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MULTILOOP CALCULATIONS: METHOD OF UNIQUENESS AND FUNCTIONAL EQUATIONS

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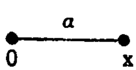
1. INTRODUCTION

In the previous paper [1] the development and application of the method of uniqueness, the method aimed at the calculation of multiloop Feynman integrals, have been given. It has been shown that despite wide possibilities the method has limitations due to nonrealization of the uniqueness conditions at every step of calculations when the number of loops is large (≥ 5).

In this paper we derive the functional equations for the coefficient functions of the diagrams of interest. Solving these equations one can evaluate integrals that are not yielded by other methods. In addition to the method of uniqueness the proposed functional equations enable us to enarge the class of exactly calculable diagrams. As an example of application of functional equations we give the evaluation of an N-like diagram in the ϕ^4 theory.

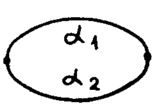
2. DERIVATION OF FUNCTIONAL EQUATIONS

Recall first the notation and necessary formulae of the method of uniqueness. All the calculations are performed in the coordinate space of dimension $D = 4 - 2\epsilon$. Integration is carried out over all internal vertices. Lines of the graphs are associated with simple powers like $1/(x^2)^a$, a being called the index of the line and depicted above the line:



$$\Rightarrow \frac{1}{(x^2)^a} .$$

In what follows the following formulae will be needed [2,1]:



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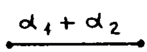
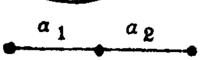
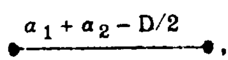


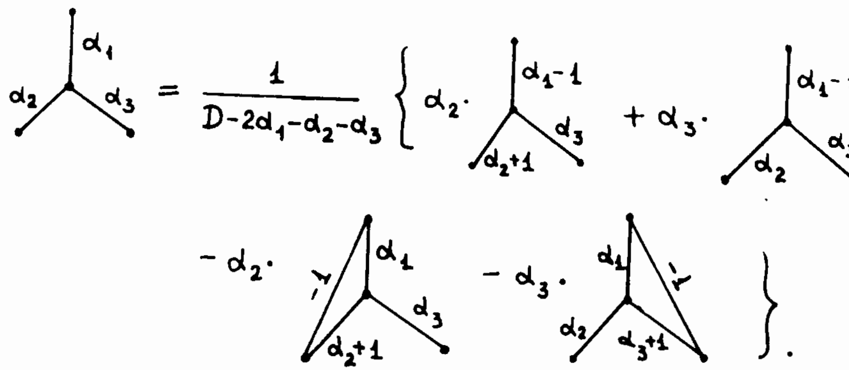


$= v(a_1, a_2, a_3)$



, (1)

$$v(a_1,a_2,a_3) = \prod_{i=1}^{3} \frac{\Gamma(D/2 - a_i)}{\Gamma(a_i)}, \quad a_3 = D - a_1 - a_2, \tag{2}$$



(3)

Consider now the characteristic two-loop diagram (Fig. 1) discussed in the literature [3,2,1]:

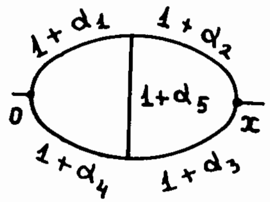
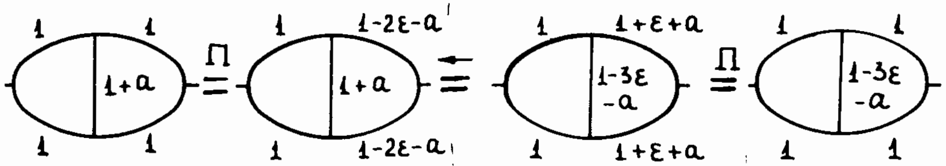


