объединенный
институт
ядерных
исследований
дубна

E2-83-758

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PHASE TRANSITIONS IN LATTICE ABELIAN HIGGS MODELS

Submitted to "Ядерная физика"

1983

1. INTRODUCTION

The formulation of gauge theories on a space-time lattice [1] supplies us with the rare opportunity to study such theories without reference to any expansion in the powers of the coupling constant. The replacement of the continuous space-time by a lattice regularizes the ultraviolet divergencies in a non-perturbative way and in the same time opens the door for some very powerful methods for calculating both gauge invariant and noninvariant quantities. At present, the theories which have been studied in detail are the pure gauge models $Z(N)$, $SU(N)$, etc. On the other hand, any realistic theory should include dynamical matter-fields (such as quarks, Higgs-bosons, etc.) as well. The presence in the theory of dynamical matter-fields could bring about significant changes in the behaviour of the quantities relevant to the gauge theory under study. An illustration of the above is the Higgs mechanism thanks to which the gauge boson acquires a mass thus rendering the forces short-ranged. At the same time the introduction of matter-fields, together with the new parameters (masses and coupling constants) which go with them, leads to a theory with a nontrivial phase structure.

According to the present understanding of gauge-scalar systems there exist at least three main phases: the confining phase, the Higgs-phase and the Coulomb phase (see for instance ref. [2]). On the phase diagram the different phases are usually separated by lines of phase-transitions and it is most intriguing to investigate the possibility that such lines terminate (have end-points). In particular, related to the conjecture of an end-point on the time which separates the Higgs phase from the confinement phase is the so-called Complementarity principle [3,4].

Quite a few papers have appeared on the interaction of gauge and matter-fields (see [5-7] and the references therein), but in the majority of these papers the Higgs fields are radially frosen:

$$|\Phi| = \text{const.} \tag{1.1}$$

The assumption (1.1), which significantly simplifies the model, is supported by the popular belief that the continuum limit, if it exists, should be taken as a critical point. At such a point the correlation length approaches infinity and, therefore, the actual size of the scalar field is irrelevant for the regularized lattice action. Unfortunately, the problem of the continuum limit for gauge-scalar systems, i.e., of whether such theories do exist, is still beyond the scope of our knowledge. Yet, provided that a continuum theory of this type exists, its properties ought to be better reflected by a lattice model with a radially varied scalar field than by its radially frozen counterpart. At the same time, as it was shown on the example of the Z(2)-gauge theory in ref. /5,6/, the radial variations of the Higgs field could significantly alter the phase picture of the theory.

This paper is devoted to the study of the phase-transitions in Abelian gauge theories with symmetry-groups Z(N), where N has been given a number of values ranging between 2 and 300, coupled to radially varied. Higgs fields in the fundamental representation of the gauge group. For sufficiently large values of N the discrete groups Z(N) are considered as a good approximation to the continuous group U(1).

In what follows we shall investigate, both by the Monte-Carlo simulations and through a mean field analysis, the phase transitions when varying the "mass" m^2 of the Higgs field ($m^2 < 0$) for zero and for infinite value of the gauge coupling $1/\beta$. For sufficiently small values of the scalar self-coupling λ we observed a first order phase transition at $\beta = 0$ for all values of N. When λ is allowed to increase it reaches a value at which system undergoes a phase transition of second order and, eventually, for even bigger values of λ, the phase transition disappears completely. It is possible, however, that in this case the system undergoes the second (or even higher) order phase transition. The demonstrated dependence of the phase structure of the theory on the scalar selfcoupling confirms the significance of the Higgs radial degree of freedom. First order phase transitions were also found in the weak gauge-coupling limit ($\beta \to \infty$).

The model we consider is described in the next paragraph; in the third paragraph we give the details of our Monte-Carlo calculations and, finally, we summarize the numerical results and compare them with some mean-field estimates in the fourth paragraph of this paper.

