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THE ELECTROMAGNETIC INTERACTIONS OF MESONS IN THE SUPERCONDUCTOR QUARK MODEL

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1. INTRODUCTION

There are a number of models which have been used to describe the electromagnetic properties of the π and K mesons *. Using the methods of the quantum chiral theory, in particular, the estimations of electromagnetic radii and electric (α) and magnetic (β) polarizabilities of the pions and kaons have been obtained [3]. The predictions of the chiral theory are in good agreement with the existing experimental data.

The nonlinear chiral theory has an analogy - the linear sigma-model in which electromagnetic interactions of pions have been also studied [4]. However, absence of σ-particles in nature makes it difficult to interpret physically this model. In the superconductor quark model [5-7], which is a further development of ideas expounded in refs. [8], the really existing in nature scalar, pseudoscalar, vector and axial-vector mesons are presented. When describing the Compton effect on a pion (on a kaon), the exchange by scalar resonances leads to the basic contributions to the electric and magnetic polarizabilities. The vector mesons give small additional contributions to the magnetic polarizabilities only. And, at last, the axial-vector mesons give small contributions to the electric polarizabilities of the charged π and K mesons. There is a remarkable consequence of taking into account the vector and axial-vector mesons. Namely, the sum $\alpha + \beta$ is not equal to zero **.

In the superconductor quark model the electromagnetic radii and polarizabilities of pions have been already obtained [5,9]. Here we make similar calculations for kaons. With regard to the pion polarizabilities we have obtained more accurate results. There are two improvements. First, the form factors of the intermediate scalar mesons are taken into account. They raise the contribution of each state by 30-40%. Second, the contributions of the axial-vector $(1^+)^+$ mesons are estimated.

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* A good review of the current models which have been used to study the electromagnetic interactions of hadrons can be found in ref. [1]. Note that the electromagnetic properties of the pseudoscalar mesons have been also studied in the nonlocal quark model [2].

** The dispersion relations for Compton scattering give a set of sum rules which in turn lead to inequality $(\alpha + \beta) > 0$ [1]. In the chiral theory we have $(\alpha + \beta) = 0$ [3].

2. ELECTROMAGNETIC RADII OF KAONS

We begin with calculating the electromagnetic radius of charged kaon. The amplitudes corresponding to the Feynman diagrams of Figs.1a,b are as follows

$$T_a = \frac{g_\rho}{2} p^\mu [(1+\frac{q^2-m_\rho^2}{8\pi^2F^2})\rho_\mu + (1+\frac{q^2-m_\omega^2}{8\pi^2F^2})\omega_\mu + \sqrt{2}(1+\frac{q^2-m_\phi^2}{8\pi^2F^2})\phi_\mu] K^+K^-, \tag{1a}$$

$$T_b = -ep^\mu (1+\frac{q^2}{8\pi^2F^2}) A_\mu K^+K^- . \tag{1b}$$

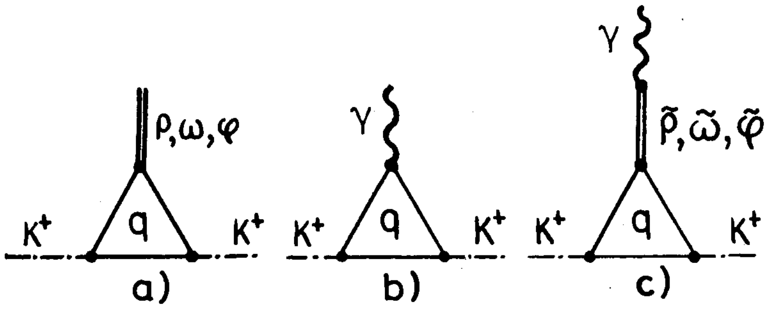

Fig.1

Here we take into account the form factors of the vector mesons and photon. (The form factors have been introduced in the papers /10–12/). g_ρ is found from $\rho \to \pi\pi$ to be $g_\rho^2/4\pi \equiv a_\rho \approx 3$. $p = p_1 + p_2$, $q = p_2 - p_1$, p_1 and p_2 are the momenta of the kaon in the initial and final states. Let us make in (1a) the replacements which are characteristic of the vector dominance model /13/.

$$\rho = \tilde{\rho} + \frac{e}{g_\rho} A, \quad \omega = \tilde{\omega} + \frac{e}{3g_\rho} A, \quad \phi = \tilde{\phi} + \frac{\sqrt{2}e}{3g_\rho} A .$$

The part of the (1a) and (1b) sum which after replacements contains photons is

$$T^\gamma_{a+b} = -\frac{ep^\mu}{(4\pi F)^2}(m_\rho^2 + \frac{m_\omega^2 + 2m_\phi^2}{3}) A_\mu K^+K^- . \tag{2}$$

Other terms of this sum contain $\tilde{\rho}$, $\tilde{\omega}$, $\tilde{\phi}$ vector mesons which in turn can be converted into photons.

$$\mathcal{L}_{VA} = \frac{e}{g_\rho}(m_\rho^2 \rho_\mu + \frac{m_\omega^2}{3}\omega_\mu + \frac{\sqrt{2}}{3} m_\phi^2 \phi_\mu) A^\mu . \tag{3}$$

For diagrams (1c) we have

$$T_c = -\frac{e}{2} p^\mu [\frac{m_\rho^2}{m_\rho^2 - q^2} + \frac{m_\omega^2}{3(m_\omega^2 - q^2)} + \frac{2m_\phi^2}{3(m_\phi^2 - q^2)} - \frac{1}{8\pi^2F^2}(m_\rho^2 + \frac{m_\omega^2 + 2m_\phi^2}{3})] A_\mu K^+K^- . \tag{4}$$

