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THE TOPOLOGICAL SUSCEPTIBILITY FROM SU(3) LATTICE GAUGE THEORY

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At present Monte-Carlo simulations in lattice gauge theories represent the most efficient tool for calculating various non-perturbative numbers such as the string tension, low-lying glueball and meson masses, the deconfinement phase transition temperature, etc. [1]. Of special interest is the extraction of vacuum expectation values for composite gluon and quark operators. Their direct computation provides an independent check of the consistency of phenomenological approaches - as, for instance, the ITEP QCD sum rule scheme [2] - as well as of our idea of the vacuum structure. As a step in this direction the SU(3) gluon condensate $\langle \alpha_s G^a_{\mu\nu} G^a_{\mu\nu} \rangle$ has been obtained recently from Wilson loop data [3] in a good agreement with the phenomenological value [2].

In this letter we want to discuss the "topological susceptibility" χ of the vacuum state in the pure SU(3) Yang-Mills theory

$$\chi \equiv - \frac{d^2 P}{d\theta^2}\Big|_{\theta=0} = \int d^4x \langle Q(x) Q(0) \rangle_{\text{no light quarks}} , \qquad (1)$$

where

$$Q(x) \equiv \frac{g^2}{64\pi^2} G^a_{\mu\nu}(x)\, \epsilon_{\mu\nu\rho\sigma}\, G^a_{\rho\sigma}(x)$$

represents the topological charge density, P and θ denote the vacuum pressure and the phase, respectively.

The quantity χ plays a fundamental role in the solution of the $U_A(1)$ problem [4,5,6]. The latter consists in the absence of a light pseudoscalar meson in the nature, which could be interpreted as a Goldstone boson corresponding to the spontaneously broken axial U(1) symmetry. The only candidate for it would be the η' meson having a significantly larger mass than the pseudoscalar octet mesons. On the basis of anomalous Ward identities and the $U_A(1)$ current anomaly it has been argued [4] that the existence of topologically non-trivial field excitations (instantons, etc.), for which $\chi \neq 0$, may provide a solution to the problem. Witten has demonstrated how in the large N_c limit a relation between χ and the η' mass can be established [5]. A more quantitative analysis within the framework of effective Lagrangians yields [6]

$$\chi = \frac{1}{2N_\ell} F_\pi^2 (m^2_{\eta'} + m^2_\eta - m^2_{K^0} - m^2_{K^+}) \simeq (182\ \text{MeV})^4 . \qquad (2)$$

($N_\ell = 3$ represents the number of light flavors, $F_\pi \simeq 95$ MeV is the pion decay constant).

For SU(2) χ has been computed recently by Monte-Carlo lattice simulations [7]. It could be shown to be non-zero; however, the numerical estimate failed approximately by two orders of magnitude.

$$\chi_{SU(2)} = (55 \pm 10\ \text{MeV})^4 . \qquad (3)$$

The calculations have been carried out by employing two different lattice definitions for the topological charge density, both of them having the same naive continuum limit Q(x). The corresponding results agreed very well. The main criticism of these lattice computations might be that the lattice definitions used in Ref. [7] are topologically not relevant. This is related to the existence of a perturbative tail to be subtracted in order to isolate the under-lying non-perturbative, renormalization group invariant quantity. Since such a procedure has proved a success in the gluon condensate case [3], we shall not disregard the proposed definitions for the time being. Rather we would like to ask, whether the relation between χ and the phenomenological value will improve in the real SU(3) case.

Let us regard here the lattice definition of the topological charge density as given by [7]

$$Q_L(n) = -\frac{1}{2^9 \pi^2} \sum_{\mu\nu\rho\sigma=\pm 1}^{\pm 4} \tilde{\epsilon}_{\mu\nu\rho\sigma}\, \mathrm{tr}(U(n)_{\mu\nu}\, U(n)_{\rho\sigma})$$
$$\xrightarrow[a \to 0]{} a^4 Q(x_n) + O(a^5), \qquad (4)$$

where $\tilde{\epsilon}_{1234} = -\tilde{\epsilon}_{2134} = -\tilde{\epsilon}_{-1234} = \ldots = 1$ and a being the lattice spacing. $U(n)_{\mu\nu}$ is the usual plaquette operator placed at site x_n within the μ-ν plane.