Fig. 1

The dependence of the integral on a single dimensional argument can be separated explicitly on dimensional grounds. Let $a_1 = a_2 = a_3 = a_4 = 0$, $a_5 = a$. Then we have

$$\text{Fig. 1} = \frac{1}{(x^2)^{1+a+2\epsilon}} \cdot F_\epsilon(1+a),$$

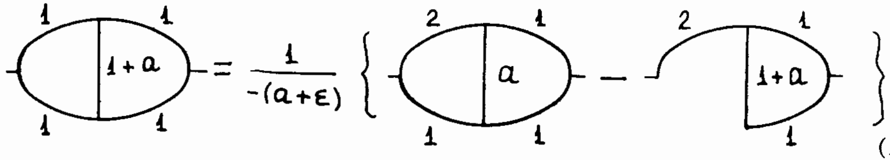
where $F_\epsilon(1+a)$ is the coefficient function of interest. We perform with the diagram the following transformations (see [2]):



In this way we obtain the first equation for $F_\epsilon(1+a)$:

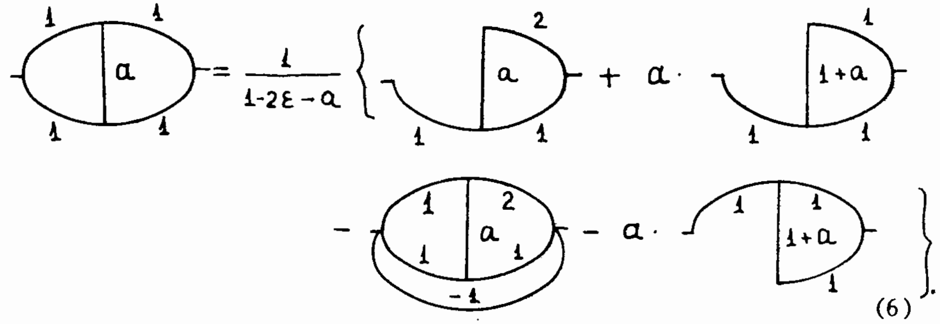
$$F_\epsilon(1+a) = F_\epsilon(1-a-3\epsilon). \qquad (4)$$

To get the second equation, we apply eq. (3) to the upper integration vertex, that gives



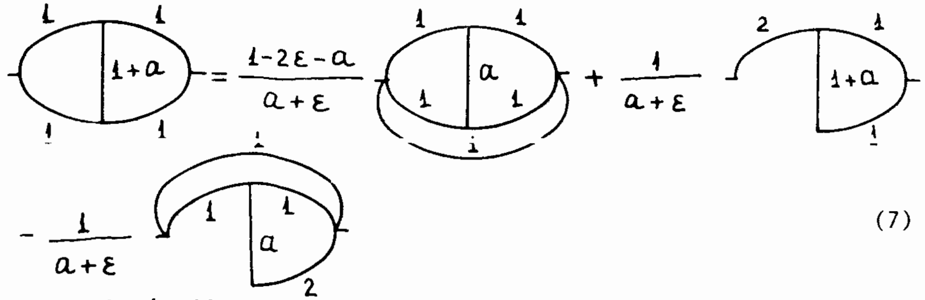
(5)

On the other hand, applying the same equation but with another isolated line we come to



(6)

Combining now eqs. (5) and (6) we obtain the desired equation



(7)

or analytically

$$F_\epsilon(1+a) = \frac{1-2\epsilon-a}{a+\epsilon} F_\epsilon(a) + \frac{2(2a-1+3\epsilon)\Gamma(-a-\epsilon)\Gamma(a+2\epsilon)\Gamma^2(1-\epsilon)}{(a+\epsilon)\Gamma(a+1)\Gamma(2-3\epsilon-a)\Gamma^2(1)}, \qquad (8)$$

where we have used eqs. (1) and (2). Equations (4) and (8) are the functional equations for $F_\epsilon(1+a)$ we are looking for.

