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CLUSTERIZATION IN A CLASS OF MODELS OF THE CLASSICAL FIELD THEORY

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Recently it was understood that various physical phenomena are most consistently described only in the framework of nonlinear theories (models). It covers models of condensed matter physics as well as nuclear and particle physics.

Linear theories are used to be constructed with the help of the Fourier transformation as the main investigation tool so that structural units are quanta of fundamental fields (oscillators). Nonlinear theories demand a new approach: the investigation tool for some of them comes to be "nonlinear Fourier transformation" (or inverse method IM) and structural units are, besides the fundamental field quanta, their peculiar bound states, i.e., solitons.

The most consistent and complete nonlinear theory is constructed in two-dimension space-time, where a great body of results was obtained analytically, especially for integrable systems. In more space dimensions $(D = 2)$ for rare exceptions only existence and stability problems can be studied analytically. Soliton dynamics, i.e., processes of their formation and interaction, remains yet the area of numerical experiments.

CONVENTIONAL RESEARCH PROGRAM

1) Studying the existence problem of soliton-like or particle-like solutions (SLS or PLS respectively).
2) SLS stability.
3) Formfactors of solitons. (1)
4) Dynamics of solitons and their bound states.

A great number of SLS bearing models possesses many common properties. We demonstrate them using as an example the simplest classical field theory with the following Lagrangian:

$$\mathfrak{L} = \mathfrak{L}_d + U\{(\bar{\phi}\phi)\}, \qquad (2)$$

with $\mathfrak{L}_d$ being the "differential" part of the Lagrangian, viz. $\bar{\phi}_\mu \phi^\mu$ in the relativistic case and $i(\bar{\phi}\phi_t - \bar{\phi}_x\phi) + \bar{\phi}_x \phi^x$ in the nonrelativistic one, and

$$U \to \epsilon[m^2(\bar{\phi}\phi) - g^2(\bar{\phi}\phi)^2 + ...]$$

when $\phi \to 0$. Here ϕ is a scalar or vector field, $\bar{\phi} = \phi^+ \gamma_0$, γ_0 is a isospace metric and $R_{D,1}$ is a configuration space.

Theory includes two classes of models:

$\epsilon = \pm 1$.

<u>The first class ($\epsilon = 1$).</u> Stable vacuum is a trivial one $\phi_v \equiv 0$ (symmetry is not broken). There are following physical models: "easy plane" ferromagnets; a nuclear model ($\phi^4 - \alpha\phi^6$ theory); α - helical molecules in biophysics and so on. Mathematical models of this class are, e.g., ϕ^4_+ and SG theories. SLS are elementary excitations over the trivial vacuum.

<u>The second class ($\epsilon = -1$).</u> Stable vacuum is nontrivial, viz. $\phi_v = \mathrm{const}$ (symmetry is spontaneously broken at $T < T_{cr}$). Such a vacuum in a number of models describes a condensate at $T = 0$. Physical models usually are "easy axis" ferromagnets. Those of structural phase transitions, superconductivity (after Ginzburg-Landau), hadron phenomenology (after T.D.Lee and coauthors) and so on. The above nonrelativistic nuclear model ($\phi^4 - \alpha\phi^6$) admitting stable nonzero vacuum (nuclear matter) is embedded in this class, too. Elementary excitations are Bogolubov spectra and soliton modes usually of the hole type. Mathematical models contain, for example, the ϕ^4 -theory and models with noncompact groups of internal symmetry.

DEFINITION

Soliton is a different from vacuum field configuration of finite energy localized in space.

Programm (1) may be fulfilled quite completely for various models of theory (2). Soliton existence and stability problems are studied in the $R_{D,1}$ space-time [1,2]. Small amplitude expansions have been obtained for certain models allowing us to calculate the form factors of solitons and their bound states - bions (pulsons) [3]. Dynamical structure factors (DSF) of the system studied are obtained [3,4] in the ideal dilute soliton gas approximation. The dynamics of solitons has been investigated via numerical experiments for a number of models [5]. For integrable models of the first class (SG and NLS with a compact isogroup) the inverse method is well developed giving a chance to study the system dynamics at $T = 0$ in detail.

Soliton statistics at small $T \neq 0$ is learned rather completely in the framework of only the SG and ϕ^4 theories via the Feynmann integral technics as well as the phenomenological approach of Krumhansl and Schrieffer [7]. The latter gives good results at small T and was used in papers [8] in deriving DSF for the SG, NLS and LL models.

Объединенный институт ядерных исследований БИБЛИОТЕКА

In the case of the second class of models, even integrable, the inverse method is developed for only NLS with the U(0,1) compact isogroup. For higher symmetry groups both compact and noncompact it faces serious mathematical problems related to the presence of the condensate in the system (analyticity of the Jost functions)[9].

I. Nevertheless, upon using the small amplitude expansions derived in works[10,11] one can prove the following general theorem of soliton stability[3]:

1) In the $R_{1,1}$ space-time SLS (if exist) are stable for infinitesimal amplitudes and hence masses M_s, their size R_s varies inversely with the mass: $R_s \sim M_s^{-1}$.