2. THE CHOICE OF THE MODEL

The action for the system of Z(N)-gauge fields and Higgs scalars in the fundamental representation of the gauge group has the form:

$$S = \beta \cdot \sum_{\square} S_{\square} + \sum_{L} S_L , \qquad (2.1)$$

where $S_{\square} = 1 - U_{ij} U_{jk} U_{k\ell} U_{\ell j}$ with the gauge variable $U_{ij} \equiv U_L$ defined on the link $L = (i,j)$ which originates from the site labelled by i and ends at site j. U_L is an element of the group Z(N):

$$U_L \subseteq Z_N , \quad \text{i.e.} \quad U_L = e^{i\frac{2\pi\sigma_L}{N}} , \quad \sigma_L = 0,1,\dots,N-1 . \qquad (2.2)$$

The first term in (2.1) is a sum over all plaquettes. The second term is a sum over all links and is of the form:

$$S_L = 2\cdot\Phi_i^*\Phi_i - [\,\Phi_i^* U_{i,i+\mu}\, \Phi_{i+\mu} + \text{h.c.}\,] + V(\Phi_i), \qquad (2.3)$$

where

$$V(\Phi_i) = \frac{1}{4}\cdot[\,\frac{m^2}{2}\cdot\Phi_i^*\Phi_i + \lambda\cdot(\Phi_i^*\Phi_i)^2\,]. \qquad (2.4)$$

The Higgs field Φ_i is defined at each site i and belongs to the fundamental representation of Z(N):

$$R_i = \sqrt{\Phi_i^*\Phi_i},$$
$$\Phi_i = R_i\, e^{i\frac{2\pi\phi_i}{N}} \quad ; \quad \phi_i = 0,1,\dots,N-1. \qquad (2.5)$$

Throughout this paper we shall assume that $m^2 < 0$. The action (2.1)-(2.4) will correspond to a U(1) gauge-scalar system as $N \to \infty$. The "naive" continuum limit of such a lattice action agrees with the well-known expression in the continuous space. Indeed, if one substitutes

$$\Phi_i \to a\cdot\Phi(x);\quad m^2 \to a^2\cdot m^2;\quad U_{i;i+\mu} \to e^{iag\cdot A_\mu(x)}$$

and takes the limit $a \to 0$; m^2, g - fixed (a is the lattice spacing) the action (2.1)-(2.4) will become:

$$S = S_G + \int d^4x\cdot\frac{1}{2}|(\partial_\mu - igA_\mu(x))\,\Phi(x)|^2 + \int d^4x\,[\,\frac{m^2}{2}\cdot\Phi^*\Phi + \lambda\cdot(\Phi^*\Phi)^2], \qquad (2.6)$$

where S_G is the action for the gauge field.

In the limit $\lambda \to \infty$ the Higgs field is radially "frozen"

$$R_i = \begin{cases} |m^2|/4\lambda, & m^2 < 0 \\ 0, & m^2 \geq 0. \end{cases}$$

At the opposite extreme ($\lambda \ll 1$) the radial fluctuations of the Higgs field could be very significant.

As for the limit $\beta \to \infty$, it leads to freezing of the gauge field: $U_{ij} = 1$.

3. THE CALCULATION METHOD

The model (2.1)-(2.4) has been studied numerically by means of the Monte-Carlo simulations. Among the Monte-Carlo algorithms two are especially widespread today: the heat bath[8] and the Metropolis algorithm[9]. While the first offers the best performance in terms of convergence per iteration, its applicability is limited to integration over a compact measure. The heat bath method could be used when the radial mode of the Higgs field is frozen: $R \equiv |\Phi_i| = \text{const}$. This was done in refs. [5,8]. If we drop the constraint (1.1), the heat bath method could not be employed without some restrictions on the integration domain of the variables. In our calculations we have used the Metropolis algorithm. The updating of the variables on each link and at each site is done after a certain number of tries which improves the convergence. The gauge field is renewed by simply generating new elements of the group while in the case of the Higgs field the updating undergoes two stages: first we try to renew the radial part of the field at a given site and then we turn to the angular part (the phase-factor). The new values of both the scalar and the gauge degrees of freedom are accepted or rejected in accordance with the prescription of the Metropolis algorithm.

