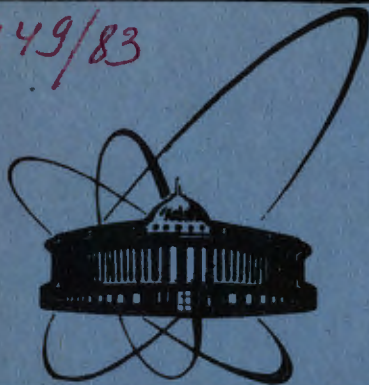


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**DYNAMICAL BREAKDOWN
OF THE ELECTROWEAK
GAUGE SYMMETRY**

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1. Introduction

Although the masses are basic characteristics of the elementary particles, their origin remains mysterious. Using vastly different coupling constants, the Higgs mechanism in the standard model describes the mass spectrum of leptons, quarks and gauge bosons, but does not explain it. This is a definition of phenomenology.

The alternative to the spontaneous breakdown of chiral and gauge symmetries via nonzero vacuum expectation values of the canonical scalar fields is known as the dynamical symmetry breakdown^{/1/}. The chiral and gauge symmetries of fermions and gauge bosons are broken in solutions of the field equations. Since these symmetries do not tolerate, in general, the bare mass terms of fermions and gauge bosons, there is no mass renormalization in this approach. Consequently, the physical masses (if they appear) are finite and calculable^{/2/}.

To our knowledge, there exist at present two types of attempts of the dynamical electroweak gauge symmetry breakdown. First, it is technicolour^{/3/}. It borrows its nonperturbative tools from QCD in strong coupling region which alone is not understood from first principles. Moreover, the technicolour is not able to offer a natural explanation of the wide fermionic mass spectrum. Second, it is the higgsless GWS model treated non-perturbatively^{/4,5,6/}. Unfortunately, it has a fundamental drawback: the combined g' and Z -boson exchange is not attractive in all desirable channels. Consequently, it cannot yield the massive d -type quarks.

In this work we solve the higgsless GWS model supplemented with a heavy neutral vector boson C in the strong coupling region and find the approximate chiral and gauge symmetry breaking solutions of the corresponding field equations.

2. Dynamical mass generation

The Lagrangian for one fermion family is given as^{/7/}

$$\begin{aligned} \mathcal{L} = & \bar{\psi}_L i \not{\partial} (\partial_\mu - i g \frac{1}{2} \vec{t} \vec{A}_\mu + i g' \frac{1}{2} B_\mu - i h \frac{1}{2} Y_H C_\mu) \psi_L + \bar{\nu}_R i \not{\partial} (\partial_\mu - i h \frac{1}{2} Y_H C_\mu) \nu_R \\ & + \bar{e}_R i \not{\partial} (\partial_\mu + i g' B_\mu - i h \frac{1}{2} Y_H C_\mu) e_R + \bar{q}_L i \not{\partial} (\partial_\mu - i g \frac{1}{2} \vec{t} \vec{A}_\mu - i g' \frac{1}{2} B_\mu - i h \frac{1}{2} Y_H C_\mu) q_L \quad (2.1) \\ & + \bar{u}_R i \not{\partial} (\partial_\mu - i g \frac{1}{2} B_\mu - i h \frac{1}{2} Y_H C_\mu) u_R + \bar{d}_R i \not{\partial} (\partial_\mu + i g' B_\mu - i h \frac{1}{2} Y_H C_\mu) d_R \\ & - \frac{1}{4} (\partial_\mu \vec{A}_\nu - \partial_\nu \vec{A}_\mu + g \vec{A}_\mu \times \vec{A}_\nu)^2 - \frac{1}{4} (\partial_\mu B_\nu - \partial_\nu B_\mu)^2 - \frac{1}{4} (\partial_\mu C_\nu - \partial_\nu C_\mu)^2 + \frac{1}{2} M_C^2 C^\alpha C^\alpha + QCD \end{aligned}$$