Then the total contribution to the kaon form factor equals the sum of (2) and (4). Assuming q^2 to be small and dropping terms higher than those of the second orher in q^2, we obtain

$$T_{K^+} = -ep^\mu [1 + \frac{q^2}{6}(\frac{3}{m_\rho^2} + \frac{1}{m_\omega^2} + \frac{2}{m_\phi^2})] A_\mu K^+K^- .$$

The electromagnetic radius of the K^+ meson is then given by

$$\langle r^2 \rangle_{K^+} = \frac{3}{m_\rho^2} + \frac{1}{m_\omega^2} + \frac{2}{m_\phi^2} = 0{,}336 \text{ fm}^2, \quad \sqrt{\langle r^2 \rangle_{K^+}} = 0{,}58 \text{ fm} .$$

By analogy with the calculation of the K^+ electromagnetic radius we obtain

$$\langle r^2 \rangle_{K^\circ} = -(\frac{3}{m_\rho^2} - \frac{1}{m_\omega^2} - \frac{2}{m_\varphi^2}) = -0{,}059 \text{ fm}^2, \quad \sqrt{|\langle r^2 \rangle_{K^\circ}|} = 0{,}24 \text{ fm} .$$

These estimations are close to the experimental data $\langle r^2 \rangle^{exp}_{K^-} = 0.26 \pm 0.07$ fm^2 /14a/, $\langle r^2 \rangle^{exp}_{K^\circ} = -0.054 \pm 0.026$ fm^2 /14b/. The results presented here are in accordance with those of the vector dominance model. Pervushin and Volkov /15/, using the quantum chiral theory, have also obtained results similar to these ones.

3. TWO-PARTICLE MESON DECAYS

As the first step we consider the processes which are useful for the discussion of Compton scattering on the π and K mesons. The strong interactions of ϵ , S^* and δ° mesons with π and K mesons are described by the Lagrangian

$$\mathcal{L} = 2g \{ m_u \vec{\pi}^2 (\epsilon \cos\gamma + S^* \sin\gamma) + \bar{K}K \{ \epsilon [(2m_u - m_s)\cos\gamma +$$

$$+ \sqrt{2}(2m_s - m_u)\sin\gamma] + S^*[(2m_u - m_s)\sin\gamma - \sqrt{2}(2m_s - m_u)\cos\gamma]\} + \tag{5}$$

$$+ (2m_u - m_s)\bar{K}\tau_3 K \delta^\circ \} .$$

Here $g = m_u/F_\pi$, F_π = 95 MeV is the pion decay constant, m_u =

= 280 MeV, m_s = 503 MeV are the constituent-quark masses obtained in the model under consideration [6b], $\gamma = 17°$ is the mixing angle of the singlet-octet components of ϵ and S* mesons

$$(\sigma_8 + \sqrt{2}\sigma_0)/\sqrt{3} = \epsilon\cos\gamma + S^*\sin\gamma, \quad (\sqrt{2}\sigma_8 - \sigma_0)/\sqrt{3} = S^*\cos\gamma - \epsilon\sin\gamma.$$

At present the mass of the ϵ meson is determined with a poor accuracy * . Therefore we use the model value m_ϵ = 800 MeV [6b] and, for comparison, m_ϵ = 700 MeV. Using Lagrangian (5) we obtain the amplitudes and widths of the scalar-meson decays

$$T_{\epsilon\to\pi^0\pi^0, \pi^+\pi^-} = 4m_u g\cos\gamma, \quad T_{S^*\to\pi^0\pi^0, \pi^+\pi^-} = 4m_u g\sin\gamma, \quad T_{\delta^0\to\bar{K}^0K^0, K^+K^-} = \mp 2g(2m_u - m_s), \qquad (6)$$

$$T_{\epsilon\to\bar{K}^0K^0, K^+K^-} = 2g[(2m_u - m_s)\cos\gamma + \sqrt{2}(2m_s - m_u)\sin\gamma],$$

$$T_{S^*\to\bar{K}^0K^0, K^+K^-} = 2g[(2m_u - m_s)\sin\gamma - \sqrt{2}(2m_s - m_u)\cos\gamma],$$

$$\Gamma_{\epsilon\to\pi\pi} = \frac{3g^2}{2\pi}\frac{m_u^2}{m_\epsilon}\cos^2\gamma\sqrt{1 - \left(\frac{2m_\pi}{m_\epsilon}\right)^2} = \begin{cases} 350 \text{ meV} & (m_\epsilon = 800 \text{ MeV}) \\ 390 \text{ MeV} & (m_\epsilon = 700 \text{ MeV}), \end{cases}$$

$$\Gamma_{S^*\to\pi\pi} = \frac{3g^2}{2\pi}\frac{m_u^2}{m_{S^*}}\sin^2\gamma\sqrt{1 - \left(\frac{2m_\pi}{m_{S^*}}\right)^2} = 27 \text{ MeV}, \quad \Gamma^{exp}_{S^*\to\pi\pi} = 26\pm 5 \text{ MeV} \text{ [18]}$$

