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A MODEL FOR BREAKDOWN OF CHIRAL SYMMETRY IN QCD

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It is a common belief that QCD is the theory of strong interactions. However, only the perturbative properties of QCD are firmly established at present : being asymptotically free, the QCD at small distances has as its particles the massive colored quarks and massless colored gluons. Origin of the current quark masses is mysterious. They are remnants of a spontaneous breakdown of the electroweak symmetry [1]. In this letter we study the QCD of light flavors in the approximation $m_u = m_d = 0$. The resulting symmetry of QCD is therefore global $SU(2)_L \times SU(2)_R$ (besides local $SU(3)_{color}$).

Since in QCD the effective coupling constant grows with distance, we may suspect that the short-distance perturbation theory will break at some critical value of α_s . This implies the breakdown of the concept of colored quarks with current masses and colored massless gluons as the corresponding particles.

Physically, one can imagine that the perturbation theory can be stabilized by forming condensates of perturbative particles due to the strong attractive QCD forces. Hopefully, a new stable perturbation theory appears, with its own particles, for which the original vacuum, filled by condensates, acts as a dressed Fock's no particle state. Unfortunately, a systematic analysis of such a situation is not available.

The natural possibility to start with is to keep the particle interpretation of the QCD fields. Then, only the masses of quarks and gluons can be dynamically generated. Perhaps it is not an accident that there is the strongest attraction namely in $\bar{\Psi}_i \Psi^i$ and $\vec{A}_\alpha \vec{A}^\alpha$ channels [2].

In this note we simply <u>assume</u> that "the possibility that massless Yang-Mills fields generate massive vector particles"[3,4,5] is indeed realized. Technically, this means that we assume that the gluon proper self-energy function $\Pi(p^2)$ in pure gluodynamics develops a single pole at $p^2 = 0$. Then the residue at this pole is equal to $-M^2$, where M is the constituent gluon mass. Assuming this to happen we <u>solve</u> the Schwinger-Dyson equation for the quark proper self-energy function $\Sigma(p^2)$. We demonstrate that the nontrivial $\Sigma(p^2) \neq 0$

that dynamically breaks the global $SU(2)_L \times SU(2)_R$ chiral symmetry of QCD down to SU(2) of isospin exists only for a large value of the effective coupling constant α_s. Necessary consequence of $\Sigma \neq 0$ is the appearance of the Goldstone pions directly coupled to the dynamically massive (constituent) quarks. We calculate the corresponding coupling constant $g_{\pi qq}$ as well as the pion decay constant f_π in a reasonable agreement with their measured values.

Needless to say that the emerging picture of constituent quarks, constituent gluons, and massless pions is approximate. The constituent objects are in fact only the "would be particles" since there is no mass shell for them if we insist on the confinement dogma, as we do. The picture is, nevertheless, successful for it provides an understanding of the nonrelativistic quark and gluon model as discussed by Manohar and Georgi [6] and by Cornwall and Soni [7].

The Schwinger-Dyson equation for $\Sigma(p^2)$ is [8]:

$$\Sigma(p^2) + ig^2\, 3\left(\frac{4}{3}\right)\int \frac{d^4k}{(2\pi)^4}\, \frac{\Sigma(k^2)}{(k^2-\Sigma^2(k^2))[(p-k)^2-M^2]} = 0. \qquad (1)$$

The factor 3 comes from using the Landau gauge and the factor (+4/3) is due to color. g^2 is assumed to be the effective coupling constant renormalized at some μ^2 [9]. Performing in (1) the angular integration after Wick's rotation we obtain [10]

$$\Sigma(-p^2) = \frac{g^2}{2\pi^2}\int_0^\infty \frac{k^2\,dk^2}{p^2+k^2+M^2+\sqrt{(p^2+k^2+M^2)^2-4p^2k^2}}\, \frac{\Sigma(-k^2)}{k^2+\Sigma^2(-k^2)}. \qquad (2)$$

We do not think that linearization of eq.(1) or (2) can shed some light on the nature of the dynamical breakdown of the chiral symmetry in QCD. We suspect a nonanalytic dependence of Σ upon the coupling constant according to the renormalization group arguments [11] (see eq.(5) below). At the expense of being only qualitative we replace the true kernel

$$f(p^2, k^2) = f(k^2, p^2) = 1/\left(p^2+k^2+M^2+\sqrt{(p^2+k^2+M^2)^2-4p^2k^2}\right)$$

by an approximate one,

$$F(p^2, k^2) = F(k^2, p^2) = M^2/2(p^2+M^2)(k^2+M^2). \qquad (3)$$

It has the properties $f(0, k^2) = F(0, k^2)$ and for $k^2 \to \infty$ $f(p^2,k^2) \to 1/2k^2$, while $F(p^2, k^2) \to [M^2/(p^2+M^2)]\cdot(1/2k^2)$. This is good. The approximation is not good for p^2, k^2 simultaneously $\to \infty$. From (3) it also becomes clear (from a technical point of view) why we need the constituent gluon mass to differ from zero.