In order to study the behaviour of the model in the vicinity of the phase transition points we have employed two different techniques:

I. Simulations from different types of initial configurations (starts).

a) A totally ordered start. This means that initially the values of the gauge and the Higgs fields are chosen uniformly on the lattice: $R_i^{(0)} = 0$; $\phi_i^{(0)} = 0$; $\sigma_{i,\mu}^{(0)} = 0$.

b) A totally disordered start. This means that all the variables on the lattice are given randomly chosen values and this is taken as the intial configuration of the simulation.

c) A variety of partially ordered (and equivalently partially disordered) starts. Here is an example of such a start: the gauge field is totally ordered while the Higgs field is angulary ordered and radially randomized.

The simulations from different initial configurations can reveal the type of the phase transition and, when it is a case of a first order phase transition, c-type starts can help us to determine which is the stable phase.

II. Thermal cycles. In a thermal cycle one of the parameters of the model, e.g., m^2 or λ is gradually varied up to a given value and back. At each intermediate step a given number of iterations is performed starting from the last configuration reached at the preceding step. If the thermal cycle carries the system across a point of phase transition this produces a typical hysteresis loop on the thermal cycle curve for the order parameter.

All our numerical experiments have been performed on a 4^4-lattice because further increase of the lattice-size does not significantly change the results. The order parameters we have used are the mean action per plaquette $\langle S_\Box \rangle$ and the mean squared radial part of the Higgs field $\langle \Phi^*\Phi \rangle$.

4. RESULTS AND DISCUSSIONS (INTERPRETATION)

The results we present in this paper were obtained for two characteristic values of β: $\beta = 0$ and $\beta = \infty$

a) $\beta = 0$.

For infinite value of the gauge coupling ($\beta = 0$) and for comparatively small values of the scalar self-coupling λ we have observed a first order phase transition when m^2 is varied and this holds for all Abelian groups Z(N) we have studied: N= = 2,3,...10,20,50,100,200,300. The dependence of the phase transition point m_c^2 on the order of the group N is shown on Fig.1a,b for two different values of λ: $\lambda = 0.1$ and $\lambda = 0.25$.

The bars on the graphics span over the intervals which contain the phase transition points $m_c^2(N)$. We can see that m_c^2 ceases to depend on N for N bigger than 5 or 6.

In order to determine the type of the phase transition we have performed both thermal cycles in m^2 and observations on the evolution of the system beginning from different initial configurations. As an example of how this works on Fig.2 we have shown a thermal cycle in m^2 for the order parameter $\langle \Phi^*\Phi \rangle$ and on Fig.3, the evolution of the same order parameter with the number of iterations for different values of m^2 and for different starts. In both cases the symmetry group is Z(200).

The dots and the crosses in Fig.3 are for ordered and disordered starts respectively. It is evident that as m^2 changes from m_1^2 to m_3^2 the order parameter $\langle \Phi^*\Phi \rangle$ undergoes a sudden change, a jump to its new value which points to a first order phase transition. We want to emphasize the fact that the type of the phase transition does not change with the order of the group. This picture is consistent with the lowest-order mean field calculations. At $\beta = 0$ and in the parametrization (2.2) and (2.5) for the field variables the action (2.1) becomes:

$$S = \sum_{i,\mu} \{(1+\frac{m^2}{8})\cdot R_i^2 + \frac{\lambda}{4}\cdot R_i^4 - R_i\cdot R_{i+\mu}\cdot \cos\frac{2\pi}{N}(\phi_i - \phi_{i+\mu} - \sigma_{i,i+\mu})\}. \quad (4.1)$$