It is $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge invariant and renormalizable (off mass shell) provided that the Adler-Bell-Jackiw anomalies cancel. They are absent for those values of the C -hypercharge Y_H

$$Y_H = (y(q_L), y(u_R), y(d_R); y(\psi_L), y(\nu_R), y(e_R)),$$

for which^{/8/}

$$y_H = \alpha Y_H^{(1)} + \beta Y_H^{(2)} \quad (2.2)$$

and $Y_H^{(1)} = (\sqrt{3}, 4/3, -2/3; -1, 0, -2)$, $Y_H^{(2)} = (0, 1, -1; 0, 1, -1)$. The real parameters α and β are arbitrary and intended to be fixed by experiment.

In fact, there is no genuine renormalization of the mass M ^{/9/}. Polarization tensor of the C -field, which must be transverse due to the current conservation, contains one logarithmic divergence which is removed by the wave function renormalization. In this respect the Lagrangian (2.1) closely resembles purely massless theory.

2.a. Fermion masses

Consider the region where the renormalized coupling constant h is large, while the other coupling constants (g , g' and g^{acd}) are small, i.e. both the electroweak interactions and QCD can be treated perturbatively as "weak external perturbations", or switched off completely. The important fermion - C -boson interaction is treated as follows.

As an example, consider the electron field ψ :

$$\mathcal{L} = \bar{\psi} i \not{\partial} \psi + h \bar{\psi} \Gamma^\alpha \psi C_\alpha - \frac{1}{4} (\partial_\mu C_\nu - \partial_\nu C_\mu)^2 + \frac{1}{2} M_C^2 C^\alpha C^\alpha - (\partial_\mu C^\alpha)^2 / 2d, \quad (2.3)$$

where

$$\Gamma^\alpha = g^\alpha(a - b\gamma_5), \quad a = \frac{1}{4}[y(\psi_L) + y(e_R)], \quad b = \frac{1}{4}[y(\psi_L) - y(e_R)]. \quad (2.4)$$

To treat the model (2.3) separately is in general legal up to the ABJ anomalies.

The use of a straightforward perturbation theory means that we a priori decide what the particles corresponding to the fields ψ and C will be not asking the dynamics whether it likes it or not. To do better (still having a particle interpretation of the corresponding fields)* we offer the system a choice:

$$\mathcal{L} = \mathcal{L}'_0 + \mathcal{L}'_{int},$$

where /10/

$$\mathcal{L}'_0 = \bar{\psi}(i\cancel{\partial} - \Sigma)\psi - \frac{1}{4}(\partial_\alpha C_\beta - \partial_\beta C_\alpha)^2 + \frac{1}{2}M^2 C_\alpha C^\alpha, \quad (2.5)$$

$$\mathcal{L}'_{int} = h\bar{\psi}\Gamma^\alpha\psi C_\alpha + \bar{\psi}\Sigma\psi. \quad (2.6)$$

\mathcal{L}'_0 , eq. (2.5) will be a starting point of a bona fide better, self-consistent perturbation theory. Self-consistency means that Σ is determined from the condition that the new interaction (2.6) gives zero contribution to \mathcal{L}'_0 in the lowest order using the propagator defined by it. In the Landau gauge ($d=0$), standard manipulations with the Feynman rules yield the self-consistency condition formulated above in the form

$$\Sigma(p^2) + i\lambda \int \frac{d^4k}{(2\pi)^4} \frac{\Sigma(k^2)}{(k^2 - \Sigma^2(k^2))[(p-k)^2 - M^2]} = 0. \quad (2.7)$$

For a nontrivial solution to exist the coupling constant

$$\lambda = \frac{3}{4}y(\psi_L)y(e_R)h^2 \quad (2.8)$$

must be positive. It is readily verified that the parameters α and β in (2.2) can be chosen so that $a^2 - b^2 = \frac{1}{4}y(\psi_L)y(e_R) > 0$ for all fermion species /11/. The quantities h and M entering into eq. (2.7) are assumed to be renormalized /11/ according to the Weinberg's scheme /12/.

