СООБЩЕНИЯ
ОБЪЕДИНЕННОГО
ИНСТИТУТА
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

ДУБНА

E2 - 8308

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RENORMALIZATION OF SUPERSYMMETRIC GAUGE THEORIES. I. QUANTUM ELECTRODYNAMICS

1974

ЛАБОРАТОРИЯ
ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

E2 - 8308

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RENORMALIZATION OF SUPERSYMMETRIC GAUGE THEORIES.

I. QUANTUM ELECTRODYNAMICS

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E2 - 8308

Перенормировка суперсимметричных калибровочных теорий.
1. Квантовая электродинамика

Построена инвариантная процедура перенормировки для суперсимметричной квантовой электродинамики. Показано, что все ультрафиолетовые расходимости устраняются общей перенормировкой волновых функций и масс полей материи и перенормировкой волновой функции калибровочного мультиплета.

Сообщение Объединенного института ядерных исследований
Дубна, 1974

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E2 - 8308

Renormalization of Supersymmetric Gauge Theories. I. Quantum Electrodynamics

An invariant renormalization procedure for supersymmetric quantum electrodynamics is constructed. It is shown that all ultraviolet divergencies may be removed by the common wave function and mass renormalization of matter fields and the wave function renormalization of the gauge multiplet.

Communications of the Joint Institute for Nuclear Research.
Dubna, 1974

The concept of supersymmetry introduced recently by Wess and Zumino /1/ seems to open new possibilities for the construction of weak electromagnetic and strong interaction models. Different aspects of this new symmetry were investigated by different authors (for references see /2/). Supersymmetry manifests itself, in particular, in the existence of strong constraints on renormalization constants. These constraints were shown to reduce in some cases the total number of renormalization constants to one /3/ . The present paper is devoted to the investigation of renormalization program for supersymmetry gauge theories. Supersymmetric generalization of quantum electrodynamics was given in paper /4/ and of non-abelian gauge theories in papers /5, 6/.

Supersymmetric gauge invariant Lagrangians appear to be highly nonlinear and therefore at the first sight nonrenormalizable. However it was shown /4,5,6/ that a special gauge exists in which all terms containing more than four fields drop out and the Lagrangian reduces to the ordinary gauge invariant terms plus some additional renormalizable interaction of scalar and spinor particles. Unfortunately, this re -

markable gauge is not supersymmetric. So it is completely unclear if it is possible to renormalize the theory in a supersymmetric way. Up to now the problem was solved only for one loop diagrams.

We propose manifestly supersymmetric renormalization procedure for gauge theories. Supersymmetric gauge will be used instead of the noninvariant Wess-Zumino gauge. It will be shown that in spite of nonlinear character of the Lagrangian the total number of independent counterterms is finite and the renormalized Lagrangian is supersymmetric and gauge invariant.

I.

In this section we use the notation of the papers /1,4/ except for our γ-matrices satisfy the relation

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu}; \quad g = \{1,-1,-1,-1\}$$

and $a^\mu b^\mu \equiv g^{\mu\mu} a^\mu b^\mu$. Matter fields are combined in scalar supermultiplets

$$S_i = (A_i, B_i, \Psi_i, \mathcal{F}_i, G_i), \quad i = 1,2 . \tag{1}$$

Multiplets S_1 and S_2 correspond to the real and imaginary parts of a complex scalar supermultiplet. Gauge fields form vector supermultiplet

$$V = (C, \chi, M, N, V_\mu, \lambda, \mathcal{D}), \tag{2}$$

where $C, M, N, \mathcal{D}$ are scalars, χ, λ – Maiorana spinors and V_μ – Hermitian vector field.