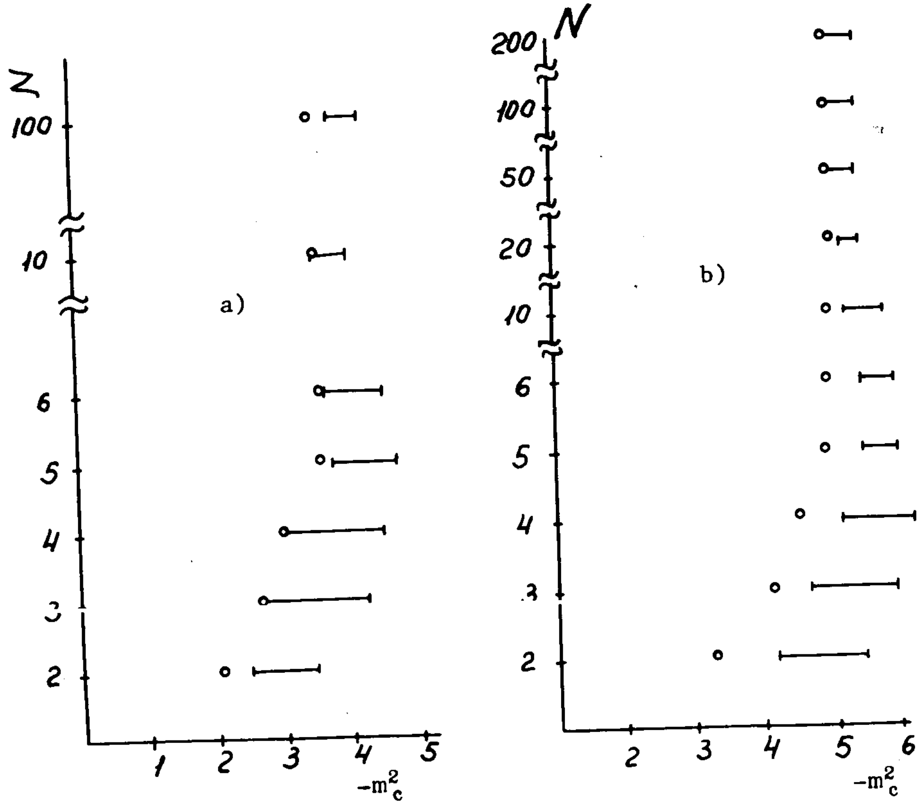


Fig.1. The position of the phase transition point m_c^2 for $\beta = 0$ as a function of the order N of the symmetry group and for two values of the scalar self-coupling constant: $\lambda = 0.1$(a) and $\lambda = 0.25$ (b). The hollow circles are the lowest order mean field predictions.

The corresponding partition function is:

$$Z = \prod_i [\int^\infty dR_i \frac{1}{N} \sum_{\phi_i=0}^{N-1}] \prod_L [\frac{1}{N}\sum_{\sigma_L=0}^{N-1}]\cdot e^{-S}. \quad (4.2)$$

After the summation over the ϕ_i, σ_L in (4.2) has been performed, we can rewrite Z in the form:

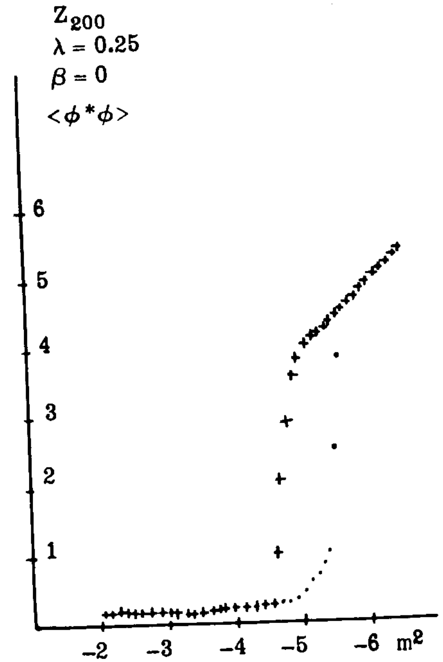


Fig.2. Thermal cycle in m^2 at $\beta = 0$ and $\lambda = 0.25$ for the group Z (200).

