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ON THE "KINEMATICAL FRAGMENTATION" IN PROTON DECAY

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The conjecture about proton non-stability is one of the most important results of grand unified theories*). If it would not be confirmed experimentally, serious theoretical consequences will follow. In such a case, however, one must exclude the possibility that the sought decay is suppressed by an additional effect.

Some possibilities of this type have been discussed recently/2-5/. Khalfin/2/ proposed that the suppression might be due to the small-time non-exponentiality of the decay law. It would imply, however, that the mass distribution of proton must differ substantially from the Breit-Wigner shape outside a tiny interval containing the peak (as small as 10^{-3} eV for the suppression time 10^{-12} s, and appropriately smaller for longer times), and this is highly unlikely**). The "Zeno's effect"/6,7/, too, cannot presumably modify essentially the proton decay law/3/. A possible exception concerns the decay of a proton bound inside a nucleus/4/, but only in the case that one accepts the questionable premise that the interaction with the "fellow nucleons" can constitute a sequence of repeated non-decay measurements.

Another effect which might modify the decay law has been considered by Fleming/5/; he called it kinematical fragmentation. Its essence is that the spreading of the proton wavepacket is eventually too fast to be neglected. However, Fleming's argument is very rough and his conclusion, namely that the proton decay must be suppressed (to $\tau \approx 10^{36}$ yr) in order to prevent the "kinematical fragmentation" during one theoretical lifetime, can be challenged. In this letter, we address ourselves with this problem.

All the above-mentioned papers use the simplest decay-law formula

$$P_{\psi}(t) = |(\psi, e^{-iHt}\psi)|^2 \quad , \tag{1a}$$

which assumes the state space $\mathcal{H}_u$ of the unstable particle to be one-dimensional and spanned by the vector ψ. However, this assumption is not fully compatible with the consistent relativistic description of the decay***). In particular, translational invariance requi-

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*) For a large review of this problem see Ref.1 .

**) This fact was stressed particularly in Refs.3,5 . For more details about decay laws of unstable quantum systems and related items see the forthcoming monograph - Ref.6 .

***) See Ref.8 or Ref.6, section 3.5 .

res $\mathcal{H}_u$ to be infinite-dimensional. A natural choice has been discussed in Ref.8 : $\mathcal{H}_u$ can consist of the functions (in p-representation)

$$\psi : \psi(m,\vec{p}) = f(m)g(\vec{p}) \quad , \tag{2}$$

or eventually of multicomponent functions of this type in the case of non-zero spin. Here g is an arbitrary square-integrable function whose support is contained in the ball B_ε of radius $\varepsilon \equiv \Delta p$. The mass distribution $|f(.)|^2$ should be obtained from an appropriate dynamical model ; in practice one mostly uses the BW-function cut according to physical thresholds, with the half-width Γ eventually calculated in the perturbation-theory framework. The mean mass M is usually fixed from experimental data.

Then the decay law (in the rest system and for an initial state ψ of the form (2)) is expressed by the formulae (3a),(13) of Ref.8. If we neglect for simplicity spin of the particle, they can be rewritten in the following explicit form

$$P_\psi(t) = \sum_{klm} \Big| \int_{m_0}^{\infty} dm\, |f(m)|^2 \int_0^{\varepsilon} \frac{p^2 dp}{2(m^2+p^2)^{1/2}}\, \overline{h_k(p)}\, \times \int_{4\pi} d\Omega_{\vec{p}}\, Y_{lm}(\Omega_{\vec{p}})\, g(\vec{p})\, \exp[-it(m^2+p^2)^{1/2}] \Big|^2 \quad , \tag{3}$$

where Y_{lm} are the spherical functions, $p \equiv |\vec{p}|$, and $\{h_k\}$ is a complete system of functions with supports in $[0,\varepsilon]$ which fulfil the orthonormality condition

$$\int_{m_0}^{\infty} dm\, |f(m)|^2 \int_0^{\varepsilon} \frac{p^2 dp}{2(m^2+p^2)^{1/2}}\, \overline{h_j(p)}\, h_k(p) = \delta_{jk} \quad . \tag{4}$$