Generalized gauge transformation looks as follows

$$\begin{aligned} \delta S_1 &= g S S_2 \\ \delta S_2 &= -g S S_1 \end{aligned} \quad ; \quad \delta V = \partial S . \tag{3}$$

Here ∂S is a vector multiplet with the components

$$C' = B,\ \chi' = \Psi,\ M' = \mathcal{F},\ N' = G,\ V'_\mu = \partial_\mu A,\ \lambda' = 0,\ \mathcal{D}' = 0 . \tag{4}$$

Symbol $S S_i$ means symmetric scalar product of S and S_i

$$\begin{aligned} S S_i &= S'_i = \{A'_i, B'_i, \Psi'_i, \mathcal{F}'_i, G'_i\} \\ A'_i &= A A_i - B B_i \\ B'_i &= A B_i + B A_i \\ \Psi'_i &= (A - \gamma_5 B)\Psi_i + (A_i - \gamma_5 B_i)\Psi \\ \mathcal{F}'_i &= \mathcal{F} A_i + \mathcal{F}_i A + G B_i + G_i B - \bar{\Psi}\Psi_i \\ G'_i &= G A_i + G_i A - \mathcal{F} B_i - \mathcal{F}_i B - \bar{\Psi}\gamma_5 \Psi_i \end{aligned} \tag{5}$$

Scalar multiplets may be also combined into vector multiplets

$$V_{I}=\frac{1}{2}\left(S_1\times S_1+S_2\times S_2\right) \tag{6}$$

and

$$V_{II}=S_1\wedge S_2 . \tag{7}$$

The first product is symmetric and the second one antysymmetric. Their explicit form may be found in /4/.

Vector multiplets in turn can be combined symmetrically into another vector multiplet $V'=V_1V_2$

$$
\begin{aligned}
C' &= C_1C_2\\
\chi' &= C_1\chi_2+C_2\chi_1\\
V_\mu' &= C_1V_{\mu 2}+C_2V_{\mu 1}+\frac{i}{2}\bar{\chi}_1\gamma_5\gamma_\mu\chi_2\\
M' &= C_1M_2+C_2M_1-\frac{1}{2}\bar{\chi}_1\gamma_5\chi_2\\
N' &= C_1N_2+C_2N_1-\frac{1}{2}\bar{\chi}_1\chi_2\\
\lambda' &= C_1\lambda_2+C_2\lambda_1-\frac{i}{2}\hat{\partial}C_1\chi_2-\frac{i}{2}\hat{\partial}C_2\chi_1+\frac{1}{2}M_1\gamma_5\chi_2+\frac{1}{2}M_2\gamma_5\chi_1+\\
&+\frac{1}{2}N_1\chi_2+\frac{1}{2}N_2\chi_1-\frac{i}{2}V_{\mu 1}\gamma_5\gamma_\mu\chi_2-\frac{i}{2}V_{\mu 2}\gamma_5\gamma_\mu\chi_1
\end{aligned}
\tag{8}
$$

$$
\begin{aligned}
\mathcal{D}' &= C_1\mathcal{D}_2+C_2\mathcal{D}_1+\partial C_1\partial C_2+V_{\mu 1}V_{\mu 2}+M_1M_2+N_1N_2-\bar{\chi}_1\lambda_2-\\
&-\bar{\chi}_2\lambda_1+\frac{i}{2}\partial_\mu\bar{\chi}_1\gamma^\mu\chi_2+\frac{i}{2}\partial_\mu\bar{\chi}_2\gamma^\mu\chi_1 .
\end{aligned}
$$

The Lagrangian invariant with respect to the generalized gauge transformations (3) is

$$
\begin{aligned}
\mathcal{L} &= \frac{1}{4}\left(V_a e^{2gV}+V_b e^{-2gV}\right)_{\mathcal{D}}+\frac{m}{2}\left(S_1S_1+S_2S_2\right)_{\mathcal{F}}-\\
&-\frac{1}{4}\left(\partial_\mu V_\nu-\partial_\nu V_\mu\right)^2-\frac{i}{2}\bar{\lambda}\hat{\partial}\lambda+\frac{1}{2}\mathcal{D}^2 ,
\end{aligned}
\tag{9}
$$

where $(\ )_{\mathcal{D}}$ and $(\ )_{\mathcal{F}}$ mean $\mathcal{D}$ and $\mathcal{F}$ components of the vector and scalar multiplets, respectively, and

$$V_a=V_{I}+V_{II} , \qquad V_b=V_{I}-V_{II} .$$

The Lagrangian (9) is invariant also with respect to supersymmetry transformations of S_i and V .

