объединенный
институт
ядерных
исследований
дубна

E2-83-795

D.Ebert

QUARK AND MESON MASSES IN A COMPOSITE-MESON MODEL WITH BROKEN SU(4)-SYMMETRY

Submitted to "Zeitschr.Phys.C - Particles and Fields"

1983

1. Introduction

At the present time the current algebra - PCAC approach to low energy hadronic interactions based on the idea of chiral symmetry and spontaneous breakdown of chiral invariance is in a remarkably satisfactory shape[1,2]. A lot of work has been done in the last years to extend these ideas from the hadronic level down to the world of quarks in order to understand the implications of chiral symmetry for quarks[1,3]. In particular, we want to understand much better such important issues like the origin of PCAC (partially conservation of axial current) which should be directly derived at the quark level, or the connection between the current quark masses in the Lagrangian and the constituent quark mass parameters used in nonrelativistic quark models. An ultimate goal is,of course, to embed the entire approach into an underlying gauge field theory of interacting quarks with approximate chiral symmetry that is,according to the present paradigm, QCD.

From the microscopic side many attempts have already been made in order to clarify the relation between the quark and hadron levels. This concerns,e.g.,1/N expansions in QCD_2[4], sum rule approaches[5] or QCD lattice calculations of condensate and hadron parameters [6]. Nevertheless, there remain many important problems that have yet to be better understood like that of the exact nature of the nonperturbative mechanism of flavour symmetry breaking, the connection between current and constituent quark masses or the construction of phenomenological chiral Lagrangians out of the functional integral of QCD. Waiting for an answer to these important questions we recall to the reader that there exists another much simpler approach to dynamical breaking of chiral symmetry proposed many years ago by Nambu-Jona Lasinio[7]. The simplicity of this model stems from the fact that the effective interaction of fermions is a local four-fermion coupling. That is, of course, not the case for QCD but there have been recently given arguments that a separable quark interaction could be a reasonable approximation to gluon and instanton mediated interactions[8]. Besides this, nonlinear quark models of this type are interesting by themselves and worth to be explored further in order to test new non-perturbative ideas and techniques.

In particular, as has been demonstrated some years ago[9], nonlinear fermion theories lead to effective composite-meson Lagrangians exhibiting a remarkable resemblance to standard Yukawa or gauge theories. In a recent paper denoted by I[10] we derived a composite-meson model containing σ, π, ρ and ω mesons starting from a U(2) invariant nonlinear quark theory. This model provides us with a dynamical explanation of vector-meson dominance of electroweak interactions, PCAC relations and interesting estimates for the bare and total masses of light quarks, $m_u^\circ \approx 5$ MeV, $m_u \approx 233$ MeV, close to or resembling the estimates for current and constituent quark masses of other approaches. In ref.[11] this type of model has been further extended to flavour SU(3) symmetry. The main purpose of this paper is to extend the analysis of quark and meson masses initiated in I to a suitably generalized SU(4) model in order to test some of the ideas mentioned in the first part of this introduction. In particular, we shall estimate and relate the bare and total masses of u, d, s and c quarks and compare them with those obtained in other approaches. Moreover, there will arise numerous predictions for coupling constants, decay constants and masses of the composite pseudoscalar 15-plet of SU(4).

The paper is organized as follows. In Sect. 2 we define the nonlinear quark Lagrangian and derive an 1/N expansion (N - number of colours) of the effective composite-meson Lagrangian by standard path-integral methods. Sect. 3 and 4 contain the meson mass formulae and a short derivation of the relevant PCAC and Goldberger-Treiman relations, respectively. Finally, Sects. 5 and 6 are devoted to numerical discussions and a summary of results.

2. Effective Quark and Meson Lagrangians

Let us consider the following effective quark Lagrangian with a global flavour symmetry $SU(4)_f$ broken by quark masses and a global colour group SU(N)

$$\mathcal{L}(q,\bar{q}) = \bar{q}_a(i\gamma\partial - \hat{m}_o)q^a + \mathcal{L}_{int}(q,\bar{q}) \tag{1}$$

$$\mathcal{L}_{int}(q,\bar{q}) = \frac{G}{2}\Big[\sum_{i=1}^{3}(\bar{q}_a\tau_i q^a)^2 + \sum_{i=1}^{15}(\bar{q}_a i\gamma_5\lambda_i q^a)^2\Big] .$$

Here the quark field $q(x)=\{q_i^a(x)\}$ ($a=1,2,\dots N;\ i=1,2,\dots 4$) is a N-dimensional vector in colour space and a quartet in flavour space, respectively (quark indices will be omitted henceforth); $\hat{m}_o$ is the diagonal mass matrix $\hat{m}_o = diag(m_u^\circ, m_u^\circ, m_s^\circ, m_c^\circ)$, λ_i are Gell-Mann $SU(4)_f$-matrices and τ_i are diagonal matrices defined by $\tau_1 = diag(1,1,0,0)$, $\tau_2 = diag(0,0,\sqrt{2},0)$ and $\tau_3 = diag(0,0,0,\sqrt{2})$. The special scalar terms in eq.(1) have been introduced to break chiral symmetry both explicitly (by $\hat{m}_o$) and dynamically (by $\langle\bar{q}\tau_i q\rangle \neq 0$).*) G is a universal four-quark coupling strength with dimension (length)^2. For simplicity, we have also disregarded nondiagonal terms in $\mathcal{L}_{int}$ responsible for singlet-octet-15 plet mixing.