Performing in (2.7) the angular integration after Wick's rotation we obtain

*) Another possibility is to require confinement: no particle corresponding to a field.

$$\Sigma(-p^2) = \frac{\lambda}{8\pi^2} \int_0^\infty \frac{k^2 dk^2}{p^2 + k^2 + M^2 + \sqrt{(p^2 + k^2 + M^2)^2 - 4p^2k^2}} \frac{\Sigma(-k^2)}{k^2 + \Sigma^2(-k^2)} \quad (2.9)$$

We argue that the linearization of eq. (2.7) or (2.9) cannot be a good approximation, since by doing it we surely loose the nonanalytic dependence of the result upon the coupling constant which we suspect to take place, according to the renormalization group arguments /13/. Instead, we replace the true kernel

$$f(p^2, k^2) = f(k^2, p^2) = 1 / (p^2 + k^2 + M^2 + \sqrt{(p^2 + k^2 + M^2)^2 - 4p^2k^2})$$

by an approximate one,

$$F(p^2, k^2) = F(k^2, p^2) = M^2 / 2(p^2 + M^2)(k^2 + M^2) \quad (2.10)$$

It has the properties $f(0, k^2) = F(0, k^2)$ and at $k^2 \rightarrow \infty$ $f(p^2, k^2) \rightarrow 1/2k^2$, while $F(p^2, k^2) \rightarrow [M^2/(p^2 + M^2)] \cdot (1/2k^2)$. The approximation is not good for p^2, k^2 simultaneously $\rightarrow \infty$. Equation (2.9) with the approximate kernel (2.10) is immediately solved:

$$\Sigma(p^2) = \frac{cM^3}{-p^2 + M^2} \quad (2.11)$$

where the dimensionless constant c is determined from the condition

$$\frac{\lambda}{16\pi^2} \int_0^\infty \frac{x dx}{x(x+1)^2 + c^2} = 1. \quad (2.12)$$

The electron mass m_e is defined as

$$m_e = \Sigma(m_e^2) = \frac{c_e M^3}{-m_e^2 + M^2} \approx c_e M. \quad (2.13)$$

It follows immediately from (2.12) that $m_e \rightarrow 0$ (i.e. $c_e \rightarrow 0$) for $\alpha = h^2/4\pi \rightarrow 4\pi/\frac{3}{4}y(\psi_L)y(e_R)$ from the above, or, that the fermion mass is generated starting from some critical, large value of the coupling constant. Renormalization group analysis of the dynamical mass generation /13/ enables us to conclude that $\alpha_{eUV}(c_e) = 4\pi/\frac{3}{4}y(\psi_L)y(e_R)$ is the nontrivial UV fixed point. For $c_e^2 \ll 1$ we find explicitly from (2.12)

$$c_e = \alpha_{eUV} / [1 + c_e^2(\ln c_e^2 - 1)].$$

According to this picture generation of different fermion masses (or fermion condensates) due to the strong C -boson interaction proceeds on different scales (M_f) depending upon the numerical values

of their C -hypercharges $y_L y_R$. Consequently, to compare different fermion masses, we must first compare the corresponding mass scales on which different fermions condense. The only tool we have at our disposal is the renormalization group: $\alpha(m_f) = 1/B \ln \frac{\Lambda}{m_f}$.

Here $B < 0$ and λ is the renormalization group invariant parameter of the C -boson-fermion interaction.

Hence

$$\frac{m_f}{m_f'} = \exp \left[-\frac{3}{16\pi B} (y_L y_R - y_L' y_R') \right]. \quad (2.14)$$

In order to compare the numerical values of different fermion masses within one family using the formulas (2.13) and (2.12), we need to renormalize explicitly our theory following ref. /12/. This is not done yet. We suspect that small differences in C -hypercharges will lead to large amplifications of fermion masses similar to those shown by eq. (2.14).

We find it quite interesting that the fermion mass ratios can be expressed in terms of the C -hypercharges, i.e. mere numbers, which do not undergo renormalizations and may be even quantized.