Then consider the ϵ , S* and δ^0 meson decays into two photons. The quark triangle diagrams of fig.2 which are calculated by using Lagrangians (3) and

$$\mathcal{L}_1 = g[(\epsilon\cos\gamma + S^*\sin\gamma)(\bar{u}u + \bar{d}d) + \sqrt{2}(\epsilon\sin\gamma - S^*\cos\gamma)\bar{s}s + \delta^0(\bar{u}u - \bar{d}d)],$$

$$\mathcal{L}_2 = \frac{g_\rho}{2}[(\omega + \rho)_\mu \bar{u}\gamma^\mu u + (\omega - \rho)_\mu \bar{d}\gamma^\mu d - \sqrt{2}\phi_\mu \bar{s}\gamma^\mu s]$$

give

$$T_{\epsilon\to\gamma\gamma} = (5\cos\gamma + \sqrt{2}\sin\gamma)t, \quad T_{S^*\to\gamma\gamma} = (5\sin\gamma - \sqrt{2}\cos\gamma)t,$$

$$T_{\delta^0\to\gamma\gamma} = 3t, \quad t = \frac{2\alpha}{9\pi F_\pi}\epsilon_\mu(q_1)\epsilon_\nu(q_2)[g^{\mu\nu}(q_1 q_2) - q_1^\nu q_2^\mu],$$

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* The recent analysis of $\pi^+ p \to p\pi^+\pi^0\pi^0$ allows existence of the narrow ϵ (700÷800) resonance [16]. On the other hand, from $\gamma\gamma\to\pi^+\pi^-$ data one gets the wide $m_\epsilon \simeq \Gamma_\epsilon \simeq$ 600 MeV scalar resonance [17].

$$\Gamma_{\epsilon\to\gamma\gamma} = \frac{m_\epsilon}{\pi}\left[\frac{\alpha m_\epsilon}{36\pi F_\pi}(5\cos\gamma + \sqrt{2}\sin\gamma)\right]^2 = \begin{cases} 2 \text{ keV} & (m_\epsilon = 800 \text{ MeV}) \\ 1.4 \text{ keV} & (m_\epsilon = 700 \text{ MeV}), \end{cases}$$

$$\Gamma_{S^*\to\gamma\gamma} = \frac{m_{S^*}}{\pi}\left[\frac{\alpha m_{S^*}}{36\pi F_\pi}(5\sin\gamma - \sqrt{2}\cos\gamma)\right]^2 = 1.6 \text{ eV}, \qquad (7)$$

$$\Gamma_{\delta^0\to\gamma\gamma} = \frac{m_{\delta^0}}{\pi}\left(\frac{\alpha m_{\delta^0}}{12\pi F_\pi}\right)^2 = 1.25 \text{ keV}.$$

Here $\alpha = e^2/4\pi = 1/137$, q_1 and q_2 are the momenta of the photons, $\epsilon_\mu(q_1)$ and $\epsilon_\mu(q_2)$ are their polarization vectors. From the reaction $\gamma\gamma\to\pi^0\pi^0$ at present only an upper limit is available $\Gamma^{exp}_{S^*\to\gamma\gamma}\cdot Br(\pi\pi) <$ 0.8 keV [19]. The characteristics of the ϵ meson are not studied well. A fit to the DCI $\gamma\gamma\to\pi^+\pi^-$ data [20] using the unitarized Born term and ϵ (700) required a $\gamma\gamma$ width for ϵ (700) of about 40 keV [21]. A considerably smaller value has been obtained by Mennessier [17], who gives $\Gamma^{exp}_{\epsilon\to\gamma\gamma}$ = 5÷9 keV. There is no experimental data for $\Gamma_{\delta^0\to\gamma\gamma}$.

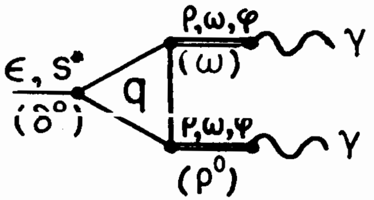

Fig.2

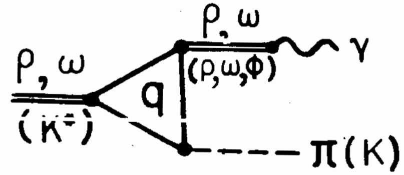

Fig.3

The radiative decays of the vector mesons $\omega\to\pi^0\gamma$, $\rho^-\to\pi^-\gamma$ and $K^*\to K\gamma$ are described using the anomal-type-triangle diagrams (fig.3). The first two processes are calculated in paper [5]. (The anomal-type-triangle diagrams have been also studied in ref. [22]). The decay widths are given by

$$\Gamma_{V\to P\gamma} = \frac{\alpha a_p}{6\pi}\left(\frac{a_V}{16\pi F_P}\right)^2\left(\frac{m_V^2 - m_P^2}{m_V}\right)^3, \qquad (8)$$

where $a_\omega = 6$, $a_\rho = 2$, $a_{K^{*}_{0}} = 2\left(1 - \frac{x}{1-x^2}\ln x^2\right) = 3.9$. $a_{K^{*}_{+}} = 1 - \frac{2x}{1-x^2}\left(3 + \frac{2+x^2}{1-x^2}\ln x^2\right) = 2.5$, $x = m_u/m_s$. The numerical estimations are presented in Table 1.