$$
\begin{aligned}
\delta A_i &= \bar{\alpha}\Psi_i\\
\delta B_i &= \bar{\alpha}\gamma_5\Psi_i\\
\delta\Psi_i &= i\left(\partial_\mu[A_i-\gamma_5B_i]\gamma^\mu\right)\alpha+\left(\mathcal{F}_i+G_i\gamma_5\right)\alpha\\
\delta\mathcal{F}_i &= i\bar{\alpha}\hat{\partial}\Psi\\
\delta G_i &= i\bar{\alpha}\gamma_5\hat{\partial}\Psi
\end{aligned}
\tag{10}
$$

$$\delta D = i\bar{\alpha}\gamma_5 \hat{\partial}\lambda$$

$$\delta C = \bar{\alpha}\gamma_5 \chi$$

$$\delta M = \bar{\alpha}\lambda + i\bar{\alpha}\hat{\partial}\chi$$

$$\delta N = \bar{\alpha}\gamma_5\lambda + i\bar{\alpha}\gamma_5\hat{\partial}\chi$$

$$\delta V_\mu = -i\bar{\alpha}\gamma_\mu\lambda + \bar{\alpha}\partial_\mu\chi$$

$$\delta\chi = i\gamma_\mu V_\mu \alpha - i\gamma_5\hat{\partial}C\alpha + (M+\gamma_5 N)\alpha \tag{11}$$

$$\delta\lambda = \frac{1}{2}(\partial_\mu V_\nu - \partial_\nu V_\mu)\gamma_\mu\gamma_\nu\alpha + D\gamma_5\alpha$$

Here α is infinitesimal Maiorana spinor independent of x. The Lagrangian (9) is highly nonlinear and nonrenormalizable in a usual sense. However gauge condition still should be imposed. Wess and Zumino noticed that one can choose the gauge

$$C = M = N = \chi = 0 \ . \tag{12}$$

In this gauge $(V^n) = 0$, for $n \geqslant 3$, and interaction term in (9) becomes

$$g V_{II} V + g^2 V_I V^2 \ . \tag{13}$$

This interaction corresponds to renormalizable theory. But the gauge condition (12) destroys the invariance under the supersymmetry transformations (10,11), and therefore the important information is lost.

In fact in this gauge we have some renormalizable Lagrangian but we know nothing about its supersymmetry properties. In particular, it is far from evident that renormalized masses and coupling constants are equal.

For this reason we abandon the Wess-Zumino gauge and impose a supersymmetric subsidiary condition. To do that we add to the Lagrangian (9) a gauge fixing term

$$\frac{1}{4\beta}\left(\partial V \times \partial V\right)_{\mathcal{F}} \ , \tag{14}$$

where ∂V is a scalar multiplet constructed from the vector multiplet V:

$$\begin{aligned} \tilde{A} &= \partial_\mu V_\mu \\ \tilde{B} &= \Box C - D \ ; \\ \tilde{\psi} &= \Box\chi - i\hat{\partial}\lambda \end{aligned} \qquad \begin{aligned} \mathcal{F} &= \Box M \\ G &= \Box N \end{aligned} \tag{15}$$

The term (14) evidently preserves the invariance under the transformations (10,11).

