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A.V.Efremov, O.V.Teryaev

THE TRANSVERSAL POLARIZATION IN QUANTUM CHROMODYNAMICS

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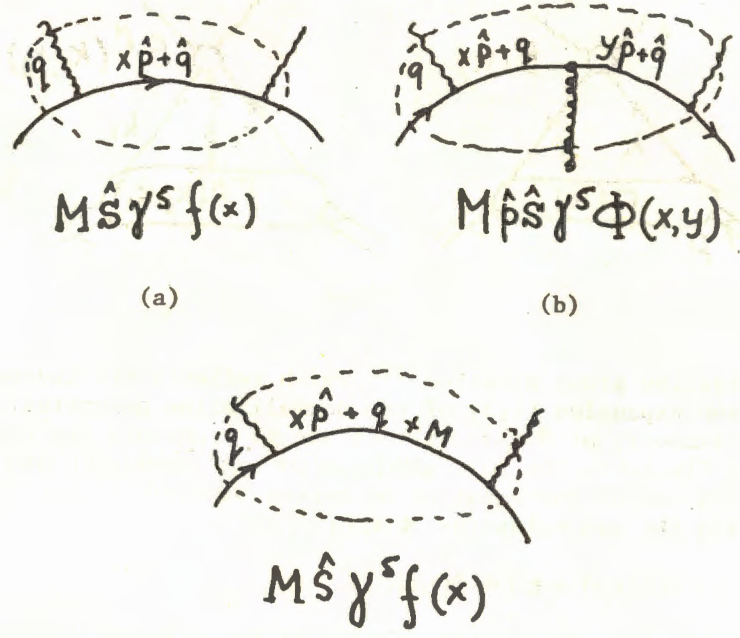
The description of hadron spin properties in hard processes in the framework of quantum chromodynamics (QCD) helps much for the experimental check of the latter as well as in understanding spin physics on the quark-gluon level. The processes are most simple for the experimental investigation in which polarization of single particle is fixed, the latter being transversal (if parity conserved) /1-5/. The study of such processes in QCD is therefore of a special interest. As has been shown earlier /6/, the study of the baryon density matrix in QCD disproves the result obtained within the parton model /7/ according to which the polarization is proportional to the (current) quark mass. On the contrary, it appears to be proportional to the polarized hadron mass, that is physically clear: the transversal polarization is proportional to some mass due to the kinematical reason (the massless particle is always longitudinally polarized). Later the underlying dynamics was established /8/ it appears that the hadron gluon fields in which a quark propagates, leads to a redefinition of the latter mass. Therefore, the effects that lead to power and logarithmic corrections only in the spin-averaged cross sections, appear to be essential in the single-asymmetry calculation. The twist 3 effects in deep inelastic scattering was considered also by Vainshtein and Shuryak /9/ and, recently, by Bukhvostov, Kuraev and Lipatov /10/ who obtained the complete evolution equation for the nonsinglet channel. As to the pioneering work of Ahmed and Ross /11/ the consideration of transversal polarization there contains mistakes.

In the present work we'll apply to the calculation of transversal polarization the factorization scheme /12/ based on the axial gauge and struck quark propagator expansion in powers of 4-momentum in vicinity of its longitudinal components. This scheme having demonstrated its convenience in the power correction calculation to processes with real hadrons /12b/ is also well accommodated to the description of polarization, and especially, of the transversal one. To avoid difficulties due to the presence of more than one hadron /13/, we'll limit ourselves to the consideration of deep inelastic scattering where we'll calculate the form factors G_1 and G_2.

Let us remind beforehand, how the described effects display themselves in the simple case of a scalar gluon (Fig.1); although the QCD formulas are more complicated, the main features of physical picture will fully preserve. In the case of massless quarks only the axial projection of a quark contri-



(a) (b) (c)

Fig.1

bution works

$$\langle p, S|\bar{\psi}_\alpha(0)\psi_\beta(z)|p, S\rangle = M\int_0^1 dx\, e^{ipzx} f(x)(\hat{S}\gamma^5)_{\beta\alpha}. \tag{1}$$

The choice of the hadron mass as a dimensional parameter is stipulated by the following sum rule which reflects the angular momentum conservation when the gluon contribution is neglected

$$\sum_f \langle p, S|\bar{\psi}_f(0)\gamma^\mu\gamma^5\psi_f(0)|p, S\rangle = MS^\mu. \tag{2}$$

The pseudotensor projection of the quark-gluon operator gives the contribution of the same twist (Fig.1b)

$$\langle p, S|\bar{\psi}_\alpha(0)G(y)\psi_\beta(z)|p, S\rangle = M\int_0^1 dx_1 dx_2 \exp[ipx_1 z + (x_2-x_1)y]\times \\ \times\Phi(x_1,x_2)(\hat{p}\hat{S}\gamma^5)_{\beta\alpha}, \tag{3}$$

$\Phi(x_1,x_2)$ being a symmetric function of its arguments (it is connected with T-invariance and will in detail be discussed later). The moments of the same structure functions enter into the re-