To derive Goldberger-Treiman relations for the 15-plet of pseudoscalar composite-mesons of $SU(4)_f$ we must further introduce electro-weak interactions into (1) by replacing ∂_μ by the covariant derivative $\mathcal{D}_\mu$ of the electro-weak gauge group $SU(2)_L \times U(1)$, i.e.,

$$\begin{aligned}\partial_\mu \to \mathcal{D}_\mu = \partial_\mu + \Big[\frac{i\kappa}{2\sqrt{2}}\frac{1+\gamma_5}{2}\Big\{\cos\theta_c\,(\lambda^{1+i2} + \lambda^{13-i14}) \\ + \sin\theta_c\,(\lambda^{4+i5} - \lambda^{11-i12})\Big\}W_\mu^+ + h.c.\Big] \\ + \frac{i\kappa}{2\cos\theta_W}\Big[(1+\gamma_5)T_3 - 2\sin^2\theta_W\, Q\Big]Z_\mu + ieQA_\mu .\end{aligned} \tag{2}$$

In eq. (2) $\kappa, \theta_W, \theta_C$ are the gauge coupling constant, Weinberg angle and Cabibbo angle, respectively; Q , T_3 are the operators of electromagnetic charge and third component of weak isospin and $W_\mu^\pm$, Z_μ , A_μ are the electro-weak gauge bosons.

The dynamical content of the nonlinear quark model (1) can be best analyzed by applying path-integral techniques which are now widely used for deriving mean-field perturbation theories or 1/N expansions[9,12]. The idea is to introduce colour-singlet composite fields for the mesons in such a way that the action is bilinear in the quark fields. One can then do the path integral over quark fields leaving a nonpolynomial action in the meson fields that can be expanded in perturbation theory. To this end, let us introduce the generating functional $Z(\varsigma,\bar{\varsigma},\dots)$ of the theory defined by

$$Z(\varsigma,\bar{\varsigma},\dots) = \mathcal{N}\int[\mathcal{D}q\,\mathcal{D}\bar{q}\dots]\exp i\int d^4x\Big(\mathcal{L}(q,\bar{q};A,W^\pm,Z) + \bar{\varsigma}q + \bar{q}\varsigma + \dots\Big), \tag{3}$$

---

*) As we are mainly interested in the 15-plet of composite pseudoscalar mesons we omitted other possible scalar interaction terms in (1). The inclusion of such terms would lead to additional scalar mesons and does not change our main results. We neglect also isospin breaking.

where $\rho, \bar{\rho}$ are quark sources and $\tilde{\mathcal{N}}$ is a normalization factor. Integration over gauge fields as well as the inclusion of suitable gauge field Lagrangians and source terms is implicitly understood in eq. (3). Introducing a convenient set of colour-singlet composite scalar or pseudoscalar fields, $\{\sigma_i\} \sim (\bar{q}\tau_i q)$, $\{\phi_i\} \sim (\bar{q} i\gamma_5 \lambda_i q)$, where

$$\{\sigma_i\} = \{\sigma_1, \sigma_2, \sigma_3\}$$
$$\{\phi_i\} = \{\pi, K, \eta_8, D, F, \eta_{15}\}$$

and performing the integration over the quark fields in (3), we get the following effective meson Lagrangian

$$\mathcal{L}_{eff}(\{\sigma\},\{\phi\}) = -\frac{\Lambda^2}{2}\Big(\sum_{i=1}^{3}\sigma_i^2 + \sum_{i=1}^{15}\phi_i^2\Big) - iN\,Tr\Big(\ln\{-S^{-1}(x,y)\}\Big)_{x=y}. \tag{4}$$

S is the quark propagator which is defined by the equation

$$S^{-1}(x,y) = -\Big\{i\gamma\mathcal{D} - \hat{m}_0 - g\Big(\sum_{i=1}^{3}\sigma_i\tau_i + i\gamma_5\sum_{i=1}^{15}\phi_i\lambda_i\Big)\Big\}\delta^4(x-y) \tag{5}$$

and the trace runs over Dirac and flavour indices.

In eq. (5) we have introduced a dimensionless coupling constant g , $g^2/4\pi$ playing the role of a (bare)"fine structure constant" of the model, by setting $G = g^2/\Lambda^2$. In the following we shall treat the distance scale Λ^{-1} as a measure characterizing the range of the short-range $q\bar{q}$ forces responsible for dynamical breaking of chiral symmetry (compare eq. (7) and remarks after eq. (9)). This implies that all momentum space integrals arising in the loop expansion of the quark determinant of (4) should also be cut off at Λ .

