$

5. Concluding Remarks

Recall briefly the results obtained in this paper. From the very beginning we aimed to achieve the super-renormalizability of the model. This aim has been achieved with the use of the summation hypothesis /3/ and the corresponding regrouping of the perturbation theory expansion. Then in the model there still were two free parameters (solutions of the system (2.9)) which we could fix uniquely by comparing the calculation results for the s-wave and experimental values. Using the nucleon Green function S_λ and the obtained values of parameters λ and W, we have calculated the lowest perturbation expansion order contribution to the neutron-proton electromagnetic mass difference.

Note that in previous models of πN-interactions the coupling constant was large and, consequently, it was impossible to employ the perturbation theory expansions for observable quantities.

Within our model this difficulty is partly removed. Indeed, if the representations (2.4) and conditions (2.9) are taken into account, then it is easy to see that the functions $S_\lambda^{(1)}$ and $S_\lambda^{(2)}$ can be represented in the form

$$S_\lambda^{(j)}(p^2) = \int_{(M+m)^2}^{\infty} \frac{dz\, \rho_\lambda^{(j)}(z)(z-M^2)}{(z-p^2)(M^2-p^2)} \tag{5.1}$$

and therefore the "cut-off" of divergent integrals occurs at the momenta

$$\Lambda^2 \sim -\int_{(M+m)^2}^{\infty} dz\, z\, \rho_\lambda^{(j)}(z) \approx 1.2M^2 \ . \tag{5.2}$$

For the low-energy effects the parameter of perturbation theory expansion will be the following quantity

$$\left(\frac{f\Lambda}{4\pi m}\right)^2 \approx \frac{1}{3}$$

therefore the obtained results (3.6) and (4.5) may be regarded not very sceptically.

It seems interesting to analyse this model in more detail. This work is now in progress.

In conclusion the authors thank G.V.Efimov and V.A.Meshcheryakov for stimulating discussions and critical remarks.

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