3. SOLUTION OF FUNCTIONAL EQUATIONS

To reduce the inhomogeneous part of eq. (8), we make the substitution

$$F_\epsilon(1+a) = 2\frac{\Gamma^2(1-\epsilon)\Gamma(-a-\epsilon)\Gamma(a+2\epsilon)}{\Gamma^2(1)\Gamma(1+a)\Gamma(1-a-3\epsilon)} G_\epsilon(1+a), \qquad (9)$$

where the function G_ϵ obeys the following system of equations

$$G_\epsilon(1+a) = G_\epsilon(1-a-3\epsilon), \qquad (10)$$

$$G_\epsilon(1+a) = -\frac{a}{a-1+3\epsilon} G_\epsilon(a) + \frac{1}{a-1+3\epsilon}\left(\frac{1}{a+\epsilon} + \frac{1}{a-1+2\epsilon}\right). \quad (11)$$

To find a solution, we consider the analytical properties of G_ϵ: It is known, e.g., on the basis of α-representation [4]

$$F_\epsilon(1+a) = \frac{\Gamma(1+a+2\epsilon)}{\Gamma(1+a)} \int_0^1 d\alpha_1 \dots d\alpha_5 \delta(1 - \Sigma\alpha_i)\alpha_5^a \left(\frac{D}{Q}\right)^{1+a+2\epsilon} \frac{1}{D^{2-\epsilon}}, \quad (12)$$

that $F_\epsilon(1+a)$ is a meromorphic function regular at $a = 0$ with simple poles at $a = \pm n - 2\epsilon$ and $a = \pm n - \epsilon$, where $n = 1, 2, \dots$. The same conclusion follows from the inhomogeneous part of eq. (11). Additional poles of $G_\epsilon(1+a)$ arise from the Γ-functions in the denominator of eq. (9). That is why we look for a solution of eqs. (10), (11) in the form of an infinite series of poles

$$G_\epsilon(1+a) = \sum_{n=1}^{\infty} f_n\left(\frac{1}{n+a+\epsilon} + \frac{1}{n-a-2\epsilon}\right) + \sum_{n=1}^{\infty} \phi_n\left(\frac{1}{n+a} + \frac{1}{n-a-3\epsilon}\right), \quad (13)$$

where we have automatically satisfied eq. (10).

Substituting now eq. (13) into eq. (11) and equalizing residues at the poles we get the equations for f_n and ϕ_n:

$$f_n = -f_{n+1}\frac{n+\epsilon}{n+1-2\epsilon}, \quad \phi_n = -\phi_{n+1}\frac{n}{n+1-3\epsilon}.$$

Their solution is

$$f_n = (-)^n \frac{\Gamma(n+1-2\epsilon)}{\Gamma(n+\epsilon)} c_1(\epsilon), \quad \phi_n = (-)^n \frac{\Gamma(n+1-3\epsilon)}{\Gamma(n)} c_2(\epsilon).$$

The inhomogeneous part of eq. (11) fixes c_1: $c_1(\epsilon) = \Gamma(\epsilon)/\Gamma(2-2\epsilon)$. Note that the first series in eq. (13) is a particular solution of the inhomogeneous equation; whereas the second one, of the homogeneous equation. To find the coefficient of the homogeneous part, i.e., $c_2(\epsilon)$, we compare the obtained solution with the known one but for a particular value of a. For this purpose we go back to $F_\epsilon(1+a)$. On uniqueness grounds it is known exactly, i.e., in all orders in ϵ, for $a = 0, -\epsilon, -2\epsilon, -3\epsilon$. Comparing eqs. (9), (13) with $F_\epsilon(1)$, we get

$$c_2(\epsilon) = -\frac{\Gamma(\epsilon)\Gamma(1-\epsilon)\Gamma(1+\epsilon)}{\Gamma(2-2\epsilon)\Gamma(1-2\epsilon)\Gamma(1+2\epsilon)}.$$