Let us recall that the bare quark masses $\hat{m}_0$ appearing in the Lagrangian (1) break chiral symmetry explicitly. Additional dynamical breaking occurs when the scalar σ fields get nonvanishing vacuum expectation values $\langle\sigma_i\rangle \neq 0$. To find the corresponding expressions one has to examine the stationarity condition $\delta\mathcal{L}_{eff}/\delta\sigma_i = 0$ for $\{\phi\} = 0$. Introducing the matrix $\hat{m}$ of total quark masses defined as the sum of bare and dynamical masses

$$\hat{m} = \hat{m}_0 + \hat{m}_{dyn} \equiv \hat{m}_0 + g\sum_{i=1}^{3}\langle\sigma_i\rangle\tau_i \tag{6}$$

the above stationarity condition takes the form of a self-consistent Hartree-Fock equation

$$m_i = m_i^0 + 8\, m_i\, G\, I_1(m_i) \qquad (i=1,2,3). \tag{7}$$

where $I_1(m_i)$ is the regularized integral

$$I_1(m_i) = \frac{iN}{(2\pi)^4}\int^{\Lambda}\frac{d^4k}{(k^2-m_i^2)} = \frac{N}{(4\pi)^2}\Lambda^2\Big\{1-\frac{m_i^2}{\Lambda^2}\ln\Big(1+\frac{\Lambda^2}{m_i^2}\Big)\Big\} \tag{8}$$

and we used the notation $(m_1, m_2, m_3) \equiv (m_u, m_s, m_c)$.

Recalling $\frac{G}{4\pi} = \frac{g^2}{4\pi}/\Lambda^2$ we find from eq. (7)

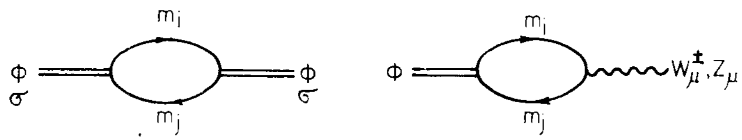
$$\frac{g^2}{4\pi} = \pi\big(1-m_i^0/m_i\big)/2N\Big\{1-\frac{m_i^2}{\Lambda^2}\ln\Big(1+\frac{\Lambda^2}{m_i^2}\Big)\Big\} \quad . \tag{9}$$

As one decreases the mass m_i to zero (with $m_i^0/m_i \to 0$), the interaction strength $g^2/4\pi$ needed to give that mass goes to a critical value, $g^2/4\pi^{(crit)} = \pi/2N$. This is just the value at which dynamical breaking of chiral symmetry sets in. To get finite quark masses, we must then have $g^2/4\pi > g^2/4\pi^{(crit)}$. Notice, that in the presence of bare quark masses which break chiral symmetry explicitly the required value of $g^2/4\pi$ needed for a given m_i will be closer to the critical value than in the case with $m_i^0 = 0$.

Proceeding further as in ref. /10/, we next perform a shift in the σ-fields, $\sigma_i \to \sigma_i' = \sigma_i - \langle\sigma_i\rangle$ and expand the quark determinant in (4) in powers of $\{\sigma_i'\}$ and $\{\phi_i\}$ (The prime on σ_i' will henceforth be omitted). Performing necessary renormalizations the resulting loop expansion will easily be recognized as an 1/N expansion with g^2N fixed.

3. Mass Formulae

In order to derive mass formulae for scalar and pseudoscalar mesons, we have to evaluate the quark loop diagram of Fig. 1a with two external meson legs.



(a) Fig. 1 (b)

This yields the following quadratic part of our composite-meson theory

$$\mathcal{L}_{eff}^{quadr} = \frac{1}{2}\sum_{i=1}^{3}\Big\{(\partial_\mu\sigma_i)^2 - M_{\sigma_i}^2\sigma_i^2\Big\} + \frac{1}{2}\sum_{i=1}^{15}\Big\{(\partial_\mu\phi_i)^2 - M_{\phi_i}^2\phi_i^2\Big\}. \tag{10}$$

where we have performed the field renormalizations

$$\sigma_i \to Z_{\sigma_i}^{1/2} \sigma_i = \left\{ 4g^2 I_2(m_i, m_i) \right\}^{-1/2} \sigma_i$$
$$\vec{\pi} \to Z_\pi^{1/2} \vec{\pi} = \left\{ 4g^2 I_2(m_1, m_1) \right\}^{-1/2} \vec{\pi}$$
$$\phi \to Z_\phi^{1/2} \phi = \left\{ 4g^2 I_2(m_i, m_j) \right\}^{-1/2} \phi \qquad ;(\phi = K, D, F)$$
$$\eta_8 \to Z_8^{1/2} \eta_8 = \left\{ \tfrac{4}{3} g^2 \left[I_2(m_1, m_1) + 2 I_2(m_2, m_2) \right] \right\}^{-1/2} \eta_8$$
$$\eta_{15} \to Z_{15}^{1/2} \eta_{15} = \left\{ \tfrac{1}{3} g^2 \left[2 I_2(m_1, m_1) + I_2(m_2, m_2) + 9 I_2(m_3, m_3) \right] \right\}^{-1/2} \eta_{15} \tag{11}$$