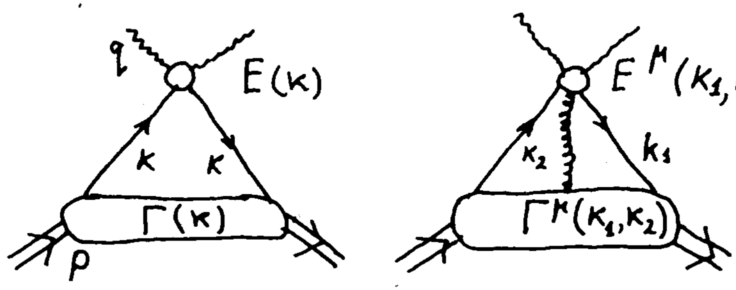


Fig.2

normalization group equation [10], which reflects the independence of Wilson expansion (1,3) of the normalization parameter.

The symmetry of $\Phi(x_1,x_2)$ allows one to transform the contribution of Fig.1b to the form $gM\Phi(x,y)/(xp+\hat{q})^2 = gM\Phi(x,y)/(yp+q)^2$. By making use of the equation of motion $\hat{\partial}\psi = g\hat{a}\psi$ it isn´t hard to obtain the sum rules for $\Phi(x,y)$

$$xf(x) = g\int dy\,\Phi(x,y) = g\int dy\,\Phi(y,x). \tag{4}$$

As a result the sum of contribution of Figs.1a and 1b gives a parton result of Fig.1c with a hadron mass in the numerator of the quark propagator.

Let us pass to the account of the factorization procedure [12] in QCD for the density matrix in scattering on a polarized target. We will limit ourselves to terms proportional to the covariant polarization of the target. Let us write in the form* (Fig.2):

$$W = \int d^4k\,\Gamma(k)\,E(k) + \int d^4k_1 d^4k_2\,\Gamma_{\mu}(k_1,k_2)\,E^{\mu}(k_1,k_2). \tag{5}$$

Here Γ´s are the hadron-parton amplitudes, the latter being quarks and gluons,

$$\Gamma_{\alpha\beta}(k) = \int \frac{dz}{(2\pi)^4}\exp(ikz)\langle p,S|\bar{\psi}_{\alpha}(0)\,\psi_{\beta}(z)|\,p,S\rangle, \tag{6a}$$

$$\Gamma^{\mu}_{\alpha\beta}(k_1,k_2) = \int\frac{dz_1dz_2}{(2\pi)^8}\exp[ik_2z_1 - i(k_1-k_2)z_2]\times \langle p,S|\bar{\psi}_{\alpha}(0)\,gA^{\mu}(z_2)\,\psi_{\beta}(z_1)|p,S\rangle, \tag{6b}$$

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*We omitted here the normalization parameter dependence (just as in the scalar case) and the colour one (it leads to a factor included in the distribution function).

E and E^{μ} are the subprocess coefficient functions, the first having quark legs and the second also the gluon one. In expression (5) the terms are omitted, which make no contribution to the leading term of twist 3 in axial gauge $n{\cdot}A = 0$. The light-like vector n is normalized by the condition $np = 1$. Note also that $p^2 = 0$, this means to neglect kinematical power corrections to polarization. The key moment of the method [12] is expansion of the 4-vector k in (5) in light-cone variables:

$$k = xp + \alpha n + k_T, \quad \text{where} \tag{7}$$

$$k_Tp = k_Tn = n^2 = p^2 = 0. \tag{8}$$

It follows that (9)

$x = kn$.

The factorization procedure is reduced now to the following formal substitution

$$d^4k \to d^4k\,dx\,\delta(x-kn). \tag{10}$$

Taking into account the terms of twists 3 only we have

$$W = \int dx\,[E_{\alpha\beta}(xp) + \partial E_{\alpha\beta}(xp)/\partial k_{\mu}\,(k-xp)^{\mu}]\,\Gamma_{\alpha\beta}(x) + \int dx_1dx_2E^{\mu}_{\alpha\beta}(x_1p_1,x_2p_2)\Gamma^{\mu}_{\alpha\beta}(x_1,x_2), \text{ where} \tag{11}$$

$$\Gamma_{\alpha\beta}(x) = \int\frac{d\lambda}{2\pi}\exp(i\lambda x)\langle p,S|\bar{\psi}_{\alpha}(0)\,\psi_{\beta}(\lambda n)|p,S\rangle. \tag{12a}$$

$$\Gamma^{\mu}_{\alpha\beta}(x_1,x_2) = \int\frac{d\lambda_1d\lambda_2}{(2\pi)^2}\exp[i\lambda_2(x_1-x_2) + i\lambda_1x_2]\times \langle p,S|\bar{\psi}_{\alpha}(0)\,gA^{\mu}(\lambda_2n)\,\psi_{\beta}(\lambda_1n)|p,S\rangle. \tag{12b}$$