The radiative decays of axial-vector mesons $A_1^+\to\pi^+\gamma$ and $Q_1^+\to K^+\gamma$ are described by the diagrams of fig.4. The total con-

Table 1

	$\Gamma^{theor.}$ (keV)	Γ^{exp} (keV)	
$\omega \to \pi^{o}\gamma$	800	861 ± 50	/18/
$\rho \to \pi\gamma$	87	71 ± 7	/23/
$\overset{*}{K}{}^{+} \to K^{+}\gamma$	47	51 ± 5	/24/
$\overset{*}{K}{}^{o} \to \bar{K}^{o}\gamma$	115	75 ± 35	/25/
$A_1^{+} \to \pi^{+}\gamma$	400	a few hundreds	/26/
$Q_1^{+} \to K^{+}\gamma$	180		

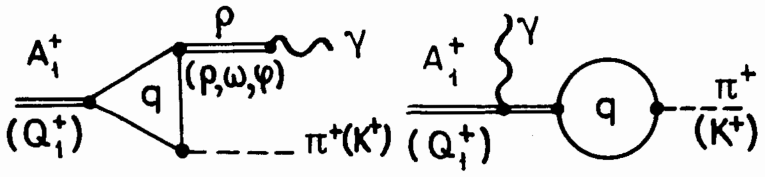

Fig.4

tribution of these diagrams to the amplitude satisfies the gauge-invariance requirement. The decay widths are given by (8), where $a_{A_1^+} = 2$, $a_{Q_1^+} = 2\frac{F_K}{F_\pi}\cos\phi$ (see also Table 1). Note, that the physical axial-vector Q_1 and Q_2 mesons are the mixture of the Q_A and Q_B pure states, belonging to the $(1^+)^+$ and $(1^+)^-$ nonets $Q_1 = \cos\phi\, Q_A + \sin\phi\, Q_B$, $Q_2 = -\sin\phi\, Q_A + \cos\phi\, Q_B$, where $\phi^{exp} = 34 \pm 3^{o}$ /27/. We neglect the contribution of the Q_2 meson for the sake of simplicity.

4. THE PION AND KAON POLARIZABILITIES

Now we proceed to the calculations of the pion and kaon polarizabilities. Besides the processes already considered here, one needs to take into account the contributions of the box quark diagrams (fig.5). Following the papers /3,9,28/ we find $T_{\square}^{\pi^o} = 10 T_{\square}^{\pi^+} = -\frac{5}{F_\pi} t$, $T_{\square}^{K^o} = 4 T_{\square}^{K^+} = -\frac{2}{F_K} t$, where $F_K = 1.26 F_\pi$ /29/ (see also (7)).

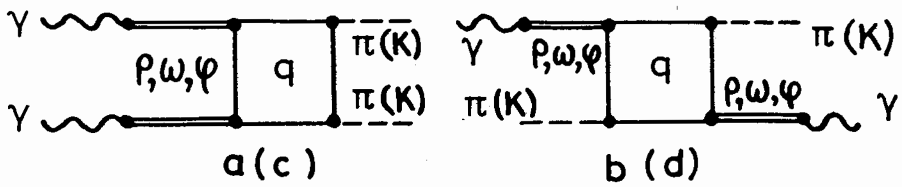

Fig.5

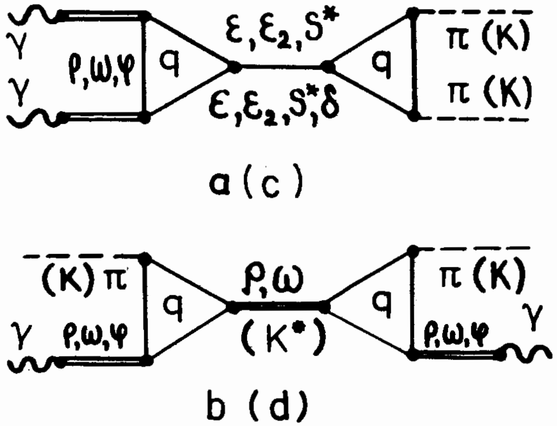

Fig.6

There are now all the components for the calculation of the pion and kaon polarizabilities. Let us consider the pions. The contributions to the electric and magnetic polarizabilities of the pions for the diagrams are presented in figs.5a,b and 6a with allowance for the scalar-meson form factors given by*

$$a_{\pi^{\pm}} = -\beta'_{\pi^{\pm}} = \frac{a}{18\pi F_\pi^2 m_{\pi^+}} \{ 8 m_u^2 [\frac{a(m_\epsilon)}{m_\epsilon^2} \cos\gamma (5\cos\gamma + \sqrt{2}\sin\gamma) + \frac{a(m_{S^*})}{m_{S^*}^2} \sin\gamma \times$$

$$\times (5\sin\gamma - \sqrt{2}\cos\gamma) + \frac{a(m_{\epsilon_2})}{m_{\epsilon_2}^2} C_1 C_3] - 1 \} = 7.86 (\frac{7.06}{8.52} + 0.03 + 1.4 - 1) \cdot 10^{-4} fm^3 =$$

$$= \begin{matrix} 5.9 \cdot 10^{-3}\, fm^3 \; (m_\epsilon = 800 \text{ MeV}) \\ 7.0 \cdot 10^{-3}\, fm^3 \; (m_\epsilon = 700 \text{ MeV}), \end{matrix}$$

---

* The coefficients C_1 , C_2 and C_3 (see formulas (9) and (11)) for the ϵ (1300) meson are calculated in the Appendix.

$$
\alpha_{\pi^\circ} = -\beta'_{\pi^\circ} = \frac{5\alpha}{9\pi F_\pi^2 m_{\pi^\circ}} \{ \frac{4}{5} m_u^2 [\frac{a(m_\epsilon)}{m_\epsilon^2} \cos\gamma (5\cos\gamma + \sqrt{2}\sin\gamma) + \frac{a(m_{S^*})}{m_{S^*}^2} \sin\gamma \times
$$

$$
\times (5\sin\gamma - \sqrt{2}\cos\gamma) + \frac{a(m_{\epsilon_2})}{m_{\epsilon_2}^2} C_1 C_3] - 1 \} = 8.15 [\frac{1}{10} (\frac{7.06}{8.52} + 0.03 + 1.4) - 1] \cdot 10^{-3}\ \mathrm{fm}^3 = \tag{9}
$$

$$
= \begin{array}{ll} -1.2 \cdot 10^{-3}\ \mathrm{fm}^3 & (m_\epsilon = 800\ \mathrm{MeV}) \\ -0.041 \cdot 10^{-3}\ \mathrm{fm}^3 & (m_\epsilon = 700\ \mathrm{MeV}). \end{array}
$$