This leads to

$$F_\epsilon(1+a) = 2\frac{\Gamma^2(1-\epsilon)\Gamma(-a-\epsilon)\Gamma(a+2\epsilon)\Gamma(\epsilon)}{\Gamma^2(1)\Gamma(1+a)\Gamma(1-a-3\epsilon)\Gamma(2-2\epsilon)} \times$$

$$\times\left\{\sum_{n=1}^{\infty}(-)^n\frac{\Gamma(n+1+2\epsilon)}{\Gamma(n+\epsilon)}\left(\frac{1}{n+a+\epsilon} + \frac{1}{n-a-2\epsilon}\right) - \frac{\Gamma(1-\epsilon)\Gamma(1+\epsilon)}{\Gamma(1-2\epsilon)\Gamma(1+2\epsilon)} \times \right. \quad (14)$$

$$\left. \times \sum_{n=1}^{\infty}(-)^n\frac{\Gamma(n+1-3\epsilon)}{\Gamma(n)}\left(\frac{1}{n+a} + \frac{1}{n-a-3\epsilon}\right)\right\}.$$

For the ultimate conclusion about the validity of eq. (14) one has to be sure that it is impossible to add any arbitrary solution of the homogeneous equation. Really, such a solution $\Delta(a)$ possesses the following properties:

(i) $\Delta(0) = 0$ due to the normalization on $F_\epsilon(1)$,

(ii) $\Delta(\pm n) = 0$, $n = 1, 2, \dots$ due to eq. (8).

(iii) $|\Delta(x+iy)| < |\Delta(x)|$, where x is in the interval between the poles. This property follows from an analogous restriction of integral (12) and the particular solution (14).

(iv) $\Delta(z)$ has no singularities since they are concentrated in the solution (14).

Hence, due to the Karlson theorem [5] $\Delta(z) \equiv 0$. Thus, eq. (14) gives us the needed solution of eqs. (4), (8).

The last sum in eq. (14) is equal to $-\Gamma(1+a)\Gamma(1-a-3\epsilon)$, so the function $F_\epsilon(1+a)$ can be represented in the form

$$F_\epsilon(1+a) = 2\frac{\Gamma^2(1-\epsilon)\Gamma(\epsilon)}{\Gamma^2(1)\Gamma(2-2\epsilon)}\left\{\frac{\Gamma(-a-\epsilon)\Gamma(a+2\epsilon)}{\Gamma(1+a)\Gamma(1-a-3\epsilon)} \times \right. \quad (15)$$

$$\left. \times \sum_{n=1}^{\infty}(-)^n\frac{\Gamma(n+1-2\epsilon)}{\Gamma(n+\epsilon)}\left(\frac{1}{n+a+\epsilon} + \frac{1}{n-a-2\epsilon}\right) + \frac{\Gamma(1-\epsilon)\Gamma(1+\epsilon)\Gamma(-a-\epsilon)\Gamma(a+2\epsilon)}{\Gamma(1-2\epsilon)\Gamma(1+2\epsilon)}\right\}.$$

Unfortunately, we have not succeeded to find a closed expression for the first sum.

For $\epsilon = 0$ eq. (15) leads to

$$F_0(1+a) = \frac{2}{a}\sum_{n=1}^{\infty}(-)^n\left[\frac{1}{(n+a)^2} - \frac{1}{(n-a)^2}\right] = \quad (16)$$

$$= -8\sum_{n=1}^{\infty}(-)^n\frac{n}{(n^2-a^2)^2} = \frac{2}{a}[\beta'(1+a) - \beta'(1-a)],$$

where [6] $\beta(1+x) = \frac{1}{2}\left[\Psi\left(1+\frac{x}{2}\right) - \Psi\left(\frac{1}{2}-\frac{x}{2}\right)\right]$.

4. CALCULATION OF AN N-LIKE DIAGRAM IN THE ϕ^4 THEORY

In the five-loop approximation of the ϕ^4 theory there remains only one diagram (Fig.2) that has not been calculated analytically. To do this, one has to find an N-like diagram (Fig. 3) up to $O(1)$.