Equation (2) with the approximate kernel (3) is immediately integrated [12]:

$$\Sigma(p^2) = \frac{cM^3}{M^2-p^2}, \qquad (4)$$

where the dimensionless constant c is determined from a nonlinear condition

$$\frac{g^2}{4\pi^2}\int_0^\infty \frac{\kappa\, d\kappa}{\kappa(\kappa+1)^2+c^2} = 1. \qquad (5)$$

It is worthwhile to compare our solution for Σ (eq.(4)) with the educated guess of refs.[13] of the same quantity and with a more intuitive quantity $\langle\bar\Psi\Psi\rangle_0$:

$$m \simeq \Sigma(p^2) \simeq g^2\langle\bar\Psi\Psi\rangle_0/p^2.$$

From (5) it is quite clear that the nontrivial solution starts to exist, i.e. $c^2 \neq 0$, only for $g^2 > 4\pi^2$. Thus, we have explicitly realized in our model that the chiral symmetry breakdown is characterized by a critical effective coupling constant

$$\alpha_\chi = g^2(\Lambda_\chi)/4\pi = \pi .$$

Consequently, Λ_χ is the corresponding scale of the chiral symmetry breakdown. Use of the perturbative formula for α_s in the strong coupling region, which is obviously a tremendous extrapolation, reveals that $\Lambda_\chi > \Lambda_{QCD}$ in agreement with what one intuitively expects [6].

Where are the Goldstone pions? The analysis which follows is standard [13,14]. The isovector axial-vector Ward-Takahashi identity, where $S^{-1}(p) = \not{p} - \Sigma(p^2)$ and $m = \Sigma(m^2)$,

$$q^\alpha \vec{\Gamma}_{\alpha5}(p+q, p) = S^{-1}(p+q)\tfrac{1}{2}\vec{\tau}\gamma_5 + \tfrac{1}{2}\vec{\tau}\gamma_5 S^{-1}(p)$$

associated with the $SU(2)_L \times SU(2)_R$ invariance of QCD Lagrangian implies that the proper vertex function $\vec{\Gamma}_5^\alpha$ develops a massless pole :

$$\vec{\Gamma}_5^\alpha(p+q,p) \underset{q\to 0}{=} \tfrac{1}{2}\vec{\tau}\gamma^\alpha\gamma^5 - \tfrac{1}{2}\vec{\tau}\gamma^5\frac{q^\alpha}{q^2}\left[\Sigma(p+q)+\Sigma(p)\right] \qquad (6)$$
$$\approx \tfrac{1}{2}\vec{\tau}\gamma^\alpha\gamma^5 - \tfrac{1}{2}\vec{\tau}\gamma^5\, 2m\,\frac{q^\alpha}{q^2} .$$

The structure of the pole term of $\vec{\Gamma}_5^\alpha$ is visualized in Fig.1.

The loop integral $J^\alpha(q)$ of Fig.1 is easily calculated :

$$J^\alpha(q) = \int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\, g_{\pi qq}(\tfrac{1}{2}\tau^i)\gamma^5 S(k)(\tfrac{1}{2}\tau^j)\gamma^\alpha\gamma^5 S(k-q) \qquad (7)$$
$$\simeq \delta_{ij}(-iq^\alpha)\, g_{\pi qq}\,\frac{3}{2}\, I(0)m ,$$

where

$$I(0) = I = \frac{1}{2\pi^2}\int_0^1 x\ln\frac{(M^2-m^2)x+m^2}{m^2}\,dx = \qquad (8)$$
$$\frac{1}{4\pi^2}\left[\ln\frac{M^2}{m^2} - \left(\frac{M^2}{m^2}-1\right)^2\ln\frac{M^2/m^2}{M^2/m^2-1} + \frac{M^2}{m^2} - \frac{3}{2}\right] .$$

The trace in (7) is taken also over the color. We should not be surprised that the integral (7) is finite. This is because the dynamically generated self-energy (4) acts as an ultraviolet as well as infrared regulator.