The multipliers $a(m) = 1 + (\frac{m}{4\pi F_\pi})^2$ are the consequence of taking into account the form factors of scalar mesons $1 + \frac{m^2 - q^2}{(4\pi F)^2}$.
From (9) it is clear that the ϵ meson gives the main contribution to the electric polarizability of the charged pion. Because there is a small multiplier $(5\sin\gamma - \sqrt{2}\cos\gamma) = 0.11$ $(\gamma = 17^\circ)$, the contribution of the S^* meson is small. The ϵ_2 (1300) meson contribution is 20% of the ϵ meson contribution. The diagrams with the intermediate $A_1^{\pm}$ axial-vector meson make contribution only to the electric polarizability of the pion, which is also small

$$
\Delta\alpha_{\pi^\pm} = \frac{\alpha\alpha_\rho}{(2\pi F_\pi)^2} \frac{m_\pi}{m_{A_1}^2 - m_\pi^2} = 4.2 \cdot 10^{-5}\ \mathrm{fm}^3.
$$

The obtained value of the electric polarizability of the charged pion is in satisfactory agreement with the experiment /30/*. $\alpha_{\pi^-}^{exp} = (8.5 \pm 1.8) \cdot 10^{-3}\ \mathrm{fm}^3$. The estimate of the neutral pion polarizability is approximate because here the sum of the box diagram contributions, on the one hand, and of the contributions of the diagrams with the intermediate scalar mesons, on the other hand, is almost equal to zero. If one neglects in (9) the form factors $(a(m) = 1)$, excludes the contribution of the ϵ_2 (1300) meson and considers the chiral-symmetric limit ($\gamma = 0$, $m_\epsilon^2 = m_\pi^2 + 4m_u^2$, $m_u \to \infty$), one obtains the results of paper /15/.

$$
\alpha_{\pi^\pm}^{CL} = -\beta_{\pi^\pm}^{CL} = \frac{\alpha}{2\pi F_\pi^2 m_\pi} = 7.1 \cdot 10^{-3}\ \mathrm{fm}^3, \qquad \alpha_{\pi^\circ}^{CL} = \beta_{\pi^\circ}^{CL} = 0.
$$

The diagrams with ρ and ω mesons (fig.6b) give rise to the amplitudes

$$
T_{\pi^+} = -\frac{\alpha\alpha_\rho}{(2\pi F_\pi)^2} T(m_\rho), \qquad T_{\pi^\circ} = -\frac{\alpha\alpha_\rho}{(2\pi F_\pi)^2} [T(m_\rho) + 9T(m_\omega)],
$$

---

*Our estimates are made in the case when the widths of resonances can be neglected. Taking account of the widths we obtain 10% divergence from the found one.

$$
T(m) = \epsilon_{\rho\sigma\kappa\mu} \epsilon^{\rho}_{\bar\sigma\bar\kappa\nu} p_1^\sigma p_2^{\bar\sigma} [\frac{\epsilon^\mu(q_1)\epsilon^\nu(q_2) q_1^\kappa q_2^{\bar\kappa}}{m^2 - (q_1 + p_1)^2} + \frac{\epsilon^\nu(q_1)\epsilon^\mu(q_2) q_1^{\bar\kappa} q_2^\kappa}{m^2 - (q_2 - p_1)^2}], \tag{10}
$$

where $\epsilon_{\rho\sigma\kappa\mu}$ is the totally antisymmetric symbol, p_1, p_2, q_1, q_2 are the momenta of mesons and photons in the initial and final states. In the initial-pion-at-rest system we get

$$
\epsilon_{\rho\sigma\kappa\mu} \epsilon^{\rho}_{\bar\sigma\bar\kappa\nu} p_1^\sigma p_2^{\bar\sigma} q_1^\kappa q_2^{\bar\kappa} \epsilon^\mu(q_1) \epsilon^\nu(q_2) = [m_\pi\omega_1 - (m_\pi^2 + m_\pi\omega_1)\cos\theta] \times \omega_1\omega_2 \vec\epsilon_1 \vec\epsilon_2 +
$$

$$
+ (m_\pi^2 + m_\pi\omega_1)(\vec\epsilon_1 \vec q_2)(\vec\epsilon_2 \vec q_1).
$$

Therefore (see the Appendix /31/), the diagrams with vector mesons give the contributions only to the magnetic polarizabilities of the pions

$$
\beta''_{\pi^+} = \frac{\alpha\alpha_\rho}{(2\pi F_\pi)^2} \frac{m_\pi}{m_\rho^2 - m_\pi^2} = 1.2 \cdot 10^{-4}\ \mathrm{fm}^3,
$$

$$
\beta''_{\pi^\circ} = \frac{\alpha\alpha_\rho}{(2\pi F_\pi)^2} m_\pi (\frac{9}{m_\omega^2 - m_\pi^2} + \frac{1}{m_\rho^2 - m_\pi^2}) = 1.1 \cdot 10^{-3}\ \mathrm{fm}^3.
$$