The region, in which $\Gamma(x_1,x_2)$ is defined, was discussed in [12b]. The use of the Ward´s identities

$$\frac{\partial E(xp)}{\partial k^{\mu}} = E^{\mu}(xp,\,xp) \tag{13}$$

permits us to unite the second and the third terms in (11) into the single, gauge-invariant expression

$$\int dx_1dx_2E^{\mu}_{\alpha\beta}(x_1p,x_2p)\,\omega^{\mu'}_{\mu}\,\Gamma^{\mu'}_{\alpha\beta}(x_1x_2), \tag{14}$$

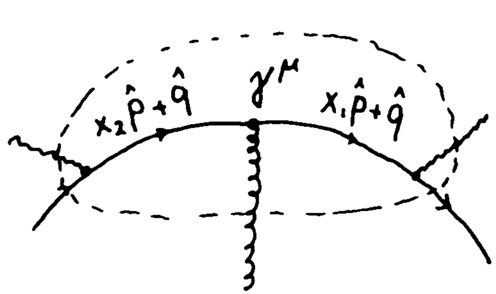

Fig.3

where $\omega_\mu^{\mu'}$ is a projector onto the transversal to p_μ direction

$$\omega_\mu^{\mu'} = \delta_\mu^{\mu'} - p_\mu n^{\mu'}, \tag{15}$$

the gauge condition $An = 0$ is taken into account; and the amplitude $\Gamma^\mu_{\alpha\beta}$ has the form

$$\Gamma^\mu_{\alpha\beta} = \int \frac{d\lambda_1 d\lambda_2}{(2\pi)^2} \exp[i\lambda_1 x_2 + i\lambda_2 (x_1 - x_2)] \times \langle p, S|\bar\psi_\alpha(0) D^\mu(\lambda n)\psi_\beta(\lambda_1 n)|p, S\rangle, \tag{16}$$

$$D^\mu(\lambda n) = i\overrightarrow{\partial}^\mu + gA^\mu(\lambda n) = -i\overleftarrow{\partial}^\mu + gA^\mu(\lambda n). \tag{17}$$

The next stage is the standard usage of the Fiertz identity with respect to indices α and β. If the quarks are massless, only vector and axial projections will give a nonzero contribution

$$W = \int dx\{[\![E(xp)\gamma^\rho]\!]\Gamma_\rho^V(x) + [\![E(xp)\gamma^5\gamma^\rho]\!]\Gamma^A_\rho(x)\} + \int dx_1 dx_2\{[\![E^\mu(x_1p, x_2p)\gamma^\rho]\!]\Gamma^V_{\rho\mu}(x_1, x_2) + [\![E^\mu(x_1p, x_2p)\gamma^5\gamma^\rho]\!]\Gamma^A_{\rho\mu}(x_1, x_2)\}. \tag{18}$$

$$[\![A]\!] \equiv \frac{1}{4}\,\mathrm{Sp}\,A, \tag{19}$$

$$\Gamma^V_\rho(x) = \int \frac{d\lambda}{(2\pi)} \exp(i\lambda x)\langle\bar\psi(0)\gamma^\rho\psi(\lambda)\rangle, \tag{20a}$$

$$\Gamma^A_\rho(x) = \int \frac{d\lambda}{2\pi} \exp(i\lambda x)\langle\bar\psi(0)\gamma^\rho\gamma^5\psi(\lambda)\rangle, \tag{20b}$$

$$\Gamma^V_{\rho\mu} = \int \frac{d\lambda_1 d\lambda_2}{(2\pi)} \exp[i\lambda_1(x_1 - x_2) + i\lambda_2 x_2]\langle\bar\psi(0)\gamma^\rho D^\mu(\lambda_1)\psi(\lambda_2)\rangle, \tag{20c}$$

$$\Gamma^A_{\rho\mu} = \int \frac{d\lambda_1 d\lambda_2}{(2\pi)^2} \exp[i\lambda_1(x_1 - x_2) + i\lambda_2 x_2]\langle\bar\psi(0)\gamma^\rho\gamma^5 D^\mu(\lambda_1)\psi(\lambda_2)\rangle.$$

We use the following notation

$$A(\lambda) \equiv A(\lambda n), \tag{21a}$$

$$\langle A\rangle \equiv \langle p, S|A|p, S\rangle. \tag{21b}$$

(Note, that usage of the Fiertz identity for $SU(3)_c$ leads to the redefinition of $D^\mu \equiv i\partial^\mu + gA^\mu_a t^a$ and to the appearance of colour-averaging factors 1/3 in $E(xp)$ and 1/4 in $E(x_1p_1; x_2p_2)$).