Now the question is under which circumstances the decay law (3) may be approximated by the formula

$$P_\psi(t) = \Big| \int_{m_0}^{\infty} e^{-imt}\, |f(m)|^2 dm \Big|^2 \quad , \tag{1b}$$

which is expected to follow from (1a) with the appropriate choice of H, ψ . It appears/8/ that this approximation is possible for small enough times ; later the decay curve has to be calculated from (3), and therefore it becomes dependent on the shape of the function g , i.e., on the initial momentum distribution. Of course, the lifetime $T_\psi = \int_0^\infty P_\psi(t)\, dt$ may differ then from $\hbar/\Gamma$. Notice that we have used no hypothesis about the decay products besides the choice of the subspace $\mathcal{H}_u$ in the carrier space of the representation of Poincaré group associated with the unstable particle under consideration.

Hence we see that the decay law can be influenced by the kinematical characteristics of the particle. At the same time, one can hardly expect that the deviation of (3) from (1b) will always mean enhancement of the non-decay probability, or even suppression of the decay.

Let us ask now whether the described effect could be observed. We have shown in Ref.8 that the approximation (1b) may be used in the whole region where the decay law can be actually measured, i.e., up to few $\hbar/\Gamma$, if the condition $\varepsilon c \ll (M\Gamma)^{1/2}$ is valid. Equivalently, we require

$$\Delta q \gg \hbar c (M\Gamma)^{-1/2} \tag{5}$$

for the initial spread Δq of the wavepacket ψ in the rest system of the particle. In distinction to all the other metastable particles, the condition (5) represents a drastic restriction when applied to the proton decay. In fact, it corresponds to the Fleming's requirement/5/ about absence of kinematical effects during one lifetime.

This is not needed, however. Nobody is able to watch the decay of a proton during the period of 10^{31} years . What we really want to know is whether the kinematical effects can play an essential role during the time interval $[0,T]$ involved in an experiment. Using the uncertainty relation, we can rewrite the condition (24) of Ref.8 for such a case as

$$\Delta q \gg c(\hbar T/M)^{1/2} \tag{6a}$$

or

$$\Delta q_{[cm]} \gg 0.025\, T_{[s]}^{1/2} = 7.4\, T_{[d]}^{1/2} = 141\, T_{[yr]}^{1/2} \quad . \tag{6b}$$

This conclusion applies, of course, only in the case when the proton can be regarded as free in a reasonable approximation. Hence it has no implications for the nuclear and radiochemical methods/9/, because they generally look for the decay of a proton bound inside a suitable nucleus (^{232}Th , ^{2}H , ^{130}Te , ^{39}K etc.). On the other hand, the protons may be treated as free in the direct methods using large underground water tanks, though with a certain amount of idealization. Let us consider a typical experiment of this type, such as IBM or HPW /9,10/. If we take the total duration of the measurement for T , and assume that the localization Δq is determined a posteriori from the kinematics check of the processes in the tank, then the inequality (6) can be violated, its both sides being of the same order of magnitude. In this optimal case therefore, shape of the wavepacket could affect the decay. In order to see the effect, however, one would have to measure the decay curve exactly enough, while today we are hunting after single decay events.

In conclusion, we can say that the "kinematical suppression" of the proton decay is highly improbable. The influence of the wavepacket size and shape might be eventually of some importance, but at most for a precise determination of the proton half-width Γ .

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Экснер П. E2-83-671

О "кинематической фрагментации" в распаде протона

Обсуждается проблема "кинематической фрагментации" в распаде, недавно выдвинутая Флемингом. Показано, что подавление распада протона в проводимых экспериментах с подземными водяными баками за счет этого эффекта является маловероятным.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1983

Exner P. E2-83-671

On the "Kinematical Fragmentation" in Proton Decay

The problem of "kinematical fragmentation" in decay recently considered by Fleming is reexamined. It is shown that this effect is not likely to suppress the proton non-stability in the underground water-tank experiments presently performed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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