χ can be extracted by means of the Monte-Carlo simulation for a finite lattice (here with size of 4^4 and 6^4 lattice points, respectively, and with periodic boundary conditions) by "measuring" the correlator

$$a^4 \chi_L \equiv \sum_n \langle Q_L(n) Q_L(0) \rangle . \qquad (5)$$

At sufficiently small (bare) coupling χ_L should behave as

$$\pi^4 2^{18} a^4 \chi_L = c_1 g_0^6 + c_2 g_0^8 + \dots + \pi^4 2^{18} (a(g_0)\Lambda_L)^4 \frac{\chi}{\Lambda_L^4}, \tag{6}$$

where for SU(3) and a 4^4 lattice $c_1 = 400.9$ /7/. The lattice spacing is expected to behave as dictated by the renormalization group

$$(a\Lambda_L)^{-2} = (\beta_0 g_0^2)^{\beta_1/\beta_0^2} \exp\left(\frac{1}{\beta_0 g_0^2}\right) \tag{7}$$

with $\beta_0 = 11/16\pi^2$ and $\beta_1 = 102/256\pi^4$. For the scale parameter Λ_L we take $\Lambda_L = (.007+.001)\sqrt{\sigma}$ /8/ and the accepted value of the string tension $\sqrt{\sigma} \simeq 420$ MeV, although, strictly speaking, the latter corresponds to a world with light quarks included. At strong coupling one finds

$$\pi^4 2^{18} a^4 \chi_L = 768\left(1 + \frac{2}{3} g_0^{-2} + \frac{7}{3} g_0^{-4} + O(g_0^{-6})\right). \tag{8}$$

Our data shown in the Figure have been produced applying Pietarinen's SU(3) heat bath procedure /9/ with a random upgrading (in the 4^4 case) of the lattice links. Typically, for any given g_0 we average over 180 and 130 sweeps through a 4^4 and 6^4 lattice, respectively. We have checked at small g_0^{-2} that the MC calculation really reproduces the high temperature behaviour (8).

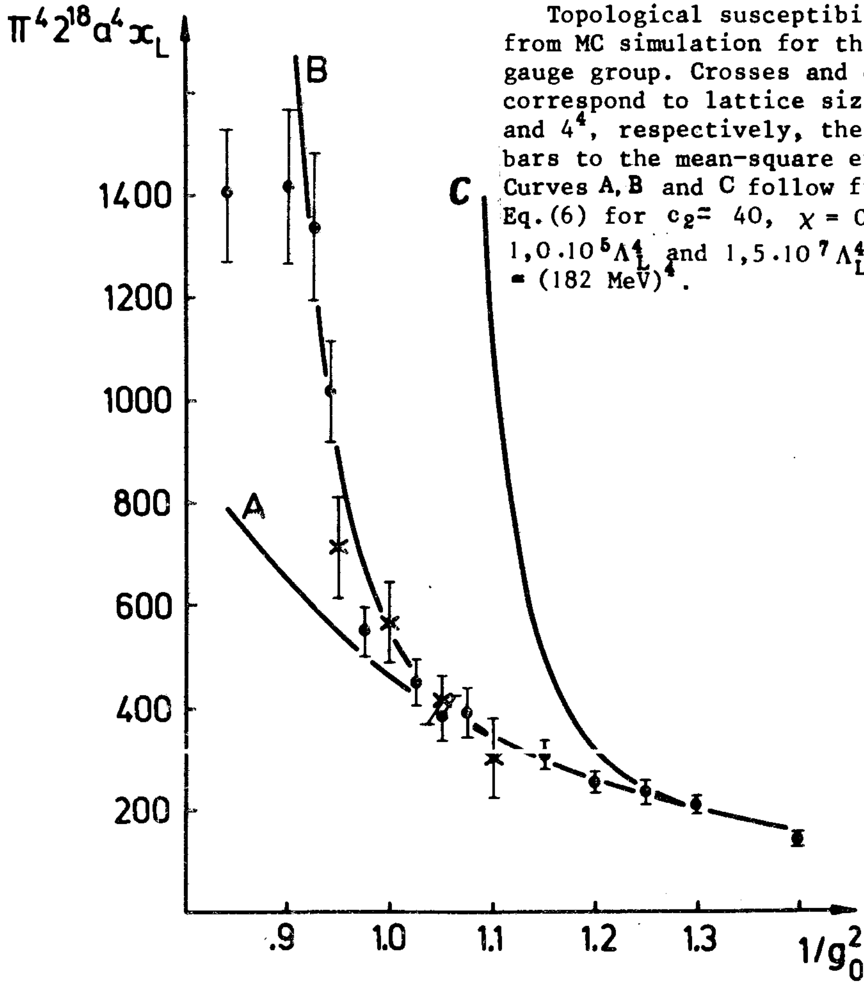
One finds a distinct non-perturbative signal in the region where a scaling behaviour is usually expected ($0.9 \leq g_0^{-2}$). The perturbative tail is rather well described already by the lowest order contribution $O(g_0^6)$. Thus, a χ^2 fit of the second coefficient yields a relatively small number $c_2 = 42 \pm 20$ (cf. Curve A). From Eq. (6) we determine (Curve B)