and $I_2(m_i, m_j)$ is another regularized integral

$$I_2(m_i, m_j) = -\frac{iN}{(2\pi)^4} \int^\Lambda \frac{d^4k}{(k^2 - m_i^2)(k^2 - m_j^2)}$$
$$= \frac{N}{16\pi^2} \left(1 - \left(\frac{m_j}{m_i}\right)^2\right)^{-1} \left\{ \ln\left(1 + \frac{\Lambda^2}{m_i^2}\right) - \left(\frac{m_j}{m_i}\right)^2 \ln\left(1 + \frac{\Lambda^2}{m_j^2}\right) \right\}. \tag{12}$$

It is straightforward to get following mass formulae for scalar and pseudoscalar mesons,

$$M_{\sigma_i}^2 = \frac{m_i^0}{m_i} \frac{g_{\sigma_i}^2}{G} + 4 m_i^2$$
$$M_\pi^2 = \frac{m_1^0}{m_1} g_\pi^2 / G$$
$$M_\phi^2 = \tfrac{1}{2}\left(\frac{m_i^0}{m_i} + \frac{m_j^0}{m_j} \right) g_\phi^2 / G + (m_i - m_j)^2 \quad ; \ (\phi = K, D, F)$$
$$M_{\eta_8}^2 = \tfrac{4}{3}\left(M_K^2 - (m_1 - m_2)^2\right)(g_8/g_K)^2 - \tfrac{1}{3} M_\pi^2 (g_8/g_\pi)^2 \tag{13}$$
$$M_{\eta_{15}}^2 = \tfrac{1}{6}\left(M_K^2 - (m_1 - m_2)^2\right)(g_{15}/g_K)^2 + \tfrac{3}{2}\left(M_D^2 - (m_1 - m_3)^2\right)(g_{15}/g_D)^2 - \tfrac{2}{3} M_\pi^2 (g_{15}/g_\pi)^2 .$$

In deriving (13) use has been made of the Hartree-Fock equations (7). Moreover, g_{σ_i} and g_ϕ are induced renormalized meson-quark coupling constants defined by

$$g_{\sigma_i} = Z_{\sigma_i}^{1/2} g = O(1/\sqrt{N}) \quad ; \quad g_\phi = Z_\phi^{1/2} g = O(1/\sqrt{N}) . \tag{14}$$

Notice that the nonperturbative SU(4) breaking in the total quark masses of (7) has its sole origin in the bare masses m_i^0. Thus, in

the chiral limit $m_i^0 \to 0$, $m_i/m_j \to 1$, and the 15-plet of 0^--mesons will indeed become the expected multiplet of massless Goldstone bosons.

It is worth mentioning that in composite-meson theories $Z = 1/O(g^2)$ due to the lack of a free kinetic part in the meson Lagrangian. Therefore, in contrast to field theories with elementary particles, the bare coupling constant g drops out in the renormalized coupling constants and the latter become calculable functions of the quark masses m_i and the intrinsic cut-off scale Λ. In the following we shall take the length scale Λ^{-1} characterizing the range of chiral symmetry breaking $q\bar{q}$ forces approximately equal for all quark flavours*). Λ was determined in I by fitting the experimental width $\Gamma(\rho \to 2\pi)$ through the induced $\rho\pi\pi$-coupling constant.

As is obvious from the explicit expressions for the Z-factors, $SU(4)_f$ breaking of quark masses will induce a corresponding symmetry breaking pattern of renormalized meson-quark coupling constants. This question will be explored in more detail in Sect. 5.

4. PCAC and Goldberger-Treiman Relations

For a subsequent discussion of the quark and meson mass formulae we need also the explicit expressions for the meson decay constants F_ϕ. To this end, let us consider the weak transition diagrams shown in Fig. 1b. Straightforward evaluation and renormalization leads to the interaction Lagrangians

$$\mathcal{L}_{int}^{ch} = \frac{K}{2\sqrt{2}} W_\mu^+ \left\{ \cos\theta_c \left[\sqrt{2} F_\pi \partial^\mu \pi^- + \sqrt{2} F_F \partial^\mu F^- \right] + \sin\theta_c \left[\sqrt{2} F_K \partial^\mu K^- - \sqrt{2} F_D \partial^\mu D^- \right] \right\} + h.c. \tag{15a}$$

$$\mathcal{L}_{int}^{neut} = \frac{K}{2\cos\theta_W} Z_\mu \left\{ F_\pi \partial^\mu \pi^0 + \frac{F_8}{\sqrt{3}} \partial^\mu \eta_8 - \sqrt{\tfrac{2}{3}} F_{15} \partial^\mu \eta_{15} \right\}, \tag{15b}$$

where

$$F_\pi = m_u / g_\pi \quad , \quad F_K = (m_u + m_s)/2 g_K$$
$$F_D = (m_u + m_c)/2 g_D \quad , \quad F_F = (m_s + m_c)/2 g_F \tag{16a}$$