2b. Gauge boson masses

Up to now we have treated the C -boson interaction separately. Now we take into account the $SU(2) \times U(1)_Y$ electroweak symmetry together with its Ward-Takahashi identities. It is important that these identities are consequences of the symmetry of the Lagrangian rather than of the symmetry of solution of the theory. Since we have dynamically generated the fermion masses, we have simultaneously spontaneously broken the $SU(2)_L \times U(1)_Y$ symmetry down to $U(1)_{em}$. According to the Goldstone theorem three massless spinless bosons should exist as physical particles provided that the $SU(2)_L \times U(1)_Y$ symmetry is considered global. In reality the "would be Goldstone bosons" are visualized as massless poles in the proper vertex functions Γ_W^α and Γ_Z^α due to the WT identities /15/, which we assume renormalizably maintained /11,13/:

$$\begin{aligned} \Gamma_W^\alpha(p+q, p) &= \frac{g}{2\sqrt{2}} g^\alpha (1-\delta_5) - \frac{g}{2\sqrt{2}} \frac{g^\alpha}{q^2} [(1-\delta_5) \Sigma_U(p+q) - (1+\delta_5) \Sigma_D(p)] \\ &\approx \frac{g}{2\sqrt{2}} g^\alpha (1-\delta_5) - \frac{g}{2\sqrt{2}} \frac{g^\alpha}{q^2} [(1-\delta_5) m_U - (1+\delta_5) m_D], \end{aligned} \quad (2.15)$$

$$\begin{aligned} \Gamma_Z^\alpha(p+q, p) &= \frac{g}{2 \cos \Theta_W} \left\{ t_3^F g^\alpha (1-\delta_5) - 2Q_F g^\alpha \sin^2 \Theta_W - \frac{g^\alpha}{q^2} t_3^F [\Sigma_F(p+q) + \Sigma_F(p)] \delta_5' \right\} \\ &\approx \frac{g}{2 \cos \Theta_W} \left\{ t_3^F g^\alpha (1-\delta_5) - 2Q_F g^\alpha \sin^2 \Theta_W - \frac{g^\alpha}{q^2} t_3^F 2m_F \delta_5' \right\}. \end{aligned}$$

The indices U and D in (2.15) abbreviate the upper (U) and down (D) fermion in an $SU(2)$ doublet and the index F stands for U and D of the same doublet.

From the pole term of Γ_W^α we extract the effective vertex between the fermion-antifermion $\bar{U}D$ pair and the charged dynamical Goldstone boson /1,14/:

$$F_{\bar{U}D} = \frac{(1-\delta_5) m_U - (1+\delta_5) m_D}{\sqrt{m_U^2 I_{U,D} + m_D^2 I_{D,U}}}, \quad (2.16)$$

where the dimensionless quantities $I_{U,D}$ and $I_{D,U}$ will be found explicitly in the following. The pole term of Γ_W^α can be split according to Fig. 1. The only thing to be calculated is the loop integral

$$\begin{aligned} J_{U,D}^\alpha(q) &= \frac{g}{2\sqrt{2}} \int \frac{d^4 k}{(2\pi)^4} \text{tr} \left[S_U(k) g^\alpha (1-\delta_5) S_D(k-q) \right] \\ &= \frac{g}{2\sqrt{2}} (-i q^\alpha) \sqrt{m_U^2 I_{U,D} + m_D^2 I_{D,U}}, \end{aligned} \quad (2.17)$$

where $I_{U,D}$ is the elementary integral

$$I_{U,D} = \frac{1}{2\pi^2} \int_0^1 x \ln \frac{(m_U^2 - m_D^2)x + m_D^2}{(m_U^2 - m_D^2)x + m_U^2} dx. \quad (2.18)$$

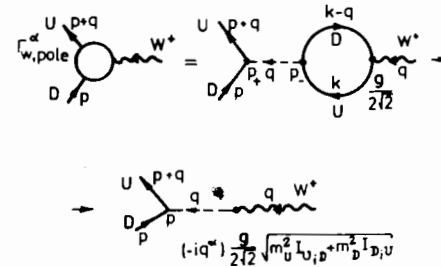


Fig. 1. The effective coupling of the charged Goldstone bosons with fermions and W boson.