$$\chi_{SU(3)} = (1.0 \pm 0.2)\cdot 10^5 \Lambda_L^4 \simeq (52 \pm 8\ \mathrm{MeV})^4. \tag{9}$$

The agreement with $\chi_{SU(2)}$ supports the view that $\chi_{SU(N_c)}$ is of order $O(N_c^0)$ provided σ does not depend on N_c. The disagreement with the phenomenologically expected value (2) is obvious (compare with Curve C). It is not an artifact of a too small lattice as one might argue. Our data taken on a 6^4 lattice in a coupling region, where the departure from the perturbative behaviour is seen, show this clearly.

We conclude with the following remarks.

(i) Before one is justified in rejecting the solution of the $U_A(1)$ problem mentioned above, one should check as a next step, whether the value (9) survives a calculation with a lattice topological charge definition avoiding a perturbative tail (see Ref. /10/, e.g.).



Topological susceptibility from MC simulation for the SU(3) gauge group. Crosses and dots correspond to lattice sizes 6^4 and 4^4, respectively, their bars to the mean-square errors. Curves A, B and C follow from Eq. (6) for $c_2 = 40$, $\chi = 0{,}0$; $1{,}0\cdot10^5\Lambda_L^4$ and $1{,}5\cdot10^7\Lambda_L^4 = (182\ \mathrm{MeV})^4$.

(ii) For a further test it is interesting to evaluate the topological susceptibility (1) within lattice QCD, i.e., taking virtual quark loops into account. The latter quantity should be related to $m_\pi^2 F_\pi^2$ (without relying on any $1/N_c$ arguments in contrast to Eq. (2)) and therefore has to vanish in the chiral limit.

(iii) Unfortunately continuum calculations based on instanton contributions so far give no hint, whether the estimate (9) is reliable or not. On the one hand, the instanton gas model with hard core and dipole-like interactions was shown /11/ to provide reasonable numbers for low-dimensional gluon condensation parameters; however, it yields only an approximate upper bound for $\chi_{SU(3)}$ being somewhat smaller than (2). On the other hand, Lüscher[12] has argued for a lower bound satisfied by our lattice result (9), as well. In contrast to a conclusion drawn in Ref.[13] we expect that lattice calculations - such as discussed in this letter - involve implicitly the effect of instantons. Within the scaling region their typical scale size $\bar{\rho}$ is slightly larger than the lattice unit a and small enough to "put" one instanton into a 4^4 lattice volume.

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Махалдиани Н.В., Мюллер-Пройскер М. E2-83-69
Вычисление топологической восприимчивости SU(3) калибровочной теории на решетке

Вычислен коррелятор для плотности топологического заряда при нулевом импульсе в случае калибровочной группы SU(3) на решетке методом Монте-Карло. Соответствующее континуальному пределу значение топологической восприимчивости находится в согласии с ранее найденным результатом для группы SU(2), но отличается от значения, которое ожидается из принятого решения $U_A(1)$ проблемы, на два порядка.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

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Makhaldiani N.V., Müller-Preussker M. E2-83-69
The Topological Susceptibility from SU(3) Lattice Gauge Theory

The SU(3) topological susceptibility is extracted from lattice Monte-Carlo data. Our result agrees with the value found recently in the SU(2) case and is approximately by two orders of magnitude smaller than it is expected from the widely accepted solution of the $U_A(1)$ problem.

The investigation has been performed at the Laboratory of Computing Technique and Automation, JINR.

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