As a result, for the sum $\beta = \beta' + \beta''$ we have

$$
\beta_{\pi^\pm} = \begin{array}{ll} -5.8 \cdot 10^{-3}\ \mathrm{fm}^3 & (m_\epsilon = 800\ \mathrm{MeV}) \\ -6.9 \cdot 10^{-3}\ \mathrm{fm}^3 & (m_\epsilon = 700\ \mathrm{MeV}), \end{array}
$$

$$
\beta_{\pi^\circ} = \begin{array}{ll} 2.3 \cdot 10^{-3}\ \mathrm{fm}^3 & (m_\epsilon = 800\ \mathrm{MeV}) \\ 1.1 \cdot 10^{-3}\ \mathrm{fm}^3 & (m_\epsilon = 700\ \mathrm{MeV}). \end{array}
$$

For the sum of the electric and magnetic polarizabilities of the neutral pion we find the value

$$
(\alpha + \beta)_{\pi^\circ} = \frac{\alpha\alpha_\rho}{(2\pi F_\pi)^2} m_\pi (\frac{9}{m_\omega^2 - m_\pi^2} + \frac{1}{m_\rho^2 - m_\pi^2}) =
$$

$$
= 3.5 (0.28 + 0.03) \cdot 10^{-3}\ \mathrm{fm}^3 = 1.1 \cdot 10^{-3}\ \mathrm{fm}^3
$$

which is close to the dispersion estimate /32/ $(\alpha + \beta)_{\pi^\circ}^{disp} = 1.3 \cdot 10^{-3}\ \mathrm{fm}^3$. At the same time in the case of the charged pion our estimate

$$
(\alpha + \beta)_{\pi^\pm} = \frac{\alpha\alpha_\rho}{(2\pi F_\pi)^2} m_\pi (\frac{1}{m_\rho^2 - m_\pi^2} + \frac{1}{m_{A_1}^2 - m_\pi^2}) = 3.5 (0.34 + 0.12) \cdot 10^{-4}\ \mathrm{fm}^3 =
$$

$$
= 1.6 \cdot 10^{-4}\ \mathrm{fm}^3
$$

is smaller than the dispersion one $(\alpha + \beta)_{\pi^\pm}^{disp} = 4.4 \cdot 10^{-4}\ \mathrm{fm}^3$ /32/.

The Compton scattering on a kaon is described using the diagrams of figs. 5c,d and 6c,d. From the box diagrams and diagrams with the scalar ϵ, ϵ (1300), S* and δ° mesons we find

$$\alpha_{K^\pm} = -\beta'_{K^\pm} = \frac{\alpha}{18\pi F_\pi F_K m_{K^+}} \{2(m_u + m_s)[\frac{\tilde{a}(m_\epsilon)}{m_\epsilon^2}(5\cos\gamma + \sqrt{2}\sin\gamma) \times$$

$$\times[(2m_u - m_s)\cos\gamma + \sqrt{2}(2m_s - m_u)\sin\gamma] - \frac{\tilde{a}(m_{S^*})}{m_{S^*}^2}(5\sin\gamma - \sqrt{2}\cos\gamma) \times$$

$$\times[\sqrt{2}(2m_s - m_u)\cos\gamma - (2m_u - m_s)\sin\gamma] + \frac{\tilde{a}(m_{\delta^\circ})}{m_{\delta^\circ}^2} 3(2m_u - m_s) +$$

$$+ \frac{\tilde{a}(m_{\epsilon_2})}{m_{\epsilon_2}^2} m_s C_2 C_3] - 1\} = 1.77(\frac{5.77}{7.19} - 0.24 + 0.40 +$$

$$+ 0.6 - 1) \cdot 10^{-4}\ fm^3 = \begin{matrix} 9.8 \cdot 10^{-4} fm^3 & (m_\epsilon = 800\ MeV) \\ 1.2 \cdot 10^{-3} fm^3 & (m_\epsilon = 700\ MeV) \end{matrix} \qquad (11)$$

$$\alpha_{K^\circ} = -\beta'_{K^\circ} = \frac{2\alpha}{9\pi F_\pi F_K m_{K^\circ}} \{\frac{m_u + m_s}{2}[\frac{\tilde{a}(m_\epsilon)}{m_\epsilon^2}(5\cos\gamma + \sqrt{2}\sin\gamma) \times$$

$$\times[(2m_u - m_s)\cos\gamma + \sqrt{2}(2m_s - m_u)\sin\gamma] - \frac{\tilde{a}(m_{S^*})}{m_{S^*}^2}(5\sin\gamma - \sqrt{2}\cos\gamma) \times$$

$$\times[\sqrt{2}(2m_s - m_u)\cos\gamma - (2m_u - m_s)\sin\gamma] - \frac{\tilde{a}(m_{\delta^\circ})}{m_{\delta^\circ}^2} \times$$

$$\times 3(2m_u - m_s) + \frac{\tilde{a}(m_{\epsilon_2})}{m_{\epsilon_2}^2} m_s C_2 C_3] - 1\} = 7.01[\frac{1}{4}(\frac{5.77}{7.19} - 0.24 - 0.40 + 0.6) - 1] \times$$

$$\times 10^{-4} fm^3 = \begin{matrix} 3.1 \cdot 10^{-4} fm^3 & (m_\epsilon = 800\ MeV) \\ 5.5 \cdot 10^{-4} fm^3 & (m_\epsilon = 700\ MeV), \end{matrix}$$

where $\tilde{a}(m) = 1 + (\frac{m}{4\pi F_K})^2$.

