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ON MATTER COUPLINGS IN N=1 SUPERGRAVITIES

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1. Introduction

There are several versions of N=1 supergravity: minimal [1] (12+12 fields), non-minimal [2,3] (20+20 fields), new minimal [4] (12+12) and, finally, the flexible (28+28) one which we propose in this paper (see also [5]). They differ in their auxiliary field sets as well as in their intrinsic geometries [5]. In the absence of matter the auxiliary fields vanish on -shell, and it is clear that all the versions are equivalent. In the presence of matter, however, the auxiliary fields are expressed in terms of the matter fields in various ways. Does this lead to different effective interactions? Are there essentially different mechanisms of supersymmetry breaking in those versions of supergravity?

These questions are intensively discussed [6-8], now in connection with the possible applications of N=1 supergravity in the phenomenological models of grand unification (see [9] and references therein).

The present paper is also devoted to this problem. The geometric approach [5,10] (see also [3]) developed earlier allows one to discuss and compare the different versions of N=1 supergravity on a common basis. The main results of the paper are listed here.

i) The most restrictive version of N=1 supergravity is the new minimal one. It demands R-symmetry in matter couplings because of its local U(1) invariance (or, equivalently, because of a rigid constraint on the supergravity prepotentials [5]). We propose here a new version with 28+28 fields which we shall refer to as the flexible one. It is obtained by relaxing the new minimal one. In other words, a Lagrange multiplier is introduced producing the above-mentioned constraint for the new minimal version when there is no matter [5]. In the presence of matter, however, the Lagrange multiplier appears in the matter sector as well. This leads to a modified ("self-adjusting") constraint the form of which is influenced by matter couplings. This explains why the flexible version allows the same types of matter couplings as the minimal and non-minimal ones. Notice that the Lagrange multiplier introduced is at the same time a gauge compensator for the local U(1) symmetry of the new minimal version. This is another explanation of the increased versatility of the flexible version.

ii) It has recently been shown that the old and new minimal and the non-minimal versions are mutually equivalent for R-symmetric matter Lagrangians[6]. However, the importance of R-noninvariant matter couplings is evident because the R-symmetry can be broken by anomalies in the quantum case. At the same time there is a common belief [6,11,12] that in the non-minimal supergravity R-non-invariant matter couplings are impossible because of the absence of a proper chiral density for the superpotentials despite such a density has been mentioned several years ago[3]. In the present paper we rederive a density of this type and obtain the corresponding superfield equations of motion for a general R-non-invariant superpotential. The consistency of these equations is proved. The auxiliary fields do not propagate and are expressible as combinations of matter fields in a Lorentz-invariant manner. So, the non-minimal version is not more restrictive in matter couplings than the minimal one.

iii) In the flexible supergravity a similar chiral density can be constructed with the help of the Lagrange multiplier (just therefore the matter fields become involved in the constraint mentioned above). Therefore the flexible version also admits general R-non--invariant matter couplings.

It is wrothwhile to make the following comment concerning both ii) and iii) statements. The chiral densities obtained have some superfield in the denominator. Then one must have a non-vanishing supercosmological term. At the same time the cosmological terms in X-space can vanish because it obtains also a contribution opposite in sign from a spontaneous supersymmetry breaking term in the matter sector [13].

iv) It has been shown[14,15] that the inclusion of the Fayet-Iliopoulos term in supergravity in a certain way implies R-invariance. Here we propose another possible way of incorporating the FI-term with its local U(1) invariance but without R-invariance. This can be done in the flexible and non-minimal supergravity but not in minimal and new minimal. Careful checks are still required to make sure the construction suggested does not lead to difficulties.

The paper is planned as follows. In section II we recall some basic elements of the geometric approach to supergravity[10,5]. Then we introduce the very useful concept of "building blocks". There are some simple determinants of derivatives of the supergravity prepotentials having homogeneous transformation laws. Playing with the

blocks one can easily construct densities, actions, etc.,avoiding the use of the elaborate machinery of differential geometry. Section III examines the various types of matter terms in the different supergravity versions. In particular, the chiral densities for the non-minimal and flexible supergravity are discussed, and the new FI-term is described. Section IV is devoted to a detailed examination of the superfield equations of motion for non-minimal supergravity coupled to general non-R-invariant matter. Finally, section V contains a summary.

II. Geometry, "Blocks", and Action Formulas

II.A. Geometry of N=1 supergravity

Here we shall briefly recall some necessary information about the geometric formulations of the various N=1 supergravity models. More details can be found in Refs.[5,10].