We should not be surprised that the integral (2.17) is finite. This is because the dynamically generated self-energy (2.11) acts as an ultraviolet (as well as infrared) regulator. In evaluating the integral (2.17) we have used the fermion propagator in the form $S_F(p) = [\not{p} + \Sigma_F(p)] / (p^2 - m_F^2)$.

The polarization tensor of the W boson is found easily in accordance with Fig. 2:

$$i\Gamma_{W,\rho\sigma}^{\alpha\beta}(q) = i \frac{q^\alpha q^\beta}{q^2} \frac{1}{g^2} (m_U^2 I_{U,D} + m_D^2 I_{D,U}).$$

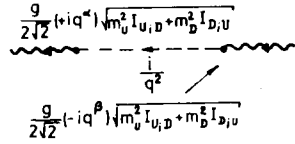


Fig. 2. Pole term in the vacuum polarization tensor of the W boson.

This means ^{115/} that the charged vector boson acquires a mass, $m_W^2 = \frac{1}{4} g^2 (m_U^2 I_{U,D} + m_D^2 I_{D,U})$. The generation of the Z boson mass proceeds quite analogously. Since both lepton and quark doublets operate in this mechanism incoherently, we can write down the finite W and Z boson masses in the form of sum rules:

$$m_W^2 = \frac{1}{4} g^2 \sum_{U,D} (m_U^2 I_{U,D} + m_D^2 I_{D,U}), \quad (2.19)$$

$$m_Z^2 = \frac{1}{4} (g^2 + g'^2) \sum_{U,D} (m_U^2 I_{U,U} + m_D^2 I_{D,D}). \quad (2.20)$$

Thus we predict the definite deviation from the canonical GWS relation

$$m_W^2 / m_Z^2 \cos^2 \Theta_W = 1. \quad (2.21)$$

We obviously get the GWS relation (2.21) by setting $m_U = m_D$ in (2.19) and (2.20).

3. Conclusions

We have shown how to dynamically calculate the fermion masses within one family as resulting from the strong attraction between left- and right-handed components of the originally massless fermion fields due to exchange of the heavy neutral vector boson C. Fermion mass ratios can be expressed in terms of the C-hypercharges, i.e. mere numbers which do not undergo renormalizations and may be even quantized. To complete this program, it remains to check the quality of the separable approximation (2.10), to perform the explicit renormalization of (2.3) following ref. ^{112/}, and to analyze the fermion mixing (more families).

Masses of W and Z bosons are generated by the Schwinger mechanism and are expressed in terms of fermion masses and fermion-Goldstone boson coupling constants in the form of sum rules (2.19) and (2.20). Saturation of these sum rules necessitates the existence of heavy fermions in the model. In order to obtain $m_W \sim (\frac{2\alpha}{\pi})^{1/2} m_U \ln^{1/2}(M/m_U) \sim 80$ GeV, it is sufficient to have one U-type fermion with $m_U = 500$ GeV and $M = 10^5$ GeV. The large value of M is dictated by the fact that the neutral current coupled to the C-boson changes flavour when more families are taken into account (see eq. (3.3)). This estimate serves simultaneously as an upper bound on the fermion masses in the electroweak model.

With $m_U = 500$ GeV and $M = 10^5$ GeV we have $(I_{U,D} \sim \frac{1}{2\pi^2} \ln(M/m_U), I_{U,U} \sim \frac{1}{2\pi^2} [\ln(M/m_U) - \frac{1}{2}])$ $m_W^2 / m_Z^2 \cos^2 \Theta_W = 1.104$.