It is convenient to single out the invariant Lorentz structure with scalar coefficients in the coordinate representation

$$\langle\bar\psi(0)\gamma^\rho\psi(\lambda)\rangle = C^V(\lambda)\epsilon^{\rho Spn}, \tag{22a}$$

$$\langle\bar\psi(0)\gamma^\rho\gamma^5\psi(\lambda)\rangle = C^A(\lambda)S^\rho + C^A(\lambda)p^\rho(Sn), \tag{22b}$$

$$\langle\bar\psi(0)\gamma^\rho D^\mu(\lambda_1)\psi(\lambda_2)\rangle = B^V_{LT}(\lambda_1, \lambda_2)p^\rho\epsilon^{\mu Spn} + B^V_{TL}(\lambda_1, \lambda_2)p^\mu\epsilon^{\rho Spn}, \tag{22c}$$

$$\langle\bar\psi(0)\gamma^\rho\gamma^5 D^\mu(\lambda_1)\psi(\lambda_2)\rangle = B^A_{LT}p^\rho S^\mu_T + B^A_{TL}p^\mu S^\rho_T + B^A_{LL}p^\rho p^\mu(Sn), \tag{22d}$$

where

$$\epsilon^{\rho Spn} \equiv \epsilon^{\rho\alpha\beta\gamma}S_\alpha p_\beta n_\gamma, \tag{23a}$$

$$S^\rho_T \equiv S^\rho - p^\rho(Sn). \tag{23b}$$

The choice of this structures was made to simplify the determination of the latter by projecting the matrix element. Terms with (Sn) correspond to the longitudinal polarization, whereas with S_T, to the transversal one.

Note that all these coefficients have a dimension of mass, the latter being that of the order of polarized hadron as follows from the angular momentum conservation [6]. The three structures can be present in the expansion of the matrix element $\langle\bar\psi\gamma^\rho D^\mu\psi\rangle$ which obey the following equation

$$\epsilon^{\rho\mu Sp} = p^\mu\epsilon^{\rho Spn} - p^\rho\epsilon^{\mu Spn}. \tag{24}$$

It can be obtained starting with the obvious identity

$$\gamma^5\gamma^\rho\hat S\hat p\hat n\hat p\gamma = 2\gamma^5\gamma^\rho\hat S\hat p\gamma^\mu \tag{25}$$

by calculating traces of both sides and also from formula (51) in [12b]

$$\epsilon^{\mu\nu np}p^\rho - \epsilon^{\mu\rho np}p^\nu + \epsilon^{\nu\rho np}p^\mu = \epsilon^{\mu\nu\rho p}. \tag{26}$$

Let us choose the structures in r.h.s. of (24) as independent, because they are eigenvectors (in the index μ) of projector (15) with eigenvalues 0 and 1, respectively. Analogously, when the projector is acting on the matrix element (22d), only the 1st term in r.h.s. survives (and doesn't change). Let us come to the above-mentioned determination of coefficients in (22). We'll put $S^2 = -1$ (the polarization or asymmetry is the coefficient of S independent of its normalization [16]). We have

$$C^V(\lambda) = -\langle\bar\psi(0)\gamma^\rho\psi(\lambda)\rangle\epsilon^{\rho Spn}, \tag{27a}$$

$$C^A_T = -\langle\bar\psi\hat S\gamma^5\psi\rangle, \tag{27b}$$

$$C^A_L = \frac{1}{(Sn)}\langle\bar\psi\hat n\gamma^5\psi\rangle, \tag{27c}$$

$$B^V(\lambda_1, \lambda_2) = -\langle\bar\psi(0)\gamma^5\gamma^\rho(D(\lambda\ )n)\psi(\lambda\ )\rangle\epsilon^{\rho Spn}, \tag{27d}$$

$$B^V_{LT} = -\langle \bar{\psi} \hat{n} D^\mu \psi \rangle \epsilon^{\mu S p n}, \tag{27e}$$

$$B^A_{LT} = -\langle \bar{\psi} \hat{n} \gamma^5 (DS) \psi \rangle, \tag{27f}$$

$$B^A_{TL} = -\langle \bar{\psi} \hat{S} \gamma^5 (Dn) \psi \rangle, \tag{27g}$$

$$B^A_{LL} = \frac{1}{(Sn)} \langle \bar{\psi} \hat{n} \gamma^5 (Dn) \psi \rangle. \tag{27h}$$

Moreover, the correlators B^A_{TL}, B^V_{TL}, B^A_{LL} don't depend on their first argument. This dependence enters through the gluon field only and vanishes in axual gauge. The gauge condition and definition (15a) give

$$D(\lambda_1) A(\lambda_2) = i \frac{dA(\lambda_2)}{d\lambda_2} \tag{28}$$

that makes possible to obtain the following relations:

$$B^V_{TL}(\lambda_1, \lambda_2) = i dC^V(\lambda_2)/d\lambda_2, \tag{29a}$$

$$B^A_{TL}(\lambda_1, \lambda_2) = i dC^A_T(\lambda_2)/d\lambda_2, \tag{29b}$$

$$B^A_{LL}(\lambda_1, \lambda_2) = i dC^A_L(\lambda_2)/d\lambda_2. \tag{29c}$$

Let us turn now to the restrictions provided by the equations of motion:

$$\hat{D}(\lambda) \psi(\lambda) = \bar{\psi}(0) \hat{D}(0) = 0. \tag{30}$$

For this purpose let us explore the relations

$$\langle \bar{\psi}(0) \gamma^5 \hat{S} \hat{n} \hat{D}(\lambda) \psi(\lambda) \rangle = 0, \tag{31a}$$

$$\langle \bar{\psi}(0) \hat{D} \hat{S} \hat{n} \gamma^5 \psi(\lambda) \rangle = 0, \tag{31b}$$

expand here the matrices sandwiched between the quark fields into vector and axial projection

$$-i \langle \bar{\psi}(0) \gamma^\rho D^\mu (\lambda) \psi(\lambda) \rangle \epsilon^{\rho \mu S n} - \langle \bar{\psi}(0) \gamma^\rho \gamma^5 D^\mu(\lambda) \psi(\lambda) \rangle ((Sn) g^{\mu\rho} + S^\rho n^\mu - n^\rho S^\mu) = 0, \tag{32a}$$

$$-i \langle \bar{\psi}(0) D^\mu(0) \gamma^\rho \psi(\lambda) \rangle \epsilon^{\rho \mu S n} + \langle \bar{\psi}(0) D^\mu(0) \gamma^\rho \gamma^5 \psi(\lambda) \rangle ((Sn) g^{\mu\rho} + S^\rho n^\mu - n^\rho S^\mu) = 0, \tag{32b}$$

and use the formula obtained from (25) under the change $p \leftrightarrow n$

$$n^\rho \epsilon^{\mu S p n} - n^\mu \epsilon^{\rho S p n} = \epsilon^{\rho \mu S n}. \tag{33}$$

Using once more equations of motion (that leads to vanishing terms $\sim (Sn)$) and (29a,b), we transform (26) to the form

$$i B^V_{LT}(\lambda, \lambda) + \frac{dC^V(\lambda)}{d\lambda} + i \frac{dC^A_T(\lambda)}{d\lambda} - B^A_{LT}(\lambda, \lambda) = 0, \tag{34a}$$

$$i B^V_{LT}(0, \lambda) + \frac{dC^V(\lambda)}{d\lambda} - i \frac{dC^A_T(\lambda)}{d\lambda} + B^A_{LT}(0, \lambda) = 0. \tag{34b}$$

After the Fourier transformation we obtain the following sum rules for distribution functions

$$\int dx_1 dx_2 \sigma(x_1) (i B^V_{LT}(x_1, x_2) - B^A_{LT}(x_1, x_2)) = \int dx \, x \sigma(x) (i C^V(x) - C^A_T(x)) \tag{35a}$$

$$\int dx_1 dx_2 \sigma(x_2) (i B^V_{LT}(x_1, x_2) + B^A_{LT}(x_1, x_2)) = \int dx \, x \sigma(x) (i C^V(x) + C^A_T(x)), \tag{35b}$$

where $\sigma(x)$ is an arbitrary test function.

Let us revert to the factorized formula for polarized contribution in the hard process cross section, writing explicitly the fractions of parton momenta connected with a polarized hadron only. The formula obtained therefore is directly related to deep inelastic scattering

$$W = - \int dx [C^V a^V - C^A_L a^A_L - C^A_T a^A_T] + \int dx_1 dx_2 [B^V_{LT} b'^V_T - B^A_{LT} b'^A_T], \tag{36}$$

where

$$a^A_T(x) = [\![E(xp) \hat{S}_T \gamma^5]\!], \tag{37a}$$

$$a^A_L(x) = [\![E(xp) \hat{p} \gamma^5]\!] (Sn), \tag{37b}$$

$$a^V(x) = [\![E(xp) \gamma^\rho]\!] \epsilon^{\rho S p n}, \tag{37c}$$

$$b'^A_T(x_1, x_2) = [\![E^\mu(x_1 p, x_2 p) \hat{p} \gamma^5]\!] S_{T\mu}, \tag{37d}$$

$$b'^V_T(x_1, x_2) = [\![E^\mu(x_1 p, x_2 p) \hat{p}]\!] \epsilon^{\mu S p n} \tag{37e}$$

(recall that $[\![\ldots]\!]$ means $\frac{1}{4} Sp(\ldots)$).

The gauge invariance demands the cancellation of all the terms, explicitly depending on the gauge vector [12b]. To single them out, let us beforehand transform (37e), making use of (24)

$$b'^{V}_{T} = [[\, E^{\mu}\gamma^{\rho}]] p_{\mu}\epsilon_{\rho S p n} - b^{V}, \quad \text{where} \tag{38}$$

$$b^{V}_{T}(x_1, x_2) = [[E^{\mu}(x_1 p,\ x_2 p)\gamma^{\rho}]]\epsilon_{\rho\mu S p}\ . \tag{39}$$

Now we'll substitute (38) into (36) and use the collinear Ward identities [12b] everywhere it is possible:

$$p_{\mu} E^{\mu}(x_1 p, x_2 p) = \frac{E(x_1 p) - E(x_2 p)}{x_1 - x_2}\ . \tag{40}$$