Consider the complex superspace

$$\mathbb{C}^{4|4} = \{ Z_L^M = (x_L^m, \theta_L^\mu, \bar\varphi_L^{\dot\mu}) \} \quad \text{or}$$
$$= \{ Z_R^M = (x_R^m, \bar\theta_R^{\dot\mu}, \varphi_R^\mu) \} ; \qquad Z_R^M = (Z_L^M)^\dagger . \tag{2.1}$$

Here Z_L^M and Z_R^M are left- or right-handed parametrizations of $\mathbb{C}^{4|4}$. Further, introduce the following "triangular" gauge group:

$$\begin{aligned} \delta x_L^m &= \lambda^m(x_L, \theta_L), \\ \delta \theta_L^\mu &= \lambda^\mu(x_L, \theta_L), \\ \delta \bar\varphi_L^{\dot\mu} &= \bar\rho^{\dot\mu}(x_L, \theta_L, \bar\varphi_L) . \end{aligned} \tag{2.2}$$

It leaves invariant the "chiral" superspace

$$\mathbb{C}^{4|4} / \mathbb{C}^{\bar\varphi} = \mathbb{C}^{4|2} ,$$

$$\mathbb{C}^{4|2} = \{ \zeta_L^M = (x_L^m, \theta_L^\mu) \} \quad \text{or}$$
$$= \{ \zeta_R^M = (x_R^m, \bar\theta_R^{\dot\mu}) \} ; \qquad \zeta_R^M = (\zeta_L^M)^\dagger . \tag{2.3}$$

The real (physical) superspace is defined as a hypersurface in $\mathbb{C}^{4|4}$:

$$\mathbb{R}^{4|4} = \{ Z^M = (x^m, \theta^\mu, \bar\theta^{\dot\mu}) \} , \tag{2.4}$$

where

$$
\begin{aligned}
&X^{m} = \mathrm{Re}\, X_L^{m}, \quad \theta^{\mu} = \theta_L^{\mu}, \quad \bar\theta^{\dot\mu} = \bar\theta_R^{\dot\mu};\\
&\mathrm{Im}\, X_L^{m} = H^{m}(x,\theta,\bar\theta),\\
&\varphi_R^{\mu} - \theta_L^{\mu} = H^{\mu}(x,\theta,\bar\theta),\\
&\bar\varphi_L^{\dot\mu} - \bar\theta_R^{\dot\mu} = \bar H^{\dot\mu}(x,\theta,\bar\theta).
\end{aligned}
\tag{2.5}
$$

The superfunctions H^m, H^μ, $\bar H^{\dot\mu}$ determine the shape of the hypersurface. After imposing a dynamical postulate (action principle) they become the supergravity superfields (prepotentials).

The group (2.2) corresponds to conformal supergravity. Using superdeterminants (Berezinians) one can impose the following relations between the transformations of the volume elements of $C^{4/4}$ and $C^{4/2}$:

$$
\left[Ber\left(\frac{\partial z_L'}{\partial z_L}\right)\right]^{3n+1} = \left[Ber\left(\frac{\partial \zeta_L'}{\partial \zeta_L}\right)\right]^{2n}. \tag{2.6}
$$

These define subgroups of (2.2) corresponding to the various N=1 Einstein supergravities/1-4/. Depending on the value of the Gates-Siegel parameter n/3/ one can distinguish the following three cases:

i) $n=-\frac{1}{3}$. According to (2.6) in this case

$$
Ber\left(\frac{\partial \zeta_L'}{\partial \zeta_L}\right) = 1, \tag{2.7}
$$

i.e. the supervolume of $C^{4/2}$ is preserved. The parameter $\bar\rho^{\dot\mu}$ in (2.2) remains unrestricted, so the prepotentials H^μ, $\bar H^{\dot\mu}$ can be gauged away, and one is left with H^m only. This is the case of minimal supergravity (12+12 fields)/1,3,10/.

ii) $n=0$. Now

$$
Ber\left(\frac{\partial z_L'}{\partial z_L}\right) = 1, \tag{2.8}
$$

i.e. the supervolume of $C^{4/4}$ is preserved. In this case there remains local U(1) invariance in the Wess-Zumino gauge/5,16/. It causes problems when trying to write down an invariant action. The first way to deal with this problem is to impose a constraint on the prepotentials $H^m, H^\mu, \