"Who ordered" the like fermions $(\nu_e, \nu_\mu, \nu_\tau, \dots)$, (e, μ, τ, \dots) (u, c, t, \dots) and (d, s, b, \dots) organized into families or generations, is not understood at present. The like fermions behave differently only in the sector that is responsible for their masses. In the standard model the like fermions differ in their Yukawa couplings, in the present approach they differ in C-hypercharges. We have analyzed also the dynamical mass generation with the fermion mixing, including the possible generation of the Majorana masses of the neutrinos. It is easy to derive the matrix "gap equations" analogous to eq. (2.7), but we have not succeeded to solve them explicitly.

For m_W and m_Z the following sum rules hold:

$$m_W^2 = \frac{1}{4} g^2 \sum_{ij} [(m_U U^\dagger(u_R) U(u_L))_{ij} (m_U U^\dagger(u_R) U(u_L))^*_{ij} I_{U,D_j} + U \leftrightarrow D], \quad (3.1)$$

$$m_Z^2 = \frac{1}{4} (g^2 + g'^2) \sum_{ij} [(m_U U^\dagger(u_R) U(u_L))_{ij} (m_U U^\dagger(u_R) U(u_L))^*_{ij} I_{U,U_j} + U \leftrightarrow D]. \quad (3.2)$$

Here $U(u_{L,R})$ and $U(d_{L,R})$ are the unitary matrices which diagonalize the general mass matrices of the upper (U) and lower (D) fermions. These matrices enter also into the neutral current coupled to the C boson:

$$2J_\alpha^C = \bar{\nu}_L U^\dagger(u_L) \gamma(u_L) U(u_L) \delta_\alpha \nu_L + \bar{\nu}_R U^\dagger(u_R) \gamma(u_R) U(u_R) \delta_\alpha \nu_R + \dots \quad (3.3)$$

$\nu_{L,R}$, ... abbreviate columns of the left- and right-handed components of the mass eigenstates of neutrinos, ... and $\gamma(u_L)$, $\gamma(u_R)$, ... are diagonal nondegenerate matrices of the left- and right-handed C-hypercharges of the neutrinos, ... Thus the neutral current (3.3) is

flavour changing and not universal (the mixing matrices in it are not unitary). This imposes severe restriction on M , which we take $M = 10^5$ GeV. In contrast with the standard model, the right-handed mixing angles are observable in this approach.

Referring to the experience with QCD, we allow ourselves an extrapolation that at small distances the strong coupling nonconfining C-boson-fermion interaction should produce a rich spectrum of quite new heavy bound states. In particular, the combinations orthogonal to the dynamical would be Goldstone bosons are of this sort.

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Динамическое нарушение калибровочной симметрии
электрослабого взаимодействия

Массы фермионов и калибровочных бозонов вычисляются динамически в модели Глшоу-Вайнберга-Салама без хиггсовских полей, дополненной тяжелым нейтральным векторным бозоном C. Массы фермионов определяются значениями C-гиперзарядов левых и правых компонент фермионных полей. Массы W и Z бозонов выражаются в терминах масс фермионов и констант связи между фермионами и динамическими голдстоуновскими бозонами в форме правил сумм. Предсказывается малое отклонение от соотношения $m_W^2/m_Z^2 \cos^2\theta_w = 1$. За счет смешивания фермионов физический нейтральный ток, связанный с C-бозоном, является неуниверсальным и меняющим аромат.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Dynamical Breakdown of the Electroweak
Gauge Symmetry

Fermion and gauge boson masses are calculated dynamically in the higgsless Glashow-Weinberg-Salam model supplemented with a heavy neutral vector boson C. Fermion masses are determined by C-hypercharges of the left- and right-handed fermion fields. The W and Z-boson masses are related to the fermion masses and to the calculated fermion-would-be-Goldstone boson coupling constants by sum rules. Small deviation from the canonical relation $m_W^2/m_Z^2 \cos^2\theta_w = 1$ is predicted. Fermion mixing is briefly discussed. Its necessary consequence is that the physical neutral current coupled to the C boson is nonuniversal and flavour changing.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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