An explicit expression for the quadratique form defining free propagators is

$$\mathcal{L}_0 = \frac{1}{2}\sum_{i=1,2}\left\{(\partial A_i)^2 + (\partial B_i)^2 + \mathcal{F}_i^2 + G_i^2 - i\bar{\psi}_i\hat{\partial}\psi_i + m\left(\mathcal{F}_i A_i + G_i B_i - \frac{1}{2}\bar{\psi}_i\psi_i\right)\right\} +$$

$$+\frac{1}{2\beta}\left\{\left[\partial_\mu(\Box C-\mathcal{D})\right]^2+\left[\partial_\mu(\partial_\rho V_\rho)\right]^2-i(\Box\bar{\chi}-i\partial_\mu\bar{\lambda}\gamma_\mu)\hat{\partial}(\Box\chi-i\hat{\partial}\lambda)+\right.$$

$$\left.+(\Box M)^2+(\Box N)^2\right\}-\frac{1}{4}(\partial_\mu V_\rho-\partial_\rho V_\mu)^2-\frac{i}{2}\bar{\lambda}\hat{\partial}\lambda+\frac{1}{2}\mathcal{D}^2 . \tag{16}$$

The propagators have the following asymptotical behaviour

$$\mathcal{D}_{\mathcal{F}\mathcal{F}}\sim\mathcal{D}_{GG}\sim\mathcal{D}_{DD}\sim 1\ ,\quad G_{\psi\psi}\sim G_{\lambda\lambda}\sim K^{-1}$$

$$\mathcal{D}_{AA}\sim\mathcal{D}_{BB}\sim\mathcal{D}_{A\mathcal{F}}\sim\mathcal{D}_{BG}\sim\mathcal{D}_{\chi\lambda}\sim\mathcal{D}_{VV}\sim K^{-2}\sim\mathcal{D}_{C\mathcal{D}} \tag{17}$$

$$\mathcal{D}_{MM}\sim\mathcal{D}_{NN}\sim K^{-4},\quad \mathcal{D}_{\chi\chi}\sim K^{-5},\quad \mathcal{D}_{CC}\sim K^{-6} .$$

In fact introducing more derivatives in the condition (14) one may obtain $\mathcal{D}_{CC}$, $\mathcal{D}_{MM}$, $\mathcal{D}_{NN}$ and $\mathcal{D}_{\chi\chi}$ propagators decreasing as K^{-n} , with arbitrary n .

Therefore any diagram including at least one such line is superficially convergent. This fact simplifies enormously the analysis of primitive divergences. The interaction Lagrangian is an infinite sum of the terms, which may be presented symbolically as

$$\mathcal{L}_I\sim\sum_{n,m}A^2(C^{n_1}\mathcal{D}+\partial^2C^{n_2}+C^{n_3}V_\mu^2+C^{n_4}V_\mu\chi^2+C^{n_5}\chi^4+\partial C^{n_6}\chi^2+\partial C^{n_7}V_\mu)+$$

$$+\bar{\Psi}\Psi(\partial C^{m_1}+C^{m_2}V_\mu)+\mathcal{F}^2C^{m_3}+A\mathcal{F}(C^{m_4}+C^{m_5}\chi^2)+\mathcal{F}\bar{\Psi}C^{m_6}\chi+$$

$$+A\bar{\Psi}(C^{m_7}\lambda+\partial C^{m_8}\chi+C^{m_9}\chi^3+C^{m_{10}}V_\mu\chi) \tag{18}$$

Being at present interested only in the calculation of the degree of divergency we omit in this formula all constants and tensor structure, and denote symbolically $A=\{A,B\}$, $\mathcal{F}=\{\mathcal{F},G\}$.

Some vertices may also have additional factors M and N , which we did not write explicitly because the diagrams with MM or NN internal lines are superficially convergent.

Using (18) one can easily calculate degree of divergency for arbitrary diagram. The answer is

$$n\leq 4-\ell_A-2\ell_D-\ell_V-2\ell_{\mathcal{F}}-\frac{3}{2}\ell_\psi-\frac{3}{2}\ell_\lambda . \tag{19}$$

Here ℓ_I denotes the number of external I -lines. So only the diagrams with at most 2 $\mathcal{D}$, $\mathcal{F}$, ψ or λ external lines, and 4 A or V_μ external lines diverge. The number of C, M, N, χ external lines, which correspond to gauge degrees of freedom may be arbitrary, so there is an infinite number of primitively divergent diagrams and the theory is nonrenormalizable in a usual sense. But as we shall show below, generalized Ward identities allow to express Green functions with n external C, M, N or χ lines in terms of Green functions with $n-1$ lines. Therefore it is sufficient to eliminate divergencies in a finite number of "basic" diagrams (i.e., diagrams without C, M, N, χ external lines). It will automatically make all Green functions finite.

