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TWO-LOOP CONTRIBUTION TO THE EVOLUTION OF THE PION WAVE FUNCTION

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1. INTRODUCTION

In the last few years there has been a considerable progress in understanding the behaviour of the pion form factor at asymptotically large momentum transfer [1-4]. In particular, it was shown that in perturbative QCD one can factorize the contributions corresponding to short and long distances. For the form factor $F_\pi(Q^2)$ this leads to the relation

$$F_\pi(Q^2) = \frac{1}{Q^2}\int_0^1 dx \int_0^1 dy\ \phi^*(y,\mu^2)\, E\left(\frac{Q^2}{\mu^2}, x, y, g(\mu)\right) \phi(x,\mu^2) + O(1/Q^4) \tag{1}$$

(see fig. 1). Here E/Q^2 is the hard scattering amplitude, in leading order in $\alpha_s(\mu) = \bar{g}^2(\mu)/4\pi$ given by [1-3]:

$$E^{(0)}(Q^2/\mu^2, x, y, g) = \frac{2\pi\alpha_s(\mu)\, C_F}{N_c\, xy} \tag{2}$$

($C_F = 4/3$ and $N_c = C_G = 3$ are the usual color factors), whereas $\phi(x,\mu^2)$ is the pion wave function. Its evolution with μ^2 is determined by the equation [3]

$$\mu \frac{d}{d\mu}\phi(x,\mu^2) = \int_0^1 dy\ V(x,y,g(\mu))\, \phi(y,\mu^2). \tag{3}$$

The leading order contribution to the perturbative expansion of the kernel V

$$V(x,y,g) = \frac{\alpha_s}{2\pi} V^{(0)}(x,y) + \left(\frac{\alpha_s}{2\pi}\right)^2 V^{(1)}(x,y) + \dots, \tag{4}$$

was presented in ref. [3]:

$$V^{(0)}(x,y) = C_F \left\{ \frac{x}{y}\left(1 - \frac{1}{x-y}\right)\theta(y-x) + \left[\begin{matrix} x \to 1-x \\ y \to 1-y \end{matrix}\right] \right\}_+, \tag{5}$$

where the "+" - prescription is defined in the usual way:

$$\{f(x,y)\}_+ = f(x,y) - \delta(x-y)\int_0^1 f(z,y)\, dz. \tag{6}$$

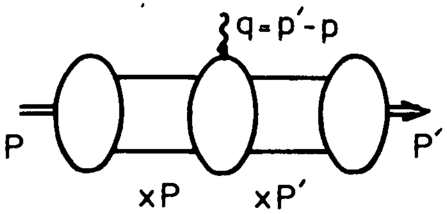

Fig. 1. The factorized representation for the pion form factor.

It is well known, however, that perturbation theory only allows reliable predictions when higher order corrections are taken into account. This is especially important in cases where already the Born term is proportional to $\alpha_s(\mu)$: leading order calculations say nothing about how to choose the parameter μ. Therefore, to get quantitative predictions for the asymptotic pion form factor, we have to calculate both the amplitude E and the kernel V at least up to second order in α_s. The first step of this program was performed in refs./5/ and /6/ where the $O(\alpha_s^2)$-correction to E was presented. The second step is the evaluation of the $O(\alpha_s^2)$-contribution to the evolution kernel V. The function $V^{(1)}(x,y)$ is expected to cancel the renormalization and factorization scheme dependence of E in order α_s^2. Moreover, calculating $V^{(1)}(x,y)$, we can study beyond the leading order how the renormalization machinery for composite QCD operators works. In particular, it is of great interest whether the close connection between the eigenfunctions of the kernel V and the conformal-invariant operators, known from leading order /1,2/, is broken by radiative corrections.

In this note we briefly describe the method and present the result of the calculation of $V^{(1)}(x,y)$ in the $\overline{\mathrm{MS}}$-scheme used already in ref./5/ to calculate the $O(\alpha_s^2)$-correction to the amplitude E. Furthermore, we describe the connection between the obtained results and the two-loop calculations in deep inelastic scattering and discuss some new peculiarities of the lightlike axial gauge.

