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ON THE AZIMUTHAL DEPENDENCE OF THE INCLUSIVE ELECTROPRODUCTION

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1. The process of deep inelastic electroproduction has been of dominant interest in particle physics in recent years. The fundamental investigations by N.Bogolubov, A.Tavkhelidze and V.Vladimirov /1/ on automodel asymptotics in quantum field theory have provided a rigorous basis for the light-cone analysis of this process.

A recent progress in the experimental technique for observing inelastic lepton scattering makes it interesting a systematic study of the inclusive electroproduction, i.e., the process where in the final state in addition to lepton also one hadron is detected /2/ .Among the theoretical descriptions of this process in deep inelastic region the following two main approaches are singled out: the parton model /3/ and the light-cone analysis /4,5/. In the parton model the asymptotic behaviour of all structure functions of the inclusive electroproduction has been investigated, and thus, the information about the azimuthal dependence of the cross section has been obtained /6/ .

In this note it is shown, that the same information in the target fragmentation region can be obtained in the framework of the second approach, using the generalization of the Mueller's theorem, suggested in /7/ , for the inclusive processes with polarized particles.

2. Consider the process of inclusive electroproduction

$$\ell + a \to \ell' + a' + \ll \text{"anything"} \gg . \quad (1)$$

In the one-photon approximation the kinematic conventions are shown in fig. 1.

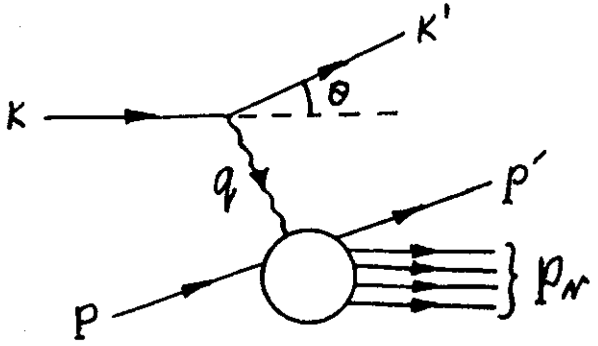

Fig. 1. Kinematics of inclusive electroproduction.

Here $k(k')$ are the 4-momenta of the incident and final electron, p is the 4-momentum of the target, p' is the 4-momentum of the hadron singled out in the final state, $q = k - k'$ is the 4-momentum of the virtual photon and θ is the electron scattering angle in lab.frame.

The differential cross section is written as follows /1/

$$d\sigma = \frac{1}{4pk}\,\frac{(4\pi\alpha)^2}{(q^2)^2}\,W_{\mu\nu}\,\ell^{\mu\nu}\,\frac{d\vec{k}'}{(2\pi)^3 4k_0'}\,\frac{d\vec{p}'}{(2\pi)^3 2p_0'}\,. \quad (2)$$

For ultrarelativistic electrons with longitudinal polarization η the leptonic tensor is:

$$\ell^{\mu\nu} = 2[k^\mu k'^\nu + k^\nu k'^\mu + \frac{q^2}{2} g^{\mu\nu} - i\eta\epsilon^{\mu\nu\alpha\beta} q_\alpha k'_\beta]. \quad (3)$$

By definition, the hadronic tensor equals

$$W_{\mu\nu} = \overline{\sum_N} (2\pi)^4\, \delta(p + q - p' - p_N) \langle p|J_\mu(0)|p'N\rangle_c \langle p'N|J_\nu(0)|p\rangle_c\,. \quad (4)$$

We expand this tensor over the structure corresponding to the definite helicities of the virtual photon /9/

$$W_{\mu\nu} = \sum_{a,b=+,0,-,L} \epsilon^a_\mu \overset{*}{\epsilon}{}^b_\nu H_{ab}\,. \quad (5)$$

Taking into account the properties of the polarization vectors ϵ^μ_a the expansion (5) could easily be conversed

$$H_{ab} = \overset{*}{\epsilon}{}^\mu_a \epsilon^\nu_b W_{\mu\nu}\,. \quad (6)$$