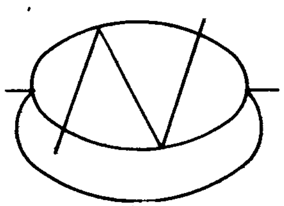


Fig. 2

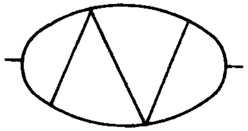
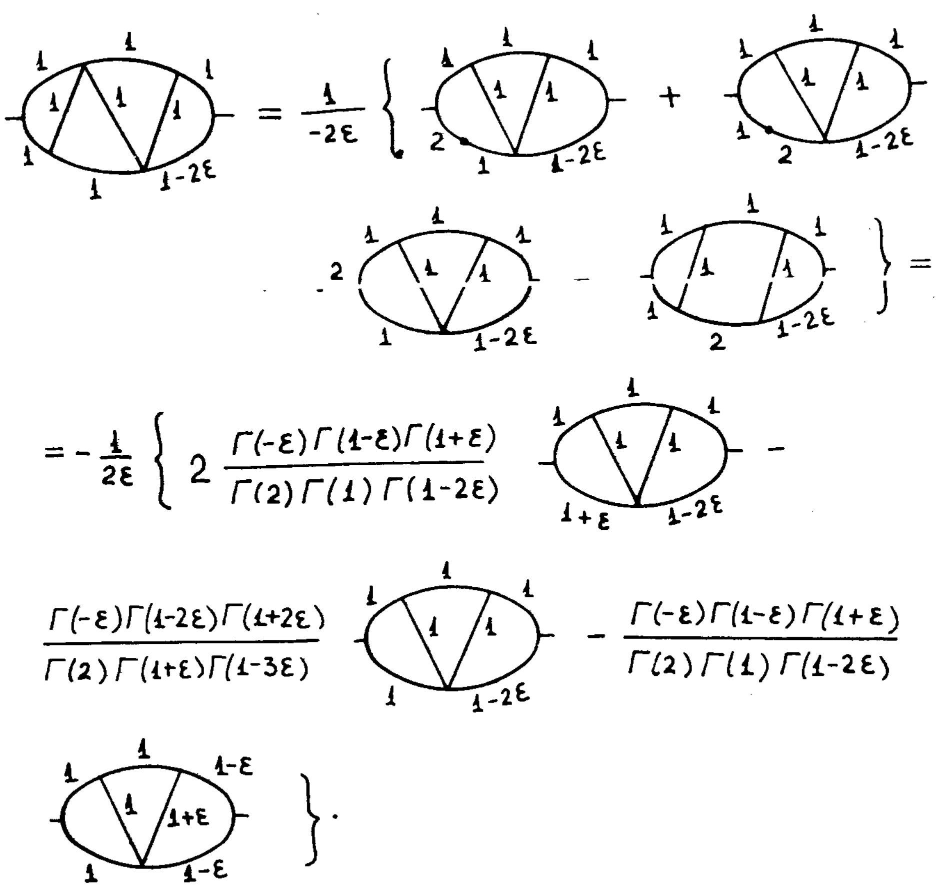


Fig. 3

It was calculated numerically in [7] and in [1] the answer was predictied to be $441/8\,\zeta(7)$. The formula (16) enables us to perform a correct evaluation.

To do this, we choose indices of the lines in the diagram (Fig. 3) in the following way and apply eq. (3) to the lower triple vertex:



We have used here eqs. (1-3).

Hence for the evaluation of the N-like diagram up to O(1) one has to know a V-like diagram up to $O(\epsilon^2)$ or a two-loop diagram (Fig. 1., $\alpha_i = a_i\epsilon$) up to $O(\epsilon^4)$. At the same time the tables obtained in [1] contain expansions up to $O(\epsilon)$ and $O(\epsilon^3)$, respectively. To continue the tables, we use the solution (16). For this purpose let us consider the expansion of $F_\epsilon(1+a\epsilon)$ taking into account the symmetry properties (4).

$$F_\epsilon(1+a\epsilon) = c_0 + c_1\epsilon + [c_2(a+1)(a+2) + c_3 a(a+3)]\epsilon^2 + [c_4(a+1)(a+2) + c_5 a(a+3)]\epsilon^3 + [c_6(a+1)(a+2) + c_7 a(a+3) + c_8 a(a+1)(a+2)(a+3)]\epsilon^4 + O(\epsilon^5).$$