We obtain

$$\begin{aligned} W = &- \int dx\, C^{A}_{T} a^{A} - \int dx_1\, dx_2 [\, B^{A} b^{A} + B^{V} b^{V}] + \\ &+ \int dx\, [C^{A}_{T} - C^{A}_{L}] a^{A}_{L} + \int dx_1 dx_2 B^{A} \frac{a^{A}_{L}(x_1) - a^{A}_{L}(x_2)}{x_1 - x_2} + \\ &+ \int dx\, C^{V} a^{V} + \int dx_1 dx_2\, B^{V} \frac{a^{V}(x_1) - a^{V}(x_2)}{x_1 - x_2}, \quad \text{where} \end{aligned} \tag{41}$$

$$a^{A} = [[\, E(xp)\, \hat{S}\gamma^{A}]], \tag{42a}$$

$$b^{A} = [[\, E^{\mu}(x_1 p,\ x_2 p)\, \hat{p}\gamma^{5}]] S_{\mu}\ , \tag{42b}$$

and the indices L, T of the remaining two-argument functions B^A and B^V are omitted. Note that all but first three terms in (41) contain the explicit gauge vector dependence. Therefore the terms with a^A and a^V must turn to zero separately with arbitrary a^A_L and a^V. This leads to the following sum rules

$$\int dx (C^{A}_{L}(x) - C^{A}_{T}(x))\, \sigma(x) = \int dx_1 dx_2\, B^{A}(x_1, x_2) \frac{\sigma(x_1) - \sigma(x_2)}{x_1 - x_2}, \tag{43a}$$

$$\int C^{V}(x)\, \sigma(x)\, dx = - \int dx_1 dx_2 B^{V}(x_1, x_2) \frac{\sigma(x_1) - \sigma(x_2)}{x_1 - x_2}, \tag{43b}$$

where $\sigma(x)$ is the test function as before. The final expression for the polarized contribution has the form

$$W = - \int dx\, C^{A}_{T}(x)\, a^{A} - \int dx_1 dx_2 (B^{A}(x_1, x_2)\, b^{A} + B^{V}(x_1, x_2)\, b^{V})\ . \tag{44}$$

It is remarkably simplified in the case of longitudinal polarization $S^{\mu} = p^{\mu}/M$. The last term turns to zero and the second is transformed with the help of (40) and (43a) to the form

$$- \int dx\, a^{A} (C^{A}_{L} - C^{A}_{T})\ . \tag{45}$$

As a result, the longitudinal-polarization contribution to the cross section takes the form

$$W_{L} = - \int dx\, C^{A}_{L}(x)\, a^{A}(x)\ . \tag{46}$$

Therefore, for the longitudinal polarization the simple parton picture is justified; for the transversal one, as it is seen from (44), the situation is more complicated: the standard parton model [14] is not true.

The sum rules (43) by which the parton formula (44) was obtained have another application. The second of them permits us to eliminate the distribution function C^V, not entering into (44) from the sum rules (35) also. It is convenient to insert (43b) into the sum of equations (35a) and (35b).

$$\begin{aligned} i \int dx_1 dx_2 \{ B^{V}(x_1, x_2)\ \sigma(x_1) + \sigma(x_2) + 2 \frac{x_1\sigma_1(x_1) - x_2\sigma(x_2)}{x_1 - x_2} - \\ - B^{A}(x_1, x_2)(\sigma(x_1) - \sigma(x_2)) \} = 0\ . \end{aligned} \tag{47}$$

Subtracting (35a) from (35b) it is possible to obtain another (as we'll see, the most important) sum rule that contains only physical (i.e., included in (44)) distribution functions.

$$\begin{aligned} \int dx_1 dx_2 \{ (\sigma(x_1) - \sigma(x_2))\, B^{A}(x_1, x_2) - i(\sigma(x_1) + \sigma(x_2))\, B^{V}(x_1, x_2) \} = \\ = 2 \int dx\, x\, \sigma(x)\, C^{A}_{\perp}(x)\ . \end{aligned} \tag{48}$$

The T-invariance, which is convenient to take into account just now, provides further simplifications. Note that in T-invariant theories the phase of hadron-parton amplitudes is fully determined by the Born approximation, because the coupling constant is real (and the cuts providing the imaginary part are absent after taking the discontinuity in M^2_X). By this reason, B^A is real and B^V is pure imaginary. (This fact also provides the absence of single asymmetries in Born approximation [15]). On the other hand, making use of translational invariance it is easy to obtain for the complex conjugated matrix element

$$\langle \bar{\psi}(0)\gamma^{\rho}\gamma^{5} D^{\mu}(\lambda_1)\psi(\lambda_2)\rangle^{*} = \langle \bar{\psi}(0)\gamma^{\rho}\gamma^{5} D^{\mu}(\lambda_1 - \lambda_2)\psi(-\lambda_2)\rangle, \tag{49a}$$

$$\langle \bar{\psi}(0)\gamma^{\rho} D^{\mu}(\lambda_1)\psi(\lambda_2)\rangle^{*} = \langle \bar{\psi}(0)\gamma^{\rho} D^{\mu}(\lambda_1 - \lambda_2)\psi(-\lambda_2)\rangle \tag{49b}$$

that after calculating the Fourier-transform gives

$$B^{A*}(x_1, x_2) = B^{A}(x_2, x_1), \quad B^{V*}(x_1, x_2) = B^{V}(x_2, x_1), \tag{50}$$

consequently

$$B^{A}(x_1, x_2) = B^{A}(x_2, x_1), \quad B^{V}(x_1, x_2) = - B^{V}(x_2, x_1)\ . \tag{51}$$