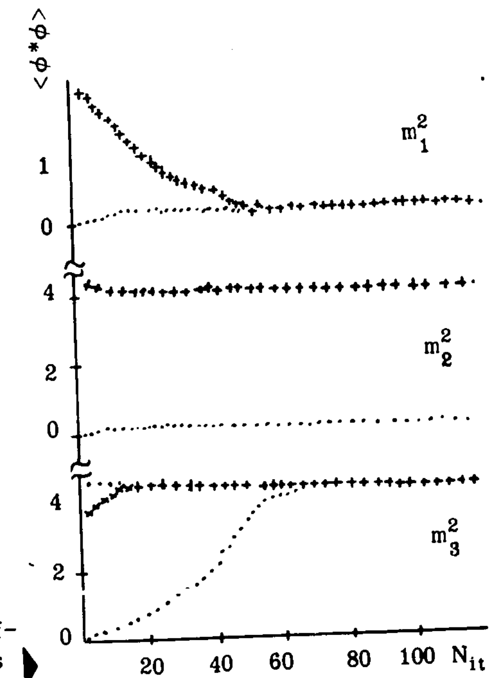


Fig.3. Equilibration from different initial configurations and for different m^2; β, λ and the group are as in Fig.2.

$$Z = \int_0^\infty \dots \int \prod_i dR_i \cdot e^{-S_{eff}(\{R_i\})}, \quad (4.3)$$

where

$$S_{eff} = \sum_{i,\mu} \{(1+\frac{m^2}{8})R_i^2 + \frac{\lambda}{4}R_i^4 - W(R_i R_{i+\mu})\} \quad \text{and} \quad (4.4)$$

$$W(x) = \ln[\frac{1}{N}\cdot\sum_{j=0}^{N-1} e^{x\cdot\cos\frac{2\pi j}{N}}]. \quad (4.5)$$

Next, we follow the recipe given in ref./10,11/ and obtain the effective potential $V_{eff}(R)$. We underline that in the above S_{eff} was used instead of S. This means that the effects of the angular part of Φ have been accounted for in full. This way we get the following expression for the V_{eff} in the lowest mean field approximation:

$$V_{eff}(R) = (1 + \frac{m^2}{8}) \cdot R^2 + \frac{\lambda}{4} \cdot R^4 - W(R^2). \qquad (4.6)$$

The characteristic behaviour of the effective potential is illustrated on Figs.4a-c.

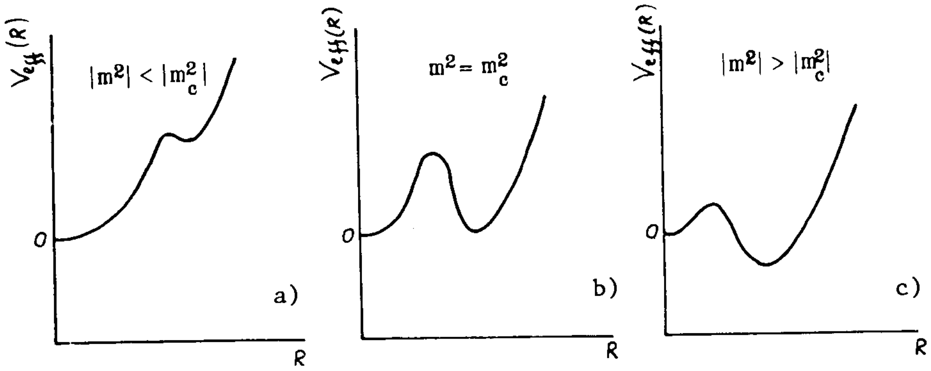


Fig.4. The effective potential (lowest order approximation) for $\beta = 0$ and different m^2.

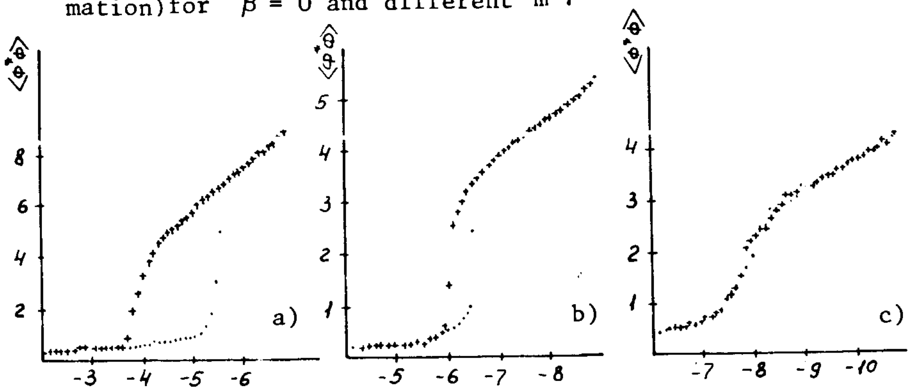


Fig.5. Thermal cycles in m^2 at $\beta = 0$ and for different λ. The group is Z(5).