2. METHOD AND RESULT

Our method is based on the technique developed by Curci et al./7/ to calculate the evolution of parton densities beyond the leading order. The approach of ref./7/ can be extended in a straightforward way to handle exclusive processes /8/. Its main ingredients are:

- the choice of a lightlike gauge $n\cdot A = 0$, $n^2 = 0$;
- the decomposition of the perturbative expansion in generalized ladders;
- the use of dimensional regularization, both for ultra-violet (UV) and for collinear singularities;

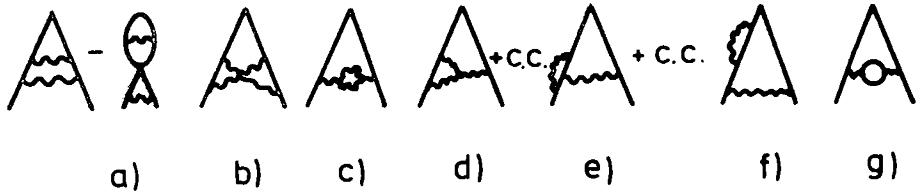

Fig. 2. Diagrams contributing to $\hat{V}^{(1)}(x,y)$.

- the use of the $\overline{\mathrm{MS}}$-subtraction scheme for UV-divergences (including spurious ones) and of renormalization group invariance.

The diagrams contributing to $V^{(1)}(x,y)$ are shown in fig.2 (diagrams renormalizing external legs should be added). A detailed description of their calculation (using the algebraic computer programme "SCHOONSCHIP" /9/) was given by one of the authors in ref./8/ where also some partial results were presented. While completing this work we obtained the paper by Sarmadi /10/ in which an analogous calculation was performed. To compare our result with that of ref./10/, we present it in a similar form and use also Sarmadi's notation (interchanging, however, $x \leftrightarrow y$). Our final result for the two-loop contribution to the evolution kernel V (x, y) is:

$$V^{(1)}(x,y) = [\hat{V}^{(1)}(x,y)]_+ \tag{7}$$

$$\hat{V}^{(1)}(x,y) = C_F^2\,\hat{V}_F(x,y) + \frac{1}{2}C_F C_G\,\hat{V}_G(x,y) + \frac{1}{2}N_F C_F\,\hat{V}_N(x,y) \tag{8}$$

$$\hat{V}_N(x,y) = \{\theta(y-x)\,[-\frac{10}{9}F - \frac{2}{3}\frac{x}{y} - \frac{2}{3}F\ln\frac{x}{y}]\} + [\begin{smallmatrix} x\to\bar{x} \\ y\to\bar{y}\end{smallmatrix}] \tag{9}$$

$$\hat{V}_G(x,y) = \{\theta(y-x)\,[\frac{67}{9}F + \frac{17}{3}\frac{x}{y} + \frac{11}{3}F\ln\frac{x}{y} + 2\bar{F}\ln y\,\ln\bar{x}] \tag{10}$$

$$+ G(x,y)\} + [\begin{smallmatrix} x\to\bar{x} \\ y\to\bar{y}\end{smallmatrix}]$$

$$\hat{V}_F(x,y) = \{\theta(y-x)\,[-\frac{\pi^2}{3}F + \frac{x}{y} - (\frac{3}{2}F - \frac{x}{2\bar{y}})\ln\frac{x}{y} -$$

$$-(F-\bar{F})\ln\frac{x}{y}\ln(1-\frac{x}{y}) + (F + \frac{x}{2\bar{y}})\ln^2\frac{x}{y} - 2\bar{F}\ln y\,\ln\bar{x}]\}$$

$$-G(x,y) - \frac{x}{2\bar{y}} \ln x\,(1 + \ln x - 2\ln \bar{x}) \; + [\begin{smallmatrix} x \to \bar{x} \\ y \to \bar{y} \end{smallmatrix}], \tag{11}$$

where

$$\begin{aligned} G(x,y) = \theta(x+y-1)\,[2(F-\bar{F})\,\mathrm{Li}_2(1-\tfrac{x}{y}) + (F-\bar{F})\ln^2 y - \\ -2F \ln x \ln y] + 2F\,\mathrm{Li}_2(1-y)\,[\theta(x+y-1) - \theta(y-x)] - \\ -2F\,\mathrm{Li}_2(x)\,[\theta(x+y-1) - \theta(x-y)]\,, \end{aligned} \tag{12}$$