We shall write down generalizad Ward identities associated with the transformations (3) and (10,11) and show that due to these identities only three independent counterterms are needed – overall wave function and mass renormalization for matter fields, and wave function renormalization for gauge multiplet.

So we claim that Lagrangian

$$\mathcal{L}_R=\frac{Z}{4}\left(V_a e^{2gV}+V_b e^{-2gV}\right)_D+Z\,\frac{m+\delta m}{2}\left(S_1 S_1+S_2 S_2\right)_{\mathcal{F}}- \tag{20}$$
$$-\frac{Z_3}{4}\left[\left(\partial_\mu V_\nu-\partial_\nu V_\mu\right)^2-2i\bar{\lambda}\hat{\partial}\lambda+2\mathcal{D}^2\right],$$

where Z, Z_3 and δm are suitably chosen counterterms, leads to the finite S-matrix. This Lagrangian evidently possesses the same invariance properties as the original one. This invariance manifests itself in the generalizad Ward identities for the Green function generating functional

$$Z(J_V, J_{S_i})=N^{-1}\int \exp\Big\{i\int\Big[\mathcal{L}_R(x)+\frac{1}{4\beta}\left(\partial V\times\partial V\right)_{\mathcal{F}}+ \tag{21}$$
$$+\left(J_V\cdot V\right)_D+\sum_{i=1,2}\left(J_{S_i}\cdot S_i\right)_{\mathcal{F}}\Big]dx\Big\}\,d\mu\,.$$

Here J_{S_i} and J_V are scalar and vector supermultiplets of sources.

Invariance of the functional (21) under the change of variables (3) leads to the first set of identities which may be written in a compact form:

$$\int\Big\{\frac{1}{2\beta}\left(\partial V\times\Box S\right)_{\mathcal{F}}+g\sum_{i,k=1,2}^{i\neq k}(-1)^k\left[J_{S_i}\cdot\left(S\cdot S_k\right)\right]_{\mathcal{F}}+\left(J_V\cdot\partial S\right)_D\Big\}\times \tag{22}$$
$$\times\exp\Big\{i\int\Big[\mathcal{L}_R(x)+\frac{1}{4\beta}\left(\partial V\times\partial V\right)_{\mathcal{F}}+\sum_{i=1,2}\left(J_{S_i}\cdot S_i\right)_{\mathcal{F}}+\left(J_V\cdot V\right)_D\Big]dx\Big\}\,d\mu=0\,.$$

Putting coefficients of A, B, Ψ, $\mathcal{F}$, G equal to zero one can easily obtain the explicit form of these equations. Thus the condition $\delta Z/\delta A=0$ gives the relation:

$$\frac{1}{\beta}\Box^2\partial_\mu\frac{\delta Z}{\delta J_{V_\mu}(x)}+i\,\partial_\mu J_{V_\mu}(x)-g\sum_{i,k=1,2}^{i\neq k}(-1)^k\Big\{J_{A_i}(x)\frac{\delta Z}{\delta J_{A_k}(x)}+ \tag{23}$$
$$+J_{B_i}(x)\frac{\delta Z}{\delta J_{B_k}(x)}+\bar{J}_{\Psi_i}\frac{\delta Z}{\delta\bar{J}_{\Psi_k}(x)}+J_{\mathcal{F}_i}\frac{\delta Z}{\delta J_{\mathcal{F}_k}(x)}+J_{G_i}\frac{\delta Z}{\delta G_k(x)}\Big\}=0$$

which is nothing but the usual Ward identity for electromagnetic field V_μ interacting with spinor Ψ and scalars A, B. By the usual arguments it follows from

(23) that

$$\partial_\mu \Pi_{\mu\nu} = 0, \quad \Gamma^{\mu}_{\nu\bar{\Psi}\Psi}(p) = \frac{\partial \Gamma_{\bar{\Psi}\Psi}}{\partial p^\mu}, \text{ etc.} \tag{24}$$

Remaining identities (22) express Green functions with arbitrary number of M , N , C , X external lines in terms of lower Green functions.