*) The flavour-independence of the distance scale Λ^{-1} suggests itself if one identifies it, for example, with the short-range core of gluon exchange. This can be compared with the flavour-independence of the bag radii of π, F and D mesons, $R_\pi \simeq R_D \simeq R_F = 3.5 \text{ GeV}^{-1}/137$. Note $R_{chir} = \Lambda^{-1} < R_{conf.} = 2 R_\pi$.

and

$$F_8 = m_s\, g_8/g_{\sigma_2}^2$$

$$F_{15} = \tfrac{1}{4} m_s\, g_{15}/g_{\sigma_2}^2 \; + \; \tfrac{3}{4} m_c\, g_{15}/g_{\sigma_3}^2 \quad . \tag{16b}$$

Note that $F_i = O(\sqrt{N})$. As required, the charged and neutral axial currents in (15) have a form respecting PCAC with decay constants satisfying the usual Goldberger-Treiman relations (16a) or a slightly modified version of it , as given by (16b). Clearly the above expressions for the decay constants include $SU(4)_f$-breaking effects from both quark masses and induced meson-quark coupling constants.

Concluding we mention that the quark determinant in eq. (4) leads also to self-interacting meson terms which can easily be evaluated along the lines of I. One gets the interesting result that interaction vertices with p meson lines ($p \geq 3$) are suppressed in the large N limit by a factor $(1/\sqrt{N})^{p-2} \sim (1/F_i)^{p-2}$ resembling the 1/F expansion of phenomenological chiral meson Lagrangians /2,4/.

In the next subsection we shall discuss the mass formulae of the model in more detail in order to get estimates for bare (current) and total $(m_0^i + m_{dyn}^i)$ quark masses, as well as predictions for meson masses and decay constants.

5. Numerical Discussion

Let us recall at the beginning that our model contains five free parameters: G, Λ, m_u^0, m_s^0 and m_c^0 (from now on we restrict us to the physical value $N=3$). To fix them it will be convenient to take the following input quantities from experiment: $g_{\rho\pi\pi}^2/4\pi \approx 3$, M_π, F_π = 93 MeV, $F_K \approx 1.2\, F_\pi$ and $m_c \approx \frac{1}{2} M_\psi$ = 1.6 GeV. With these values we then get predictions for the other quark masses and the masses, coupling and decay constants of the 15-plet of pseudoscalar mesons.

i) Light particles

Let us shortly recapitulate some results of the composite-meson model I and recall how the parameters Λ, G and m_u^0, m_u have been fixed there. In this paper we investigated a slightly extended version of nonlinear quark models containing besides scalar and pseudoscalar mesons also vector mesons. This allows one to derive an interesting relation between pion-quark and measurable $\rho\pi\pi$-coupling constants, i.e.,

$$g_{\rho\pi\pi} = \sqrt{6}\, g_\pi \tag{17}$$

Taking $g_{\rho\pi\pi}^2/4\pi$ as determined from the experimental value of $\Gamma(\rho\to 2\pi)$ then yields $g_\pi^2/4\pi \approx \frac{1}{2}$. With (16a) and F_π = 93 MeV we then estimated the total u-quark mass as 233 MeV which is somewhat smaller than the usual constituent quark mass $(m_u)_{con} \approx 330$ MeV derived from magnetic moments of hadrons or by setting $(m_u)_{con} \approx M_N/3$ /1,3/. Moreover, from (14) the intrinsic cut-off Λ has been fixed as $\Lambda^2 \approx 22\, m_u^2 = (1093\ \text{MeV})^2$. Finally, taking into account the experimental value of M_π, we estimated m_u^0 and G from eqs. (7), (13) as

$$m_u^0 = m_u \left(1 + 8 g_\pi^2\, I_1(m_u)/M_\pi^2\right)^{-1} \approx 5\ \text{MeV} \quad , \tag{18}$$
$$G/4\pi \approx 0.6/\Lambda^2 .$$

The above value for m_u^0 is in agreement with a corresponding estimate of the current-quark mass, $(m_u)_{curr} \approx 5$ MeV, obtained in a current-algebra approach supplemented by linear baryon mass formulae /1,3/.