We see that V_{eff} has two minima. For $m^2 = m_c^2$ (Fig.4b) the two minima are on the same level, the order parameter is discontinuous and therefore the phase transition is of the first order.

In addition to providing a qualitatively correct description, this crude mean field analysis is in good agreement with the Monte-Carlo results as it can be seen from Fig.1 (the hollow circles).

Finally, the behaviour of the system at $\beta = 0$ has proved to be very sensitive to changes of the scalar self-coupling λ. More specifically, if we consider thermal cycles in m^2 at $\beta = 0$

for a sequence of increasing values of λ we shall observe that the characteristic hysteresis loop gradually shrinks to vanish completely for some $\lambda = \lambda_0$. This analysis was carried out for some groups and the results, obtained for Z(5) are presented on Fig.5a-c.

In the case of the symmetry-group Z(5), the value of λ at which the first-order phase transition disappears is close to $\lambda_0 \approx 0.6$. Now, when we go on increasing above λ_0, we observe a characteristic change of behaviour in the order parameter (on Fig.5c this happens near $m^2 = -8$). This could indicate a higher order phase transition, possibly of second order. The analysis of the effective potential beyond λ_0 supports this point of view. On Fig.6 we show the effective potential for different values of m^2 at $\lambda = 0.9$. The gauge group is Z(5).

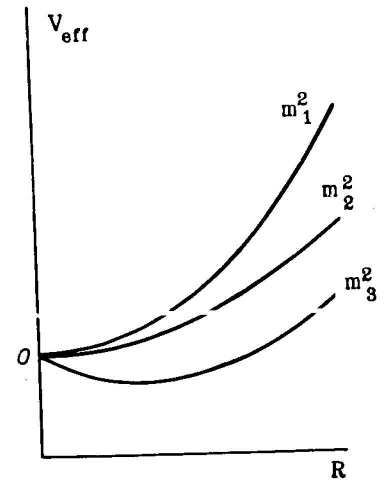


Fig.6. The effective potential for different values of m^2. The gauge group is Z(5) and $\lambda = 0.9$.

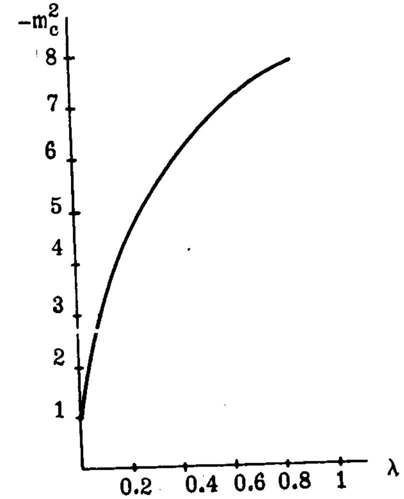


Fig.7. The position of the first-order phase transition point as obtained from eq. (4.6).

We see that, for $m^2 = m_1^2$ V_{eff} has only one minimum (at $R = 0$) which tends to get flatter $(m^2 = m_2^2)$ and, finally, it slides away the origin $(m^2 = m_3^2)$. On Fig.7 we show the λ dependence of the first order phase transition point obtained for Z(5) by means of eq. (4.6). This way, it has been found that the point where the first order phase transition terminates is near the value $\lambda \approx 0.85$.

b) $\beta = \infty$.