For example,

$$\frac{1}{\beta} \Box^2 \frac{\delta Z}{\delta J_M(x)} + \sum_{i,k=1,2}^{i\neq k} (-1)^k g \left\{ J_{\mathcal{F}_i}(x) \frac{\delta Z}{\delta J_{A_k}(x)} - J_{G_i}(x) \frac{\delta Z}{\delta J_{B_k}(x)} \right\} + \tag{25}$$

$$+ i J_M(x) = 0 .$$

In particular,

$$\beta^{-1} \Box^2 \langle M(x) M(y) \rangle_T = \delta(x-y) \tag{26}$$

$$\beta^{-1} \Box^2 \langle M(x) \mathcal{F}_i(y) A_k(z) \rangle_T = (-1)^k g\, \delta(x-y) \langle A_k(x) A_k(z) \rangle .$$

Analogous equations are valid for N , C , X Green functions (for C and X they are a little more complicated due to $C\mathcal{D}$ and $X\lambda$ mixing, but the result is the same). Therefore these Green functions need not independent renormalization and it is sufficient to renormalize only diagrams without M , N , C , X external lines. According to (19) there are a finite number of such diagrams, which

are primitively divergent. Necessary counterterms are fixed by the second set of generalized Ward identities, associated with the invariance under the supersymmetry transformations (10, 11).

The second set of identities may be written as

$$\int \exp\left\{ i \int \left[\mathcal{L}_R(x) + \frac{1}{4\beta} (\partial V \times \partial V)_D + (J_V \cdot V)_D + \sum_{i=1,2} (J_{S_i} \cdot S_i)_{\mathcal{F}} \right] dx \right\} \times$$

$$\times \int \left\{ \left[J_V(x) \delta V(x) \right]_D + \sum_{i=1,2} \left[J_i(x) \delta S_i(x) \right]_{\mathcal{F}} \right\} dx \, d\mu = 0 , \tag{27}$$

where δS_i and δV are given by the formula (10,11).

It is convenient to express eq. (27) in terms of one-particle irreducible Green functions, generated by the functional

$$\Gamma(R_V, R_{S_i}) = W - \int \left\{ (J_V \cdot R_V)_D + \sum_{i=1,2} (J_{S_i} \cdot R_{S_i})_{\mathcal{F}} \right\} dx , \tag{28}$$

where

$$W(J_V, J_{S_i}) = i \ln Z(J_V, J_{S_i})$$

$$R_{V(S_i)} = \frac{\delta W}{\delta J_{V(S_i)}} ; \quad J_{V(S_i)} = -\frac{\delta \Gamma}{\delta R_{V(S_i)}} . \tag{29}$$

Eq. (27) becomes

$$\int \Big\{ \Big(\Big(i \frac{\delta\Gamma}{\delta R_D} \gamma^5 \hat{\partial} + i \frac{\delta\Gamma}{\delta R_{V_\mu}} \partial_\mu \Box^{-1} - i \gamma_\mu \frac{\delta\Gamma}{\delta R_{V_\mu}} \Big) R_{\bar{\lambda}} + \Big[\frac{1}{2} (\partial_\mu R_\nu - \partial_\nu R_\mu) \gamma^\mu \gamma^\nu + R_D \gamma_5 \Big] \frac{\delta\Gamma}{\delta R_\lambda} +$$

$$+ \sum_{i=1,2} \Big(\frac{\delta\Gamma}{\delta R_{A_i}} + \frac{\delta\Gamma}{\delta R_{B_i}} \gamma_5 + i \frac{\delta\Gamma}{\delta R_{F_i}} \hat{\partial} + i \frac{\delta\Gamma}{\delta R_{G_i}} \gamma_5 \hat{\partial} \Big) R_{\Psi_i} + \tag{30}$$

$$+ \sum_{i=1,2} \Big[(R_{\bar{F}_i} + \gamma_5 R_{G_i}) - i \hat{\partial} (R_{A_i} - \gamma_5 R_{B_i}) \Big] \frac{\delta\Gamma}{\delta R_{\Psi_i}} \Big\} dx = 0$$