Fig.1 implies the following identifications :

$$g_{\pi qq} = \frac{2}{\sqrt{3}}\frac{1}{\sqrt{I}} , \qquad f_\pi = \sqrt{3}\, m\sqrt{I} . \qquad (9)$$

Combination of $g_{\pi qq}$ and f_π yields the Goldberger-Treiman relation at the quark level [15] :

$$g_{\pi qq} = \frac{2m}{f_\pi} .$$

Furthermore, in the nonrelativistic quark model we have [15,16] :

$$\frac{g^2_{\pi NN}}{4\pi} = \left(\frac{5}{3}\right)^2 (m_N/m)^2\,\frac{1}{4}\,\frac{g^2_{\pi qq}}{4\pi} . \qquad (10)$$

The factor 1/4 comes from our normalization of the quark-pion interaction and it is necessary if we want to compare (10) with its experimental value $g^2_{\pi NN}/4\pi \simeq 15$ ($\mathcal{L}_{int} = i g_{\pi NN}\bar{\Psi}_N\vec{\tau}\gamma_5\Psi_N\cdot\vec{\pi}$).

If we put into the formulas (9) and (10) typical numbers m = 330 MeV and M = 2m [5,7] , we get f_π = 103 MeV (to be compared with 93 MeV in our normalization) and $g^2_{\pi NN}/4\pi$ = 18 in a reasonable agreement with the experimental values. We also calculate the pion-quark coupling constant $\frac{1}{4}(g^2_{\pi qq}/4\pi)$ = 0.81 and find it to agree with its independent determination in [16] . Since our knowledge of M is very scarce at present, this numerical analysis can rather be understood as an indirect determination of the constituent gluon mass.

The dynamically generated constituent quark mass m implies the quark Dirac magnetic moment $\vec{\mu} = Q(\frac{e}{2m})\vec{\sigma}$. The p^2 dependence of Σ in (4), using the "minimal" prescription $i\partial \to i\partial - eQA$, yields the anomalous quark magnetic moment $\vec{\mu}' = (\frac{m}{M})^2\, Q\,(\frac{e}{2m})\vec{\sigma}$. These magnetic moments build up the magnetic moments of baryons. These and other consequences of the existence of the chiral phase in QCD are discussed at length by Manohar and Georgi [6] and by Cornwall and Soni [7].

We would like to emphasize that the proper self-energy functions $\Sigma(p^2)$ and $\Pi(p^2)$ discussed in this letter obviously vanish at large momenta and do not require any renormalization [17]. Consequently, their appearance is not in conflict with the local $SU(3)_{color}$ gauge invariance of QCD at small distances.

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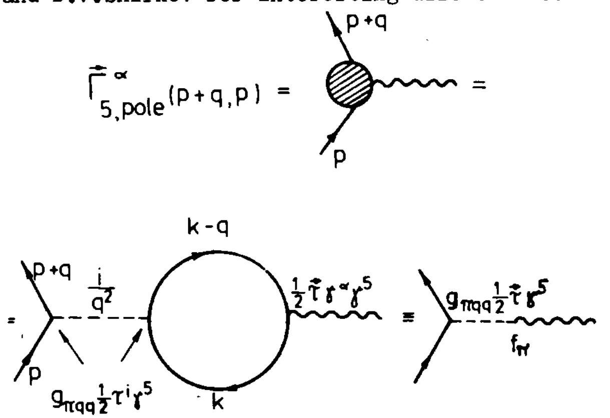

Fig.1. The effective coupling of the Goldstone pions with fermions and with the external axial-vector current.

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Хошек И. E2-83-765
Модель нарушения киральной симметрии в КХД

Показано, что $SU(2)_L \times SU(2)_R$ инвариантная КХД переходит при большом значении эффективной константы связи в киральную фазу составных кварков, составных глюонов и безмассивных пионов. Эта картина лежит в основе нерелятивистской кварковой и глюонной моделей. Вычисляются величины f_π и $g^2_{\pi NN}/4\pi$ в согласии с их экспериментальными значениями.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1983

Hošek J.
A Model for Breakdown of Chiral Symmetry in QCD

It is demonstrated that the $SU(2)_L \times SU(2)_R$ invariant QCD with a large effective coupling constant turns into a chiral phase that provides a basis for the nonrelativistic quark and gluon model. The true particles of this phase are the colored constituent quarks, colored constituent gluons, and the massless pions. We calculate the quantities $g^2_{\pi NN}/4\pi$ and f_π in a reasonable agreement with their measured values.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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