$$F \equiv F(x,y) = \frac{x}{y}[1 - 1/(x-y)], \;\; \bar{F} = F(\bar{x},\bar{y}), \;\; \bar{x} = 1-x, \;\; \bar{y} = 1-y\,,$$

and $\mathrm{Li}_2(x)$ is the Spence function

$$\mathrm{Li}_2(x) = -\int_0^x \frac{\ln(1-t)}{t}\,dt\,. \tag{13}$$

We have found that our result (7)-(12) does not agree with that one obtained by Sarmadi for two reasons: first, instead of the "+" - form (7) Sarmadi presents the relation $V^{(1)}(x,y) = \hat{V}^{(1)}(x,y) + Z^{(1)}\,\delta(x-y)$, where the expression for $Z^{(1)}$ given in ref. /10/ does not depend on y. This seems to contradict some general arguments leading to the representation (7). Moreover, explicitly calculating the simplest diagrams renormalizing external legs, one can easily show that in the axial gauge the Z-factors essentially depend on y. Second, there is some minimal discrepancy in the function $\hat{V}^{(1)}(x,y)$ itself: to get the result of Sarmadi from our one, one has to change the coefficient of the $\ln^2(x/y)$ term in eq. (11) from $(F + \frac{x}{2\bar{y}})$ to $(F + \frac{x}{2y})$. This difference is evidently due to a misprint in ref. /10/. A detailed description of our calculation will be presented elsewhere. In a subsequent publication we plan to discuss also the diagonalization of the two-loop anomalous dimension matrix and the influence of the obtained result on the asymptotic behaviour of the pion form factor. Here we want to mention only that at the two-loop level the diagonality of the anomalous dimension matrix in the conformal basis is broken. This breakdown is caused by the $\ln(x/y)$ terms in the functions $\hat{V}_N$ and $\hat{V}_G$ and the $\ln^2$-terms in $\hat{V}_F$.

3. CORRESPONDENCE TO THE DEEP INELASTIC SCATTERING RESULTS

The fact that our result for the function $\hat{V}^{(1)}(x,y)$ is (almost) identical to that one presented by Sarmadi provides a strong evidence for its correctness.

As a further independent test, we have checked that the eigenvalues of the kernel $V^{(1)}(x,y)$ coincide with the two-loop anomalous dimensions $\gamma_n^{(1)}$ of the non-singlet operators in deep inelastic scattering /11,12/. Such an agreement is expected because of the triangular form of the anomalous dimension matrix corresponding to $V^{(1)}(x,y)$.

The n-th eigenvalue of the kernel $V^{(1)}(x,y)$ can be computed numerically and compared with $\gamma_n^{(1)}$, in principle, for any given n. We have done this for the first ten values of n. There is, however, another possibility that allows us to compare the eigenvalues of $V^{(1)}(x,y)$ with the $\gamma_n^{(1)}$ analytically for all n. It makes use of the fact that the $\gamma_n^{(1)}$ are moments of the two-loop contribution to the Altarelli-Parisi kernel $P^{(1)}(t)$ /7/. This leads to the relation

$$P(t) = \frac{1}{2\pi i}\int_{-\sigma-i\infty}^{-\sigma+i\infty} t^{-n-1}\,dn\,\Big\{\frac{1}{n!}\Big(\frac{d}{dy}\Big)^n\Big|_{y=0}\int_0^1 V(x,y)\,x^n\,dx\Big\}_{AC} \tag{14}$$

connecting the kernels $V(x,y)$ and $P(t)$. (The symbol "AC" denotes analytic continuation in the complex n-plane).

Equation (14) permits us to find the corresponding counterparts in the kernel $P^{(1)}(t)$ for most of the terms contained in eqs. (9)-(11). For example, we get

$$\begin{aligned} \frac{1}{C_F} V^{(0)}(x,y) \equiv \{\theta(y-x)\,F(x,y) + \theta(x-y)\,F(\bar{x},\bar{y})\}_+ \to \\ \to \Big(\frac{1+t^2}{1-t}\Big) \;\equiv \frac{1}{C_F} P^{(0)}(t) \end{aligned} \tag{15}$$

$$\Big\{\frac{x}{y}\,\theta(y-x) + \frac{1-x}{1-y}\,\theta(x-y)\Big\}_+ \to (1-t)_+ \tag{16}$$

$$\Big\{\theta(y-x)\,F(x,y)\,\ln\frac{x}{y} + [\begin{smallmatrix} x \to \bar{x} \\ y \to \bar{y} \end{smallmatrix}]\Big\}_+ \to \Big(\frac{1+t^2}{1-t}\ln t + 1 - t\Big)_+. \tag{17}$$