The known expressions for $F_\epsilon(1+a\epsilon)$ for $a = 0, -1, -2, -3$ give us the coefficients $c_0 \div c_7$. We get

$$F_\epsilon(1+a\epsilon) = \frac{1}{1-2\epsilon}\{6\zeta(3) + 9\zeta(4)\epsilon + [21(a+1)(a+2) - 6a(a+3)]\times \zeta(5)\epsilon^2 + [45(a+1)(a+2) - \frac{15}{2}a(a+3)]\zeta(6)\epsilon^3 - [23(a+1)(a+2) - 8a(a+3)]\zeta^2(3)\epsilon^3 + [147(a+1)(a+2) - 9a(a+3)]\zeta(7)\epsilon^4 - [\frac{135}{2}(a+1)(a+2) - \frac{45}{2}a(a+3)]\zeta(3)\zeta(4)\epsilon^4 + c_8 a(a+1)(a+2)(a+3)\epsilon^4\}, \qquad (17)$$

c_8 is not determined from the particular values of a. As is easy to see, it is equal to

$$c_8 = \frac{1}{4!}\left.\frac{d^4 F_0(1+a)}{da^4}\right|_{a=0}.$$

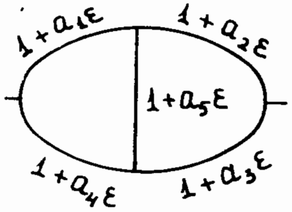
To find it, we expand the function $F_0(1+a)$ (16) into a series in a^2. We have

$$F_0(1+a) = 8\sum_{n=0}^{\infty} a^{2n}(n+1)\left(1 - \frac{1}{2^{2n+2}}\right)\zeta(2n+3). \qquad (18)$$

This leads to

$$c_8 = 189/8\cdot\zeta(7). \qquad (19)$$

The number obtained enables us to complete eq. (17) and also to construct the expansion up to $O(\epsilon^4)$ for an arbitrary two-loop diagram and up to $O(\epsilon^2)$ for an arbitrary V-like diagram, i.e., to continue the tables obtained in [1] by one order of ϵ.

They are 

$$= \frac{\exp\{-2[\gamma\epsilon + \frac{\zeta(2)}{2}\epsilon^2]\}}{1-2\epsilon}\{A_0\zeta(3) +$$

$$+ A_1\zeta(4)\epsilon + A_2\zeta(5)\epsilon^2 + A_3\zeta(6)\epsilon^3 -$$

$$- A_4\zeta^2(3)\epsilon^3 + A_5\zeta(7)\epsilon^4 - A_6\zeta(3)\zeta(4)\epsilon^4 + O(\epsilon^5)\}, \qquad (20)$$

$A_0 = 6,$

$A_1 = 9,$

$$A_2 = 42 + 30(a_1 + a_2 + a_3 + a_4) + 45a_5 + 10(a_1^2 + a_2^2 + a_3^2 + a_4^2) +$$
$$+ 15a_5^2 + 15a_5(a_1 + a_2 + a_3 + a_4) + 10(a_1a_2 + a_3a_4 +$$
$$+ a_1a_4 + a_2a_3) + 5(a_1a_3 + a_2a_4),$$

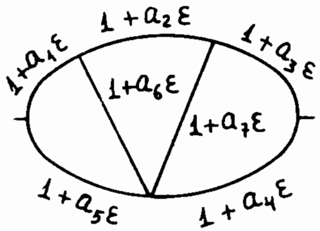
$$A_3 = \frac{5}{2}(A_2 - 6),$$

$$A_4 = 46 + 42(a_1 + a_2 + a_3 + a_4) + 45a_5 + 14(a_1^2 + a_2^2 + a_3^2 + a_4^2) +$$
$$+ 15a_5^2 + 33a_5(a_1 + a_2 + a_3 + a_4) + 50(a_1a_2 + a_3a_4) +$$
$$+ 31(a_1a_3 + a_2a_4) + 14(a_1a_4 + a_2a_3) + 6a_5(a_1^2 + a_2^2 + a_3^2 + a_4^2) +$$
$$+ 6a_5^2(a_1 + a_2 + a_3 + a_4) + 24a_5(a_1a_2 + a_3a_4) +$$
$$+ 12a_5(a_1a_3 + a_2a_4) + 12(a_1a_2a_3 + a_1a_2a_4 + a_1a_3a_4 + a_2a_3a_4) +$$
$$+ 12(a_1^2a_2 + a_2^2a_1 + a_3^2a_4 + a_4^2a_3) + 6(a_1^2a_3 + a_3^2a_1 + a_2^2a_4 + a_4^2a_2),$$