(We omitted here the terms, originated from the sources of gauge components, as they give no new information).

Differentiating eq.(30) one easily obtains

$$-\Gamma_{A_i A_i} = -\Gamma_{B_i B_i} = p^2 \Gamma^{(1)}_{\Psi_i \Psi_i} = p^2 \Gamma_{F_i F_i} = p^2 \Gamma_{G_i G_i}$$

$$\Gamma_{A_i F_i} = \Gamma_{B_i G_i} = \Gamma^{(2)}_{\Psi_i \Psi_i}, \quad \Gamma_{DD} = \Gamma^{(1)}_{\lambda\lambda} = \Pi, \quad \Gamma^{(2)}_{\lambda\lambda} = 0, \tag{31}$$

where

$$-\Gamma_{\xi\xi} = \hat{p}\, \Gamma^{(1)}_{\xi\xi} + \Gamma^{(2)}_{\xi\xi}, \quad \Pi_{\mu\nu} = (g^{\mu\nu} \Box - \partial^\mu \partial^\nu) \Pi .$$

One can choose, for example

$$\Gamma_{A_i A_i} = \Gamma_{B_i B_i} = 0 .$$

$$-\Gamma'_{A_i A_i} = -\Gamma'_{B_i B_i} = \Gamma^{(1)}_{\Psi_i \Psi_i} = \Gamma_{F_i F_i} = \Gamma_{G_i G_i} = \Gamma_{DD} = \Gamma^{(1)}_{\lambda\lambda} = 1 . \tag{32}$$

$$\Gamma^{(2)}_{\Psi_i \Psi_i} = \Gamma_{A_i F_i} = \Gamma_{B_i G_i} = m . \qquad p^2 = 0 .$$

It is a simple exercise to show that analogous relations providing supersymmetry of renormalized theory are valid for three and four-point Green functions. For example,

$$\Gamma_{D A_i B_j} = -\Gamma_{D A_j B_i} = i\, \Gamma_{V A_i A_j} = -i\, \Gamma_{A_i \lambda \Psi_j} , \tag{33}$$

where

$$\Gamma_{V_\mu A_i A_j} = i p_\mu \Gamma_{V A_i A_j} .$$

Summarizing eqs.(24), (27), (28) we see that all ultraviolet infinities may be removed by the common wave function and mass renormalization of matter field, and wave function renormalization of gauge multiplet. As in the usual electrodynamics no mass renormalization for gauge fields is needed. That proves the assumption (20).

III. We showed that performing renormalization in the explicitly supersymmetric gauge one preserves symmetry properties of the unrenormalized theory. Of course, supersymmetric and gauge invariant regularization was assumed. There is no problem in constructing such regularization for the model under consideration. One can introduce, for example, higher covariant derivatives /7/, or use dimensional regularization /8/.

The technique described above may be directly transferred to the non abelian supersymmetric gauge theories. Imposing supersymmetric subsidiary condition one can easily deduce relevant identities by the method introduced in paper /9/. Detailed calculations will be presented elsewhere.

Finally we mention that if one is interested only in on-shell S-matrix then the supersymmetry of renormalized theory may be proved more easily with the help of S-matrix generating functional proposed in our paper /10/.

Acknowledgements.

I am grateful to N.N.Bogolubov for valuable discussion. My thanks also to J.Illiopoulos for bringing my attention to the problem.

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Received by Publishing Department
on October 7, 1974.