Equations (15)-(17) allow us to obtain the contribution to $P^{(1)}(t)$ proportional to $N_F C_F$ and to establish that it coincides with the result presented in ref. /7/. However, up to now we did not succeed in convoluting in an analogous way some of the terms contained in the function $G(x,y)$. Therefore, we limited ourselves to the consideration of the sum $V_G^{(1)}(x,y) + V_F^{(1)}(x,y)$ (free of G); we have shown that after applying eq. (14) to this function, we exactly arrive at the sum $P_G(t) + P_F(t)$ of Curci et al. /7/.

4. PECULIARITIES OF THE LIGHTLIKE GAUGE

Performing the calculation of $V^{(1)}(x,y)$ we have been confronted with some new peculiarities of the lightlike gauge. It is well-known /13,14/ that the "axial denominator" $1/\ell \cdot n$ of the gluon propagator

$$D^{ab}_{\mu\nu}(\ell) = \frac{i}{\ell^2}\delta^{ab}\left[-g_{\mu\nu} + \frac{\ell_\mu n_\nu + \ell_\mu n_\nu}{\ell \cdot n}\right] \tag{18}$$

gives rise to extra divergences, both of ultra-violet and of infrared type. In the framework of ref. /7/ the former ones must be subtracted "by hand" whereas the latter ones should cancel among themselves after adding up all diagrams of the given order in α_s. Moreover, the presence of the $1/\ell \cdot n$-term in eq. (18) breaks the multiplicative renormalization of the propagators and vertex functions of the theory. In general, this leads to a spoiling of the usual structure of the R-operation. To our knowledge, however, in all calculations done so far the R-operation seems to work as usual. For example, in the two-loop calculation for deep inelastic scattering all terms breaking the multiplicative renormalization of subdiagrams can be converted into some effective renormalization constants /7/. We found that for some of the diagrams contributing to $V^{(1)}(x,y)$ this is already not the case: in the results for the diagrams c) and d) of fig.2 besides terms preserving the structure of the R-operation there remain some additional terms even after integration over the second loop is performed.

Furthermore, we found a second kind of unexpected spurious objects: The diagrams a) and b) (see fig.2) turn out to contain ultra-violet double pole terms. This indicates that in the light-like gauge, in general, even diagrams (subdiagrams) with more than three external legs are UV-divergent. Again, there are no effects of this kind in deep-inelastic scattering (at least, at the two-loop level). Probably, this is due to a greater symmetry of the corresponding diagrams in that case. It can be considered as a further independent check of our calculations that, after summing up all diagrams of fig.2, all these spurious pole terms cancel among themselves. Nevertheless, in view of the mentioned peculiarities it seems worth calculating $V^{(1)}(x,y)$ in a more regular (say, Feynman) gauge, too.

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Диттес Ф.-М., Радюшкин А.В. E2-83-666

Двухпетлевой вклад в ядро эволюции волновой функции пиона

Приведены результаты расчета фейнмановских диаграмм, дающих вклад в двухпетлевую поправку к эволюционному ядру, описывающему ренормализационные свойства волновой функции пиона в пертурбативной квантовой хромодинамике. Расчет проведен с помощью метода, основанного на использовании светоподобной аксиальной калибровки и размерной регуляризации как ультрафиолетовых, так и инфракрасных расходимостей. Описана процедура проверки результатов расчета путем сведения их к соответствующим вкладам в ядро Алтарелли-Паризи. Выявлены новые типы расходимостей, свойственные светоподобной калибровке.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1983

Dittes F.-M., Radyushkin A.V. E2-83-666

Two-Loop Contribution to the Evolution of the Pion Wave Function

The results are presented of the calculation of two-loop Feynman diagrams contributing into the evolution kernel describing the renormalization properties of the pion wave function in perturbative QCD. The calculation was performed with the help of the method based on the use of the lightlike axial gauge and dimensional regularization both for ultraviolet and infrared divergencies. The checking procedure is described basing on the reduction of the results to corresponding contributions into the Altarelli-Parisi kernel. New types of divergences inherent to lightlike gauge are revealed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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