2) In $R_{D,1}$ $(D \geq 2)$ SLS (if exist) are stable in only a certain mass region necessarily bounded below, viz. $\mu < M_s < M$ (this region may be an empty set, too) and $\mu \neq 0$, M varies in the interval $[\mu, \infty]$. The values of μ and M depend on the type of the model and even on its parameters; μ grows usually with D. It is important that μ can be considerably less than the mass of the t'Hooft-Polyakov monopole.

II. The next conclusion follows from the analysis of computer experiments on the soliton dynamics and of results obtained via IM on studying integrable models as well. In the system effected by a localized perturbation of greater than μ energy there arize soliton like objects (see, e.g.,[5,11,12]). Bound states of solitons were also found, in particular, a stable bound state from two absolutely unstable quasisolitons[13].

III. The form factors for all known solitons (kinks and antikinks and bions-breathers) have been calculated in the framework of the SG, ϕ^4 and LL models (see[14]) and upon using the Feynmann approach $Z = \int D\dot{\phi} D\phi \exp(-\beta H)$, $\beta = T^{-1}$ or the dilute soliton gas approximations their DSFs were found. The form of DSF, i.e., its central peak and satellites, gives information about the properties of soliton-like excitations occurring in the system at small temperatures. The position of the DSF satellites reflecting the internal soliton structure varies with T: they move towards the central peak with temperature growing[3]. Since in the $D > 2$ case there is a low mass limit μ for stable solitons to exist the central peak and satellites different from those of linear waves appear at finite temperatures, $T > \mu$ that may be regarded as a phase transition in the system with respect to clusterization.

IV. Is an experimental test of the results discussed possible?

1) High energy physics: monopoles, instantons, gluonium... At the moment either we do not see them (can't identify) or they correspond to higher energies. However, according to Gell-Mann the list of particles must contain "Goldstone bosons and fermions,... solitons (such as magnetic monopoles) or other particle-like solutions and bound states involving them... We must understood to what extent all these secondary objects can masquerade at present energies as elementary particles[15].

2) Nuclear physics. Clustering in nuclei: by now, on the basis of the nonrelativistic $\phi^4 - \alpha\phi^6$ model SLS have been obtained for both the problems, i.e., with zero and condensate boundary conditions. In the first case they simulate the nuclei, in the second one they model internal excitations in the medium and heavy nuclei. The latter is of two types: usual hole ones (rarefaction) and droplet ones (compression). In $R_{1,1}$ Barashenkov and the author have obtained their analytical expressions.

Note that the hole-like excitations can be responsible for a breakup of the compound nucleus produced in the heavy ion interactions. In the framework of this and some other models via numerical experiments on the nuclear (soliton) interaction there have been firstly observed "fission windows" in angular momentum (or that just the same in impact parameter) and nuclear molecules discovered in nature experiment later (see, e.g.,[16] and[13]).

Precision computer experiments on the fission windows can shed a new light on the possibility of the production of superheavy nuclei in the heavy-ion collisions.

3) Low energy physics. Ferromagnets (e.g., $CsNiF_3$). DSF have beem measured[17], the kink contributions to CP have been calculated[4] and also the bion contributions to CP and satellites[3]. In the latter paper the satellites and their positions have been obtained as functions of T. There is an obvious qualitative agreement between the experimental data and the theoretical calculations. The comparison of quantitative results comes to turn.

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Кластеризация в классе моделей классической теории поля

На примере простой нелинейной теории поля рассматриваются вопросы существования, устойчивости, динамики и статистики солитонов в пространстве $R_{D,1}$ с U(p,q) группой внутренней симметрии. Обсуждается ряд математических и физических моделей, входящих в этот класс, в частности SG, ϕ^4, НУШ, калибровочно-эквивалентная последней модель Ландау-Лифшица /континуальный магнетик Гейзенберга/ и ядерная модель $\phi^4 - \alpha\phi^6$. Показано, что наличие малоамплитудных разложений позволяет вычислить формфакторы солитонов не только в рамках точно решаемых моделей SG, НУШ и ЛЛ в пространстве $R_{1,1}$, но и для значительно более широкого класса моделей в $R_{D,1}$. Свойства устойчивости солитонов приводят к тому, что кластеризация /образование солитонов/ в рамках рассматриваемых неодномерных моделей происходит при конечной температуре и может рассматриваться как фазовый переход.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

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Clusterization in a Class of Models of the Classical Field Theory

Using as an example simple nonlinear field theory problems of soliton existence, stability, dynamics, and statistics are discussed for the U(p,q) internal symmetry group in the space-time $R_{D,1}$. A number of mathematical and physical models belonging to this class are considered, in particular, the SG, phi-four theory, a nuclear model $\phi^4 - \alpha\phi^6$, the Nonlinear Schrödinger Equation (NLS), and the Landau-Lifshitz model gauge equivalent to the last one. Existence of small amplitude expansions is shown to allow calculating the soliton formfactors for not only solvable models such as SG, NLS and LL in $R_{1,1}$ space but for a broader class of models in $R_{D,1}$ too. Soliton stability properties imply clustering (soliton creation) to occur at finite temperature in the framework of non-one-space dimensional models considered and may be regarded as a phase transition.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

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