For vanishing values of the gauge coupling constant $(\beta = \infty)$ we observe essentially the same dependence as in $\beta = 0$ case of

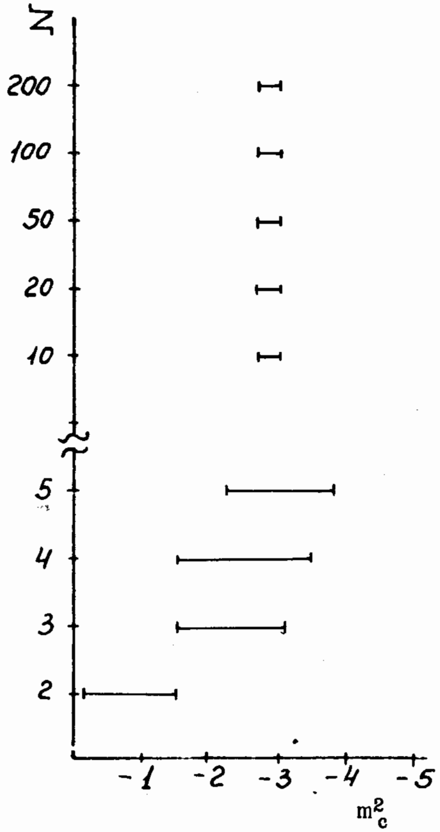


Fig.8. The dependence of the phase transition point on the order of the group for $\beta = \infty$ and at $\lambda = 0.25$.

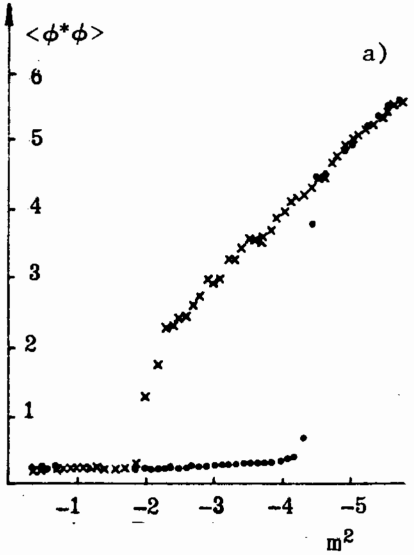
the phase transition point m_c^2 as we move from one gauge group Z(N) to another. For $\lambda = 0.25$ this dependence is shown on Fig.8. We see that from N = 10 onward the position of the phase transition does not change with N. On Fig.9 we show thermal cycles in m^2 for groups: Z(5), Z(10) Z(300).

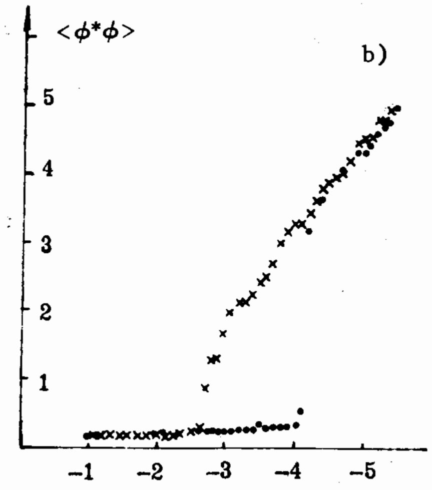
The histograms we obtained from binning the radial part of the Higgs field give a signal for the existence of two competing minima of the effective potential. An idea of how this works is given on Fig.10, where we have shown the histograms corresponding to simulations from two different initial configurations. This was done for the group Z(80) and at $m^2 = -3$, $\lambda = 0.25$. In analogy with the reasoning in the $\beta = 0$ case we conclude that here again the phase transition is of first order (compare with Fig.4).

At present it is an open question what would be the relation of all this to the continuum theory, a question the answer to which is being persued.

5. CONCLUSION

We have carried out an exhaustive analysis of the phase transitions in a number of Abelian gauge-lattice theories for two extreme values of the gauge coupling constant. In drawing our conclusions about the character of the phase transitions we have referred both to the numerical (Monte-Carlo) calculations and to a mean field analysis of the theory. We have convincingly shown that the radial variations of the Higgs field play a very important role in determining the behaviour of gauge-scalar system and we intend to study this matter further in order to get a fuller picture of the phase structure of such fields.