$$A_5 = 294 + 402(a_1 + a_2 + a_3 + a_4) + \frac{2223}{4}a_5 + 260(a_1^2 + a_2^2 + a_3^2 + a_4^2) +$$
$$+ \frac{3183}{8}a_5^2 + 516a_5(a_1 + a_2 + a_3 + a_4) + 386(a_1a_2 + a_3a_4 + a_1a_4 +$$
$$+ a_2a_3) + \frac{575}{2}(a_1a_3 + a_2a_4) + 84(a_1^3 + a_2^3 + a_3^3 + a_4^3) + \frac{567}{4}a_5^3 +$$
$$+ 168(a_1^2a_2 + a_2^2a_1 + a_3^2a_4 + a_4^2a_3 + a_1^2a_4 + a_4^2a_1 + a_2^2a_3 + a_3^2a_2) +$$
$$+ \frac{441}{4}(a_1^2a_3 + a_3^2a_1 + a_2^2a_4 + a_4^2a_2) + \frac{945}{4}a_5(a_1^2 + a_2^2 + a_3^2 + a_4^2) +$$
$$+ 252a_5^2(a_1 + a_2 + a_3 + a_4) + \frac{693}{2}a_5(a_1a_2 + a_3a_4 + a_1a_4 + a_2a_3) +$$

$$+ \frac{945}{4}a_5(a_1a_3 + a_2a_4) + 210(a_1a_2a_3 + a_1a_2a_4 + a_1a_3a_4 + a_2a_3a_4) +$$
$$+ 14(a_1^4 + a_2^4 + a_3^4 + a_4^4) + \frac{189}{8}a_5^4 + 42a_5(a_1^3 + a_2^3 + a_3^3 + a_4^3) +$$
$$+ \frac{189}{4}a_5^3(a_1 + a_2 + a_3 + a_4) + \frac{525}{8}a_5^2(a_1^2 + a_2^2 + a_3^2 + a_4^2) +$$
$$+ \frac{357}{4}a_5^2(a_1a_2 + a_3a_4 + a_1a_4 + a_2a_3) + \frac{105}{2}a_5^2(a_1a_3 + a_2a_4) +$$
$$+ 84a_5(a_1^2a_2 + a_2^2a_1 + a_3^2a_4 + a_4^2a_3 + a_1^2a_4 + a_4^2a_1 + a_2^2a_3 + a_3^2a_2) +$$
$$+ \frac{189}{4}a_5(a_1^2a_3 + a_3^2a_1 + a_2^2a_4 + a_4^2a_2) + \frac{357}{4}a_5(a_1a_2a_3 +$$
$$+ a_1a_2a_4 + a_1a_3a_4 + a_2a_3a_4) + 28(a_1^3a_2 + a_2^3a_1 + a_3^3a_4 +$$
$$+ a_4^3a_3 + a_1^3a_4 + a_4^3a_1 + a_2^3a_3 + a_3^3a_2) + 14(a_1^3a_3 + a_3^3a_1 +$$
$$+ a_2^3a_4 + a_4^3a_2) + 42(a_1^2a_2^2 + a_3^2a_4^2 + a_1^2a_4^2 + a_2^2a_3^2) +$$
$$+ \frac{189}{8}(a_1^2a_3^2 + a_2^2a_4^2) + 42(a_1^2a_2a_3 + a_1^2a_2a_4 + a_1^2a_3a_4 +$$
$$+ a_2^2a_1a_4 + a_2^2a_1a_3 + a_2^2a_3a_4 + a_3^2a_1a_4 + a_3^2a_2a_4 + a_3^2a_1a_2 +$$
$$+ a_4^2a_2a_3 + a_4^2a_1a_3 + a_4^2a_1a_2) + \frac{315}{4}a_1a_2a_3a_4,$$

$$A_6 = 3(A_4 - 1).$$



$$= \frac{\exp\{-3[\gamma\epsilon + \frac{\zeta(2)}{2}\epsilon^2]\}}{1-2\epsilon} \times$$