ii) Strange particles

Let us next discuss the case of strange quarks and mesons. The ratio of total quark masses m_s/m_u can be determined from the relation

$$m_s/m_u = (2F_K/F_\pi)\, g_K/g_\pi - 1 . \tag{19}$$

Using $F_K/F_\pi \approx 1.2$ and considering $g_K/g_\pi = \left(\frac{I_2(m_u,m_u)}{I_2(m_u,m_s)}\right)^{1/2}$ as a function of m_s/m_u we have solved the transcendental equation (19) numerically. This yields

$$m_s/m_u = 1.85 \quad , \quad g_K/g_\pi = 1.2 \tag{20}$$

and thus, using $m_u = 233$ MeV, $m_s = 430$ MeV. Notice, that this value of m_s is not too different from the "symmetric" estimate, $m_s \approx 460$ MeV, obtained by setting $g_K = g_\pi$ in (19) and using the standard value $(m_u)_{con} = 330$ MeV . Both these values agree within 10-15% with the constituent mass $(m_s)_{con} \approx 510$ MeV obtained from magnetic moment considerations or by setting $(m_s)_{con} \approx \frac{1}{2} M_\varphi \approx \frac{1}{3} M_\Omega$. Finally, the bare quark mass m_s^0 can be calculated from the Hartree-Fock equation (7) as $m_s^0 \approx 105$ MeV. Taking the estimate $g_8/g_\pi = 1.2$ we can then predict the masses of the K and η_8 mesons as well as F_8,

$$M_K = 480\ \text{MeV} \quad , \quad M_{\eta_8} = 500\ \text{MeV}$$
$$F_8 = 1.2\, F_\pi . \tag{21}$$

These masses correspond within an uncertainty of about 10% to the K mass and the value M_{η_8} determined from η^0-η^8 mixing*). Moreover, the value for F_8 is comparable with the estimate $F_\eta/F_\pi = 1.18$ of ref./14/ obtained for the physical η meson. Alternatively, we could insert the experimental value of the K meson mass into (13) and try to fit m_s^0. This yields

$$m_s^0/m_u^0 = \left\{\left[2M_K^2-2(m_u-m_s)^2\right]\frac{g_\pi^2}{g_K^2 M_\pi^2} - 1\right\} m_s/m_u \approx 25 \qquad (22)$$

and thus $m_s^0 \approx 125$ MeV. Again this is close to the standard value of the strange current quark mass, $m_s^{curr} \approx 150$ MeV/1,3/. In this case we get also $M_{\eta_8} \approx 520$ MeV. It is worth remarking that the ratio(22) which includes SU(4) breaking via masses and coupling constants coincides with the PCAC result for the current-quark mass ratio

$$(m_s/m_u)^{curr} = 2(M_K^2/M_\pi^2) - 1 \approx 25 \qquad (23)$$

derived from linear mass relations (cf. ref./1/)

$$M_\pi^2 = 2\, m_u^{curr}\, \xi \quad , \quad M_K^2 = (m_u + m_s)^{curr}\, \xi \, . \qquad (24)$$

In (24) the scale factor $\xi = -\frac{1}{2F_\pi}\langle 0|\vartheta^\pi|\pi\rangle = -\frac{1}{2F_K}\langle 0|\vartheta^K|K\rangle$ is assumed to be SU(3) invariant, ϑ^i being pseudoscalar quark densities.

iii) Charmed Particles

Next, we shall evaluate the relevant parameters of charmed particles. As the decay constants of D - and F mesons are not yet measured, we cannot parallel the case of strange quarks and calculate, say, m_c/m_u, as a function of $2F_D/F_\pi$. On the contrary, we will estimate instead F_D or F_F by taking some standard value of m_c, say $m_c \sim \frac{1}{2} M_\psi = 1.6$ GeV. Using the formulae

$$F_D/F_\pi = \frac{1}{2}\left(1 + \frac{m_c}{m_u}\right) g_\pi/g_D$$
$$F_F/F_\pi = \frac{1}{2}\left(\frac{m_s}{m_u} + \frac{m_c}{m_u}\right) g_\pi/g_F \qquad (25)$$

*) This uncertainty arises mainly from the corresponding uncertainties in the values of the bare and dynamical quark masses. Notice, that the relation (17) which was used for determining Λ and m_u has been derived using an off-shell renormalization procedure. Therefore, the identification $g^2_{\sigma\pi\pi}(0)/4\pi \approx g^2_{\sigma\pi\pi}/4\pi$ is only a crude approximation. Further uncertainties are related to the approximate character of the Goldberger-Treiman relation.

and inserting the coupling constant ratios evaluated with the above value of m_c,

$$g_D/g_\pi = 2.6 \quad , \quad g_F/g_\pi = 2.9 \qquad (26)$$

we finally get

$$F_D \approx F_F \approx 1.5\, F_\pi = 140 \text{ MeV} \, . \qquad (27)$$

As expected, the symmetry breaking in coupling and decay constants of charm mesons is larger than in the case of K or η_8 mesons. In particular, we find that our values of F_D, F_F are comparable to recent estimates from QCD sum rules/15/, and potential models/16/. Finally, the bare mass of charm quarks is evaluated from (7) as

$$m_c^0 = 1250 \text{ MeV}, \qquad (28)$$

which is close to the estimate by QCD sum rules,/3/ for the running mass $m_c(\mu)$, $m_c(m_c) \approx 1.27$ GeV and not too different from the constituent quark mass $m_c \approx 1.6$ GeV.*)

Taking together the values for m_c^0, m_c and the ratios of coupling constants (26), we predict the following values of D and F meson masses

$$M_D = 2140 \text{ MeV},$$
$$M_F = 2380 \text{ MeV}, \qquad (29)$$

which deviate by about 15% from the experimental values $M_D^{Exp} \approx 1869$ MeV, $M_F^{Exp} \approx 2021$ MeV.