From (11) in the chiral limit ($\gamma = 0$, $\tilde{a} = 1$, $C_i = 0$, $m^2_{\epsilon, S^*, \delta^\circ} = m_\pi^2 + 4m_q^2$, $m_u = m_d = m_s = m_q \to \infty$) we have

$$\alpha^{CL}_{K^\pm} = \frac{\alpha}{18\pi F_\pi F_K m_{K^\pm}}(5 + 2 + 3 - 1) = \frac{\alpha}{2\pi F_\pi F_K m_{K^\pm}} = 1.6 \cdot 10^{-3}\ fm^3$$

$$\alpha^{CL}_{K^\circ} = \frac{2\alpha}{9\pi F_\pi F_K m_K}(\frac{5}{4} + \frac{1}{2} - \frac{3}{4} - 1) = 0.$$

Here, in parentheses are written consequently the contributions of the ϵ, S*, δ° mesons and box quark diagrams. As in the case of the π° meson the estimates of the neutral kaon polarizabilities are approximate. The Q_1^+ meson pole diagrams make the contributions to the electric polarizabilities of the charged kaons.

$$\Delta\alpha_{K^\pm} = \frac{\alpha\alpha_\rho}{(2\pi F_\pi)^2} \frac{m_K}{m^2_{Q_1} - m^2_K} \cos^2\phi = 1.2 \cdot 10^{-4} fm^3.$$

It gives rise to small corrections for the total polarizabilities.

$$\alpha_{K^\pm} + \Delta\alpha_{K^\pm} = \begin{matrix} 1.1 \cdot 10^{-3}\ fm^3 & (m_\epsilon = 800\ MeV) \\ 1.3 \cdot 10^{-3}\ fm^3 & (m_\epsilon = 700\ MeV). \end{matrix}$$

In the case of the intermediate K* vector meson (fig.6d) we have the following amplitudes

$$T_{K^*} = -\alpha\alpha_\rho(\frac{a_{K^*}}{4\pi F_K})^2 T(m_{K^*}),$$

where a_{K^*} and $T(m_{K^*})$ are given in (8) and (10). From this

$$\beta''_K = \alpha\alpha_\rho(\frac{a_{K^*}}{4\pi F_K})^2 \frac{m_K}{m^2_{K^*} - m^2_K} = \begin{matrix} 4.2 \cdot 10^{-4} fm^3 & (K^\pm) \\ 1.0 \cdot 10^{-3} fm^3 & (K^\circ). \end{matrix}$$

The magnetic polarizabilities of the kaons are equal to

$$\beta_{K^\pm} = \begin{matrix} -0.6 \\ -0.8 \end{matrix} \cdot 10^{-3}\ fm^3, \quad \beta_{K^\circ} = \begin{matrix} 0.7 \\ 0.45 \end{matrix} \cdot 10^{-3}\ fm^3 \quad \begin{matrix} (m_\epsilon = 800\ MeV) \\ (m_\epsilon = 700\ MeV). \end{matrix}$$

For the sum of the electric and magnetic polarizabilities of the kaons we have

$$(\alpha + \beta)_{K^\pm} = \frac{\alpha\alpha_\rho}{(4\pi F_K)^2} m_K[\frac{a^2_{K^{*\pm}}}{m^2_{K^*} - m^2_K} + \frac{(2\frac{F_K}{F_\pi}\cos\phi)^2}{m^2_{Q_1} - m^2_K}] =$$

$$= (4.2 + 1.2) \cdot 10^{-4} fm^3 = 5.4 \cdot 10^{-4} fm^3,$$

$$(\alpha + \beta)_{K^\circ} = \alpha\alpha_\rho(\frac{a_{K^{*\circ}}}{4\pi F_K})^2 \frac{m_K}{m^2_{K^*} - m^2_K} = 1.0 \cdot 10^{-3} fm^3,$$

The final results are presented in Table 2.

Table 2

$\alpha(\beta) \cdot 10^{-3}$ fm^3 ($e^2/4\pi = 1/137$)								(MeV)
$\alpha_{\pi^\pm}$	$\beta_{\pi^\pm}$	α_{π°	β_{π°	$\alpha_{K^\pm}$	$\beta_{K^\pm}$	α_{K°	β_{K°	
5.9	-5.8	-1.2	2.3	1.1	-0.6	0.3	0.7	m_ϵ = 800
7.0	-6.9	-0.041	1.1	1.3	-0.8	0.55	0.45	m_ϵ = 700

5. CONCLUSION

Our results are close to those obtained in quantum chiral theory [3,15,28]. The most essential distinction of the new data is inequality of absolute values of the magnetic and electric meson polarizabilities. It is the consequence of taking into account the pole diagrams with vector and axial-vector mesons (see also [33]).

Further distinction of the model consists in making use of the mesons really existing in nature. For example, instead of the old fictitious σ-particles in the scalar-meson sector the real ϵ, S^* and δ resonances are considered.

In this paper we study π and K meson polarizabilities using the box diagrams and the diagrams with one-particle intermediate states of spins 0 and 1. The tensor mesons may also be included in the model. This problem will be solved in our next paper. The approximation under consideration gives more accurate results for the estimations of the charged meson polarizabilities. The estimations of the neutral meson polarizabilities are of qualitative character. It is the result of almost complete cancellation of the box diagram contributions and the contributions of the diagrams with scalar mesons. In this case it is important to take into account the contributions of two-particle intermediate states and, first of all, the pion-loop diagrams. According to the papers [15,34], the function describing the contribution of the pion-loop diagrams to the amplitude $\gamma\gamma \to \pi\pi$ is rapidly varying in the photon energy interval from zero up to two pion creation threshold $2m_\pi$. This anomaly should be taken into account when comparing experimental data with predictions of the calculations. The neutral π° and K° meson polarizabilities obtained here are valid at the energy of photons much below the pion mass.