Note that (51) transforms the sum rule (47) into the identity. Thus sum rule provides gauge invariance of the T-noninvariant contribution, and this simplification is quite natural. As to the sum rule (40), it takes the form

$$\int dx_1 dx_2 \sigma(x_1)(B^A(x_1,x_2) - iB^V(x_1,x_2)) = \tag{52}$$
$$= \int dx_1 dx_2 \sigma(x_2)(B^A(x_1,x_2) + iB^V(x_1,x_2)) = \int dx\, x\sigma(x)\, C_T^A(x),$$

and provides gauge-invariance of the T-invariant contribution.

Let us turn to the applications of the parton formula (44) and to the analysis of simplifications of the answer, resulting from the sum rules. We limit ourselves here to the simplest diagram of deep inelastic scattering. Coefficient functions of "nonparton" (or, more exactly, nonprimitive-parton) contributions of distribution functions with two arguments ("correlators") differ from the usual ones by the following propagator (that one interacting) with an external gluon, see Fig.3):

$$E^\mu(x_1,x_2) = \frac{(x_1\hat{p} + \hat{q})\gamma_\mu(x_2\hat{p} + \hat{q})}{(x_1 p + q)^2 (x_2 p + q)^2}, \tag{53}$$

where q is the photon momentum in deep inelastic scattering (and the integration momentum in general hard subprocess). It is convenient to write out the axial and vector terms of (53) explicitly:

$$E^\mu = V^\mu + A^\mu, \quad \text{where} \tag{54}$$

$$V^\mu(x_1,x_2) = \{-\gamma^\mu(pq(x_1+x_2) + q^2) + q^\mu(\hat{p}(x_1+x_2) + \hat{q})\} \times \tag{55a}$$
$$\times (2x_1 pq + q^2)^{-1}(2x_2 pq + q^2)^{-1} \equiv V_1^\mu + V_2^\mu,$$

$$A^\mu = -i\epsilon_{\mu\beta pq}\gamma^\beta\gamma^5(x_1 - x_2)[(2x_1 pq + q^2)(2x_2 pq + q^2)]^{-1}. \tag{55b}$$

We omitted in (55a) terms proportional to p^μ turning into zero after the multiplication by S_μ (see (39), (42b), (44)), and introduced a special notation for remaining two terms. Only the first of them will survive in the case of deep inelastic scattering on a transversely polarized proton. For a further analysis the following nominator expansion is useful

$$V_1^\mu(x_1,x_2) = -\gamma^\mu\{\frac{1}{2x_2 pq + q^2} + \frac{1}{2x_1 pq + q^2}\}, \tag{56a}$$

$$A^\mu(x_1,z_2) = -i\gamma^\beta\gamma^5\epsilon_{\beta\mu pq}\frac{1}{2pq}\{\frac{1}{2x_2 pq + q^2} - \frac{1}{2x_1 pq + q^2}\}. \tag{56b}$$

As is seen from (56), the vector term is symmetric, while the axial is antisymmetric in arguments. Therefore in the simplest

QCD (T-invariant theory) diagram the axial distribution function is convoluted only with the vector part of the dressed propagator, and vice versa.

The non-parton contribution is transformed in the following way

$$W_{\nu\sigma}^{(NP)} = \frac{1}{2}\int dx_1 dx_2 \{B^A(x_1,x_2)[(2x_1 pq + q^2)^{-1} + (2x_2 pq + q^2)^{-1}] + \tag{57}$$
$$+ iB^V(x_1,x_2)[(2x_1 pq + q^2)^{-1} - (2x_2 pq + q^2)^{-1}]\}[[\hat{p}\gamma^5\gamma^\nu\hat{S}\gamma^\sigma]].$$

It is possible, by making use of (48) with the following test function

$$\sigma(x) = (2x\, pq + q^2)^{-1} \tag{58}$$

to combine vector and axial contributions:

$$W_{\nu\sigma}^{(NP)} = \int\frac{dx\, x\, C_T^A}{(xp + q)^2}[[\hat{p}\gamma^5\gamma^\nu\hat{S}\gamma^\sigma]]. \tag{59}$$

The parton contribution, in turn, has the form

$$W_{\nu\sigma}^{(P)} = \int dx \frac{C_T^A}{(xp + q)^2}[[\hat{S}\gamma^5\gamma^\nu(x\hat{p} + \hat{q})\gamma^\sigma]]. \tag{60}$$

So, (59) coincides with the parton formula for the contribution of the quark mass, equal to that of the hadron, and cancels with the non-gauge-invariant part (the first term in parentheses) of the parton contribution. The final result appears to be gauge-invariant and parton-like:

$$W_{\nu\sigma} = \int dx\, C_T^A(x)[2x\, pq + q^2)\, i\epsilon_{\nu\sigma Sq}, \quad \text{or} \tag{60a}$$

$$M^2 G_1(x) + (pq) G_2(x) = C_T^A(x)/2pq. \tag{60b}$$

As the calculation of anomalous dimension of nonsinglet structure functions shows /10/, in higher orders of perturbation theory such cancellations of the two-argument distribution functions do not occur.