For completeness, let us also quote the values of the masses and coupling constants of the η_{15}-meson and the scalar mesons. We find

$$M_{\eta_{15}} = 1640 \text{ MeV},$$
$$g_{15}/g_\pi = 2.1 \qquad (30)$$

and

$$M_{\sigma_1} = 490 \text{ MeV} \quad , \quad g_{\sigma_1}/g_\pi = 1 \, ,$$
$$M_{\sigma_2} = 1100 \text{ MeV} \quad , \quad g_{\sigma_2}/g_\pi = 1.4, \qquad (31)$$
$$M_{\sigma_3} = 4700 \text{ MeV} \quad , \quad g_{\sigma_3}/g_\pi \approx 4 \, .$$

*) Note that in contrast with m_u^0, m_s^0 the mass m_c^0 is not small in comparison with the characteristic scale of bound states. The ratio $C = (m_c^0 - m_u^0)/(m_s^0 - m_u^0)$ which measures the size of SU(4) splitting in comparison with SU(3) splitting is rather large, $C \approx 10$, and coincides with the value of ref. /3/.

The value of $M_{\eta_{15}}$ is smaller than the estimate $M_{\eta_{15}}=2293$ MeV obtained from the mass formula $M^2_{\eta_{15}}=\frac{1}{6}(M^2_{K^0}+9M^2_{D^0})$ of a chiral SU(4)xSU(4) breaking scheme [2]. Anyway, one should keep in mind that to compare this mass with the mass of the physical η_c -meson, $M_{\eta_c}=2980$ MeV, would require to take into account η_0-η_8-η_{15} mixing which is outside the scope of this paper. By the same reasons the scalar mesons $\sigma_1, \sigma_2, \sigma_3$ cannot be identified with physical mesons.

Concluding we mention that we did not try to get a better overall fit or fine-tuning of the various parameters of the model. The reason is that they were determined by using an off-shell renormalization scheme and identification with physical parameter values cannot be but a crude approximation. To obtain a reasonable on-shell scheme one should probably first include a suitable confinement mechanism for quark propagators and/or meson wave functions into the model (comp. with the remarks at the end of Sect. 6).

6. Summary and Conclusions

In this paper we derived an 1/N expansion (N - number of colours) for a composite-meson model based on an effective nonlinear quark Lagrangian with approximate flavour $SU(4)_f$ symmetry. The model contains a variety of relations which enable one to estimate quark and meson masses, quark-meson coupling constants and decay constants. Let us summarize the parameter contents of the model. It contains five free parameters: the four-quark coupling constant G , the intrinsic cut-off scale Λ and the bare quark masses m^0_u, m^0_s and m^0_c. A convenient set of input quantities used in our calculations with 3 colours were: $g^2_\pi/4\pi=\frac{1}{2}$ (as determined from the relation $g_\pi=\frac{1}{16}g_{g\pi\pi}$ and the experimental value $g^2_{g\pi\pi}/4\pi\approx 3$), M_π , $F_\pi=93$ MeV, $F_K=1.2\,F_\pi$ and $m_c\approx\frac{1}{2}M_\psi=1.6$ GeV. With this set of input quantities we then predicted, e.g., $m^0_s\approx 105-125$ MeV, $m_s=430$ MeV, $m^0_c=1250$ MeV, $F_D\approx F_F\approx 1.5\,F_\pi$, $g_K/g_\pi=1.2$, $g_D/g_\pi=2.6$, $g_F/g_\pi=2.9$. Generally speaking, also in the case of strange and charm quarks our estimates for bare and total quark masses turn out to be reasonable and not too different from other estimates for current or constituent quark masses. Moreover, the estimates for the decay constants $F_{D,F}$ are close to predictions obtained in other approaches like QCD sum rules or potential models. Finally, the model yields a mass pattern for the 15-plet of pseudoscalar mesons which is in rough agreement (15% error) with experiments. These results look rather encouraging and support further studies in this field.

Future theoretical work using this type of models should, of course, try to incorporate some confinement mechanism in order to forbid the ionization of mesons into $q\bar{q}$ pairs. As the Goldstone properties of pseudoscalar mesons are determined by the short-range core of the $q\bar{q}$ forces at a scale $R_{chir}=\Lambda^{-1}\approx 1\ GeV^{-1}$ (recall $R_{chir}<R_{conf}=2R^\pi_{bag}\approx 7\ GeV^{-1}$), one expects the confining forces only to modify the long-range tail of the wave functions and to lead to small corrections in the meson masses. First attempts in this direction have been recently done by the authors of ref. [8] who postulated baglike confinement conditions on the $q\bar{q}$ wave function paralleling closely the MIT-bag development [17]. Clearly, it is a challenging task to construct more sophisticated versions of separable nonlinear quark models with light and heavy quarks including baglike confinement conditions.