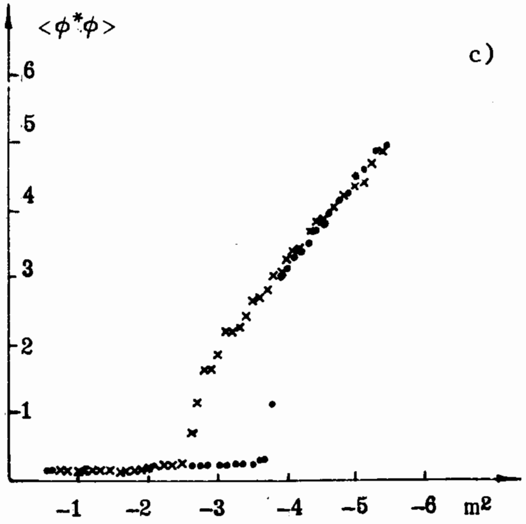


Fig.9. Thermal cycles in m^2 for $\beta = \infty$ and $\lambda = 0.25$. The gauge groups are: Z(5) - a, Z(10) - b, Z(300) - c.

Finally, we gratefully acknowledge the interest paid to our work by Drs. V.A.Matveev, M.G.Meshcheryakov, M.Müller-Preussker, D.V.Shirkov, A.N.Sissakian and A.M.Zadorozni.

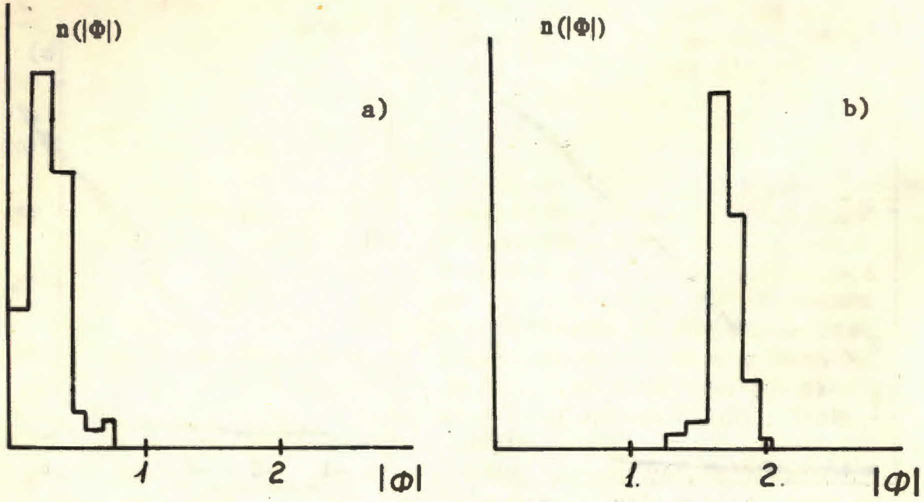


Fig.10. Histograms for the radial part of the Higgs field. The group is Z(80), $m^2 = -3$, $\beta = \infty$ and $\lambda = 0.25$. The initial configuration is ordered in (a) and disordered in (b).

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Received by Publishing Department
on November 3, 1983.

Гердт В.П., Илчев А.С., Митрюшкин В.К. E2-83-758
Фазовые переходы в абелевых
хиггсовских моделях на решетке

Исследованы фазовые переходы в абелевых калибровочных теориях на решетке с включением хиггсовских полей в фундаментальном представлении калибровочной группы. Рассмотрены два случая предельных значений константы взаимодействия калибровочных полей. Радиальная степень свободы калибровочных полей "разморожена", показано, что это приводит к существенным различиям с моделями, в которых радиальная мода хиггсовского поля фиксирована.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1983

Gerdt V.P., Ilchev A.S., Mitrjushkin V.K. E2-83-758
Phase Transitions in Lattice Abelian
Higgs Models

Abelian lattice gauge theories coupled to Higgs's fields in the fundamental representation of the gauge group are studied with reference to phase transitions at extreme values of the gauge coupling. The scalar fields are allowed to vary radially and this leads to a phase structure which is drastically different from the one in the models with radially "frozen" Higgs fields.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1983