From the current conservation, using $\epsilon^\mu_L = \frac{q^\mu}{\sqrt{-q^2}}$ it follows that $H_{aL} = H_{La} = 0$ (so further the indices a and b will denote $+, 0, -$.). The "hermiticity" of hadron tensor $W_{\mu\nu} = W^*_{\nu\mu}$ implies that $H_{ab} = H^*_{ba}$ and the parity conservation gives $H_{ab} = H_{-a,-b}$. Thus the hadron tensor can be expressed in terms of the five independent real structures which are chosen to be H_{00}, H_{++}, H_{+-}, $\mathrm{Re}\, H_{+0}$, $\mathrm{Im}\, H_{+0}$.

In the lab. frame it is convenient to take the axes x, y, z as is shown in fig. 2, so H_{ab} are now functions of the four independent Lorentz scalars $Q^2 = -q^2$, $\nu = pq$, $\nu' = p'q$, $\kappa = pp'$ and are independent of the azimuthal angle Φ.

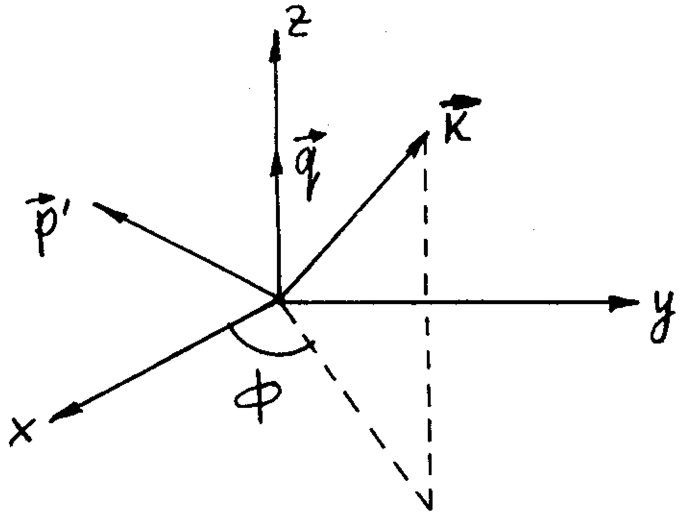

Fig. 2. Coordinate axes in the lab. frame. z axis is directed along $\vec{q}$, $\vec{p}'$ lies in xz-plane. Polarization vectors are $\epsilon^\mu_\pm = \frac{1}{\sqrt{2}}(0, 1, \pm i, 0)$, $\epsilon^\mu_0 = \frac{1}{\sqrt{-q^2}}(q^3, 0, 0, q^0)$.

Process (1) can be treated as an emission of virtual photons by electron and subsequent interaction of this photons with the target. The helicity density matrix of virtual photons is determined by leptonic part of the process (1). From (2), (4), (5) and (6) it follows that $\epsilon^{ab} \sim \ell^{\mu\nu} \epsilon^{a}_{\mu} \overset{*}{\epsilon}{}^{b}_{\nu}$. The coefficient of proportionality can be found from $\mathrm{Tr}\,\epsilon^{ab} = 1$. Thus, for the helicity density matrix of virtual photons we have

$$\epsilon^{a,b} = \frac{1}{2(1+\epsilon)} \begin{pmatrix} 1+\eta\sqrt{1-\epsilon^2}, & \sqrt{\epsilon(1+\epsilon)}\,e^{i\Phi} - \eta\sqrt{\epsilon(1-\epsilon)}\,e^{i\Phi}, & \epsilon\, e^{2i\Phi} \\ \sqrt{\epsilon(1+\epsilon)}\,e^{-i\Phi} - \eta\sqrt{\epsilon(1-\epsilon)}\,e^{-i\Phi}, & 2\epsilon, & \sqrt{\epsilon(1+\epsilon)}\,e^{i\Phi} + \eta\sqrt{\epsilon(1-\epsilon)}\,e^{i\Phi} \\ \epsilon\, e^{-2i\Phi}, & \sqrt{\epsilon(1+\epsilon)}\,e^{-i\Phi} + \eta\sqrt{\epsilon(1-\epsilon)}\,e^{-i\Phi}, & 1-\eta\sqrt{1-\epsilon^2} \end{pmatrix} \tag{7}$$

where $\epsilon = \dfrac{1}{1 + 2\left(1 + \dfrac{\nu^2}{m^2 Q^2}\right) \mathrm{tg}^2 \dfrac{\theta}{2}}$.