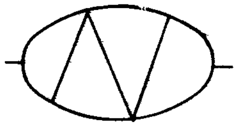
$$\times\{20\zeta(5) + \epsilon[50\zeta(6) + (20 + 6(a_4 + a_5 + a_6 + a_7))\zeta^2(3)] +$$
$$+ \epsilon^2[\zeta(7)\cdot 7(\frac{380}{7} + 20(a_1 + a_3) + 32a_2 + 17(a_4 + a_5) + 33(a_6 + a_7) +$$
$$+ 6(a_1^2 + a_3^2) + 8a_2^2 + 4(a_4^2 + a_5^2) + 8(a_6^2 + a_7^2) + 8(a_1 + a_3)a_2 + \qquad (21)$$
$$+ 2(a_1a_4 + a_3a_5) + 6(a_1a_5 + a_3a_4) + 10(a_1a_6 + a_3a_7) +$$
$$+ 6(a_1a_7 + a_3a_6) + 4a_1a_3 + 4(a_4 + a_5)a_2 + 12(a_6 + a_7)a_2 +$$

$$+ 2a_4a_5 + 4(a_4a_6 + a_5a_7) + 6(a_4a_7 + a_5a_6) + 10a_6a_7 +$$

$$+ \frac{1}{4}(a_4 + a_5 + a_6 + a_7) + \frac{1}{8}(a_4 + a_5 + a_6 + a_7)^2) +$$

$$+ \zeta(3)\,\zeta(4)\cdot 3(20 + 6(a_4 + a_5 + a_6 + a_7)) + O(\epsilon^3)\} .$$

Eq. (21) enables us to complete the calculation of the N-like diagram. The result is



$$= \frac{1}{x^2} \cdot \frac{441}{8}\zeta(7) ,$$

that coincides with the prediction made before [1]. The expansions (20), (21) can be used further like tables for the multiloop-integral evaluation.

5. CONCLUSION

We demonstrate here that functional equations can be useful in calculating multiloop Feynman integrals. Analogous equations can be obtained for more complicated diagrams. It may be that along this way we can see the general structure of the diagram. Till now all exactly calculable integrals were represented in the form of a product of Γ-functions and their derivatives and hence could be expanded into series in ζ-functions. Whether it is also true in general is not clear. Anyhow, solving functional equations one can obtain solutions in the form of a one-fold series like eq. (14).

From a practical point of view, the tables (20), (21) are sufficient for multiloop calculations up to 5 loops. The task is to reduce the diagram to the table as it was done in §4. It seems that the accuracy achieved will be sufficient in real calculations for a long period.

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Казаков Д.И. E2-83-839
Многопетлевые вычисления: метод уникальностей и функциональные уравнения

В рамках метода вычисления многопетлевых фейнмановских интегралов - метода уникальностей - получены функциональные уравнения для коэффициентных функций диаграмм. Путем решения функционального уравнения вычислена N-образная диаграмма, последняя из 5-петлевых диаграмм теории ϕ^4. Полученный результат позволяет на порядок расширить построенные ранее таблицы для вычисления многопетлевых интегралов.

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Kazakov D.I. E2-83-839
Multiloop Calculations: Method of Uniqueness and Functional Equations

In the framework of the method of uniqueness we derive functional equations for the coefficient functions of Feynman diagrams. Solving these equations we evaluate an N-like diagram, the last diagram in the five-loop approximation in ϕ^4 theory. This result enables us to expand the tables for multiloop calculations obtained earlier.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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