Acknowledgements

I would like to thank discussions with D.Amati, H.Leytwyler and O. Nachtmann at the CERN Theory Division, where this work has been started. Discussions with M. Ilgenfritz and M.K.Volkov at the Laboratory for Theoretical Physics of the Joint Institute for Nuclear Research, Dubna, are also gratefully acknowledged.

References

1. For recent reviews containing references to the literature see: Scadron M.D., Rep. Progr.Phys. 1981,44, p. 213 and Ann. of Physics, 1983, 148, p. 257.
2. Ebert D., Volkov M.K., Fortschr. Phys. 1981, 29, p. 35. This paper reviews low-energy applications of phenomenological chiral Lagrangians.
3. For a recent review on quark masses see: Gasser J., Leutwyler H., Physics Rep., 1982, 87, p. 77.
4. 't Hooft G., Nucl.Phys., 1974, B75, p. 461. Callan C.G., Cootes N., Gross D.J., Phys.Rev., 1976, D13, p. 1649. Ebert D., Pervushin V.N., Theoret. Math.Fiz., 1978, 36, p. 313.
5. Shifman M.A., Vainshtein A.I., Zakharov V.I., Nucl.Phys. 1979, B147, p. 385, 448, 519.
6. For a review on lattice qauge theories see, for example: Kogut J.B., Rev.Mod.Phys., 1983, 55, p. 775.
7. Nambu Y., Jona-Lasinio G., Phys.Rev., 1961, 122, p. 345.
8. Goldman T., Haymaker R.W., Phys.Rev. 1981, D24, p. 724, 743.

9. Eguchi T., Phys.Rev., 1976, D14, p. 2755.
Kikkawa K., Progr.Theor.Phys., 1976, 56, p. 947.
10. Ebert D., Volkov M.K., Z.Phys. C - Particles and Fields, 1983, 16, p. 205.
11. Ebert D., Proceedings XVI International Symp. Ahrenshoop GDR, 1982, p. 170.
Kreopalov D.V., Volkov M.K., JINR Prepr. E2-83-19, 1983, Dubna.
12. Ebert D., Reinhardt H., Nucl.Phys., A298, 1978, p. 60.
Ebert D., Reinhardt H., Pervushin V.N., Sov. J. Part. Nucl. 1979, 10, p. 444.
13. Izatt D., De Tar C., Stephenson M., Nucl.Phys., 1982, B199, p. 269.
14. Jones H.F., Scadron M.D., Nucl.Phys., 1979, B155, p. 409.
15. Shuryak E.V., Nucl.Phys., 1982, B198, p. 83;
16. Suzuki M., Nucl.Phys., 1981, B177, p. 413.
Krasemann H., Phys.Lett., 1980, 96B, p. 397.
17. De Grand T., Jaffe R.L., Johnson K., Kiskis J., Phys.Rev., 1975, D12, p. 2060.
Chodos A., Jaffe R.L., Johnson K., Thorn C.B., Weisskopf V.F., Phys.Rev., 1974, D9, p. 3471.

Received by Publishing Department
on November 28, 1983.

SUBJECT CATEGORIES
OF THE JINR PUBLICATIONS

Index	Subject
1.	High energy experimental physics
2.	High energy theoretical physics
3.	Low energy experimental physics
4.	Low energy theoretical physics
5.	Mathematics
6.	Nuclear spectroscopy and radiochemistry
7.	Heavy ion physics
8.	Cryogenics
9.	Accelerators
10.	Automatization of data processing
11.	Computing mathematics and technique
12.	Chemistry
13.	Experimental techniques and methods
14.	Solid state physics. Liquids
15.	Experimental physics of nuclear reactions at low energies
16.	Health physics. Shieldings
17.	Theory of condenced matter
18.	Applied researches
19.	Biophysics

Эберт Д.

E2-83-795

Кварковые и мезонные массы в модели составных мезонов с нарушенной SU(4) -симметрией

Получен эффективный лагранжиан составных мезонов для 15-плета 0^- -мезонов /допускающий $1/N$ разложение/ исходя из нелинейного кваркового лагранжиана с глобальной цветовой SU(N)- и нарушенной ароматной SU(4)-симметрией. В рамках модели получена оценка голых и полных кварковых масс, даны предсказания величин масс, а также констант связи и распадов составных мезонов.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1983

Ebert D.

E2-83-795

Quark and Meson Masses in a Composite-Meson Model with Broken SU(4)-Symmetry

Starting from a nonlinear quark Lagrangian with global colour SU(N)- and broken flavour SU(4)-symmetry, we derive an effective composite-meson Lagrangian for the 15-plet of 0^--mesons allowing for $1/N$ expansion. We estimate the bare and total quark masses of the model and give predictions for the masses, coupling and decay constants of composite mesons.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1983