Finally for the cross section of the process (1) we have

$$\frac{d\sigma}{dQ^2 d\nu\, d\nu' d\kappa\, d\Phi} = \frac{\alpha}{4\pi} \frac{1}{(mE)^2} \frac{2\nu - Q^2}{Q^2} \frac{1+\epsilon}{1-\epsilon} \frac{d\sigma^{\gamma^*}}{d\nu' d\kappa\, d\Phi}, \tag{8}$$

where the cross section of virtual photoproduction equals

$$\frac{d\sigma^{\gamma^*}}{d\nu' d\kappa\, d\Phi} = \frac{\alpha}{(2\pi)^2} \frac{1}{2\sqrt{\nu^2 + m^2 Q^2}} \frac{1}{2\nu - Q^2} \sum_{a,b} \epsilon^{ab} H_{ab} =$$

$$= \frac{\alpha}{(2\pi)^2} \cdot \frac{1}{2\sqrt{\nu\ \ + m^2 Q^2}\,(2\nu - Q^2)(1-\epsilon)} \times \tag{9}$$

$$\times \{ H_{++} + \epsilon H_{00} + \epsilon \cos 2\Phi H_{+-} + 2\sqrt{\epsilon(1+\epsilon)} \cos\Phi \,\mathrm{Re} H_{+0} - $$

$$- 2\eta \sqrt{\epsilon(1-\epsilon)} \sin\Phi\, \mathrm{Jm} H_{+0} \} .$$

Hence, the problem of studying the process (1) is completely equivalent to the problem of virtual photoproduction with one hadron singled out in the final state

$$\gamma^* + a \to a' + \ll \text{''anything''} \gg . \tag{10}$$

3. As is known /8/, the one-particle inclusive distribution is connected with a definite discontinuity of the amplitude of the forward three-particle scattering. For studying the process (1) it must be taken into account that even if we are not interested in polarization of incident and outgoing particles, the virtual photon in (10) nevertheless turns out to be polarized.

Generalization of the Mueller's theorem to the case of inclusive reactions with polarized particles has been suggested in /7/. For process (10) the generalized Mueller's theorem gives

$$H_{a,b} = \mathrm{Disc}\, \mathcal{H}_{a,b} , \tag{11}$$

where $\mathcal{H}_{a,b}$ is the amplitude of the "forward" $3 \to 3$ scattering (fig. 3).

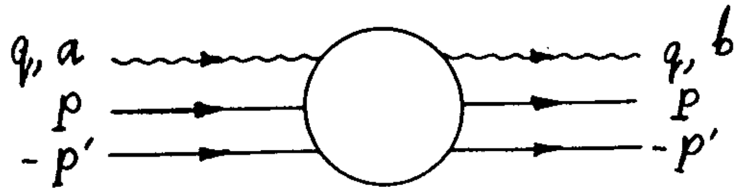

Fig. 3. 3-particle "forward" scattering amplitude. Indices a and b denote the helicities of the incident and final photons.