For completeness consider the contribution of the second term in (55a), which discriminates between the longitudinal and transversal polarization (and results in the form factor $G_2(x)$)

$$W_{\sigma\nu}^L = -i\int dx_1 dx_2 B^A(x_1,x_2)[(2x_1 pq + q^2)(2x_2 pq + q^2)]^{-1}\epsilon_{\nu\sigma Sq}, \tag{61}$$

$$2pq\, G_2 = ⨍\frac{dy\, B^A(x,y)}{x - y}. \tag{62}$$

Note that [17] $\int dx\, 2pqG(x) = 0$. It seems important to measure G_1 and G_2 independently in order to determine C_A^T, and obtain some information about $B^A(x_1, x_2)$ and $B^V(x_1, x_2)$.

In conclusion we have to note that the method proposed opens the straightforward application of perturbative QCD to the calculation of single asymmetries which is now in progress. The answer will contain a new two-argument distribution function (see Exp. (44)) the important information on which can be obtained from the experimental study of polarized deep inelastic scattering.

REFERENCES

1. Nurushev S.B. JINR, D1,2-82-27, Dubna, 1982, p.6.
2. Bunce G. et al. Phys.Rev.Lett., 1976, vol.36, p.1113.
3. Heller K. et al. Phys.Rev.Lett., 1978, 41, p.607.
4. Erhan J. et al. Phys.Lett., 1979, 82B, p.1325.
5. Antille J. et al. Phys.Lett., 1980, 94B, p.523.
6. Efremov A.V., Teryaev O.V. Yad.Fiz., 1982, 36, p.242.
7. Cane G.L., Pumplin J.P., Repko W. Phys.Rev.Lett., 1978, 41, p.1689.
8. Efremov A.V., Teryaev O.V. JINR, D1,2-82-27, Dubna, 1982, p.53.
9. Shuryak E.V., Vainshtein A.I. Nucl.Phys., 1982, B201, p.141.
10. Bukhvostov A.P., Kuraev E.A., Lipatov L.N. ZhETF Pis´ma, 1983, 37, p.406.
11. Ahmed M., Ross G.C. Nucl.Phys., 1976, 111B, p.441.

12a). Furmanski W. IC/81/152, Trieste, 1981; b) Ellis R.K., Furmanski W., Petronzio R. CERN TH 3301, Geneva, 1982.

13. Efremov A.V., Radyushkin A.V. Teor.Mat.Fiz., 1980, 44, p.327.
14. See, e.g., Feynman R.P. Photon-Hadron Interactions. W.A.Benjamin Inc., New York, 1972.
15. Christ A., Lee T. Phys.Rev., 1966, 143, p.1316.
16. Beresteckij V.B., Lifshitz E.M., Pitayevskij L.P. Relativistic Quantum Fields.
17. Burkhardt M., Cottingham W.H. Ann.Phys., 1976, 56, p.453.

Ефремов А.В., Теряев О.В. E2-83-700

Поперечная поляризация в квантовой хромодинамике

Матрица плотности поляризованного адрона в жестких процессах анализируется в рамках подхода Петронцио, Фурманского и Эллиса, использующего аксиальную калибровку и разложение пропагатора активного кварка вблизи коллинеарного направления. Подтверждена установленная ранее картина,физически эквивалентная перенормировке массы кварка при движении его во внешнем глюонном поле адрона, причем стандартная партонная картина оказывается, вообще говоря, несправедливой. Правила Фейнмана для расчета коэффициентных функций высших порядков не отличаются, по существу, от стандартных. Результаты применены к исследованию глубоконеупругого рассеяния поляризованных электронов на поляризованных протонах.

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Efremov A.V., Teryaev O.V. E2-83-700

The Transversal Polarization in Quantum Chromodynamics

The polarized hadron density matrix in hard processes is analysed in the framework of the Ellis, Furmanski and Petronzio approach that makes use of the axial gauge and struck quark propagator expansion near the collinear direction. The earlier established picture, physically equal to quark mass renormalization during the propagation in an external gluon field of the hadron, is confirmed; the standard parton picture is, generally speaking,invalid. The Feynman rules for the calculation of higher-order coefficient functions do not differ essentially from the standard ones. The results are applied to the investigation of deep inelastic scattering of polarized electrons off polarized protons.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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