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6. APPENDIX

Let us consider the ϵ (1300) meson decays: $\epsilon_2 \to \pi\pi$, $\bar{K}K$, $\gamma\gamma$. The heavier ϵ (1300) is not included in the light 0^{++} scalar nonet. Therefore, we must define three new parameters C_1, C_2 and C_3 to describe the ϵ(1300) decays in analogy with ϵ (700) case. These parameters are fixed by the experimental values of the widths $\epsilon_2 \to \pi\pi$, $\bar{K}K$, $\gamma\gamma$ as input.

$$T_{\epsilon_2 \to \pi^+\pi^-, \pi^\circ\pi^\circ} = 4m_u g C_1, \quad T_{\epsilon_2 \to K^+K^-, \bar{K}^\circ K^\circ} = 2m_s g C_2, \quad T_{\epsilon_2 \to \gamma\gamma} = tC_3.$$

From $\Gamma^{exp}_{\epsilon_2 \to \pi\pi} = 360$ MeV [18] and $\Gamma^{exp}_{\epsilon_2 \to \bar{K}K} = 40$ MeV [18] we get $C_1 = 1.2$, $C_2 = 0.5$. The parameter C_3 is fixed using the estimation $\Gamma^{exp}_{\epsilon_2 \to \gamma\gamma} Br(\pi^+\pi^-) < 1.5$ keV [35]. Assuming $\Gamma^{exp}_{\epsilon_2 \to \gamma\gamma} \sim 1$ keV, we obtain $C_3 = 1.8$.

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E2-83-921

Электромагнитные свойства мезонов
в кварковой модели сверхпроводящего типа

В кварковой модели сверхпроводящего типа вычислены электромагнитные радиусы каонов $\langle r^2\rangle_{K^\pm} = 0{,}34$ Фм^2, $\langle r^2\rangle_{K^0} = -0{,}06$ Фм^2, а также электрические (α) и магнитные (β) поляризуемости пионов и каонов при двух возможных значениях массы ϵ мезона: $m_\epsilon = 800$ МэВ и $m_\epsilon = 700$ МэВ. В первом случае получено: $\alpha_{\pi^\pm} = 5{,}9$, $\beta_{\pi^\pm} = -5{,}8$, $\alpha_{\pi^0} = -1{,}2$, $\beta_{\pi^0} = 2{,}3$, $\alpha_{K^\pm} = 1{,}1$, $\beta_{K^\pm} = -0{,}6$, $\alpha_{K^0} = 0{,}3$, $\beta_{K^0} = 0{,}7$; во втором: $\alpha_{\pi^\pm} = 7{,}0$, $\beta_{\pi^\pm} = -6{,}9$, $\alpha_{\pi^0} = -0{,}041$, $\beta_{\pi^0} = 1{,}1$, $\alpha_{K^\pm} = 1{,}3$, $\beta_{K^\pm} = -0{,}8$, $\alpha_{K^0} = 0{,}55$, $\beta_{K^0} = 0{,}45$. Данные приведены в единицах $\alpha(\beta)\cdot 10^{-3}$ Фм^3 $(e^2/4\pi = 1/137)$. При вычислении поляризуемостей мезонов учтены вклады четырехугольных кварковых диаграмм и диаграмм с промежуточными ϵ, $\epsilon(1300)$, S^*, δ, ρ, ω, K^*, A_1, Q_1 мезонами /скалярные, векторные и аксиально-векторные мезоны/. Учтены также формфакторы промежуточных мезонов.

Работа выполнена в Лаборатории теоретической физики и Лаборатории ядерных проблем ОИЯИ.

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The Electromagnetic Interactions
of Mesons in the Superconductor Quark Model

The electromagnetic radii of kaons $\langle r^2\rangle_{K^\pm} = 0.34$ fm^2, $\langle r^2\rangle_{K^0} = -0.06$ fm^2 as well as electric (α) and magnetic (β) polarizabilities of pions and kaons at two possible values of the ϵ meson mass: $m_\epsilon = 800$ MeV and $m_\epsilon = 700$ MeV, are calculated in the superconductor quark model. In the first case it is obtained that $\alpha_{\pi^\pm} = 5.9$, $\beta_{\pi^\pm} = -5.8$, $\alpha_{\pi^0} = -1.2$, $\beta_{\pi^0} = 2.3$, $\alpha_{K^\pm} = 1.1$, $\beta_{K^\pm} = -0.6$, $\alpha_{K^0} = 0.3$, $\beta_{K^0} = 0.7$. In the second one, $\alpha_{\pi^\pm} = 7.0$, $\beta_{\pi^\pm} = -6.9$, $\alpha_{\pi^0} = -0.041$, $\beta_{\pi^0} = 1.1$, $\alpha_{K^\pm} = 1.3$, $\beta_{K^\pm} = -0.8$, $\alpha_{K^0} = 0.55$, $\beta_{K^0} = 0.45$. The data are given in units $\alpha(\beta)\cdot 10^{-3}$ fm^3 $(e^2/4\pi = 1/137)$. In evaluating the meson polarizabilities the contributions of the box quark diagrams and ϵ, $\epsilon(1300)$, S^*, δ, ρ, ω, K^*, A_1, Q_1 (scalar, vector and axial-vector mesons) pole diagrams are taken into account. The form factors of intermediate mesons are also used.

The investigation has been performed at the Laboratory of Theoretical Physics and the Laboratory of Nuclear Problems, JINR.

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