The part of $\mathcal{H}_{a,b}$, we are interested in equals (averaging and summation over hadron spin are implied)

$$\mathcal{H}_{a,b} = \overset{*}{\epsilon}{}^{a}_{\mu} \epsilon^{b}_{\nu} \int dx\, e^{iqx} \langle p, -p' | \theta(x_0) [J^{\mu}(x), J^{\nu}(0)] | p, -p' \rangle . \tag{12}$$

We shall look for the asymptotic form of the structure functions in the limit

$$\nu \to \infty, \quad \frac{Q^2}{\nu}, \quad \frac{\nu'}{\nu}, \quad \kappa \quad \text{- fixed} \tag{13}$$

further on denoted by a symbol $=>$ (deep-inelastic target fragmentation region). The asymptotic form of $\mathcal{H}_{a,b}$ in the limit (13) is determined by the behaviour of the current commutators near the light-cone. Assuming /10/ that $[J^{\mu}(x), J^{\nu}(0)]$ for $x^2 \to 0$ behaves like a commutator of free quark currents we can write (all unimportant factors are included into the unknown functions F_i)

$$\langle p, -p' | \theta(x_0) [J^{\mu}(x) J^{\nu}(0)] | p, -p' \rangle \underset{x^2 \to 0}{\simeq}$$

$$\simeq s_{\mu\nu\alpha\beta} [p^{\alpha} F_1(px, p'x, \kappa) + p'^{\alpha} F_2(px, p'x, \kappa) + \tag{14}$$

$$+ x^{\alpha} F_3(px, p'x, \kappa)] \partial^{\beta} \theta(x_0) \delta(x^2),$$

where

$$s_{\mu\nu\alpha\beta} = g_{\mu\alpha} g_{\nu\beta} + g_{\mu\beta} g_{\nu\alpha} - g_{\mu\nu} g_{\alpha\beta} .$$

We introduce the Fourier transforms of the functions $F_i(px, p'x, \kappa)$

$$F_i(px, p'x, \kappa) = \int_{-\infty}^{+\infty} d\xi \, d\xi' f_i(\xi, \xi', \kappa) e^{i\xi px + i\xi' p'x} . \tag{15}$$

Taking into account (15), (14) and (12) from (11) we obtain

$$\mathcal{H}_{a,b} => \epsilon_a^{*\mu} \epsilon_b^{\nu} s_{\mu\nu\alpha\beta} \mathrm{Disc} \iint d\xi d\xi' \{ [p^{\alpha} f_1(\xi, \xi' \kappa) + \tag{16}$$

$$+ p'^{\alpha} f_2(\xi, \xi', \kappa)] \times (q + \xi p + \xi' p')^{\beta} \epsilon(q_0 + \xi p_0 + \xi' p'_0) \delta[(q + \xi p + \xi p')^2] +$$

$$+ f_3(\xi, \xi', \kappa) \frac{\partial}{\partial q_{\alpha}} (q + \xi p + \xi' p')^{\beta} \epsilon(q + \xi p + \xi' p') \delta[(q + \xi p + \xi' p')^2] .$$

From (16) it is easy to obtain the following automodel behaviour of the structure functions:

$$\begin{aligned} H_{++} &=> \Phi_{++}\left(\frac{Q^2}{\nu}, \frac{\nu'}{\nu}, \kappa\right) \\ H_{00} &=> \frac{1}{\nu} \Phi_{00}\left(\frac{Q^2}{\nu}, \frac{\nu'}{\nu}, \kappa\right) \\ H_{+-} &=> \frac{1}{\nu} \Phi_{+-}\left(\frac{Q^2}{\nu}, \frac{\nu'}{\nu}, \kappa\right) \\ H_{+0} &=> \frac{1}{\sqrt{\nu}} \Phi_{+0}\left(\frac{Q^2}{\nu}, \frac{\nu'}{\nu}, \kappa\right) . \end{aligned} \tag{17}$$

The function Φ_{ab} cannot be found explicitly in the framework of this approach.

The automodel behaviour of the structure functions (17) was obtained assuming that the asymptotic form of the discontinuity of amplitudes coincide with that of the amplitudes itself. Some grounds for the assumptions can be found in /4/.

Hence, in the target fragmentation region light-cone analysis predicts the same azimuthal behaviour of the process (1) as the parton model, i.e., absence of the dependence of the cross section on Φ.

The detailed experimental examination of these predictions has not yet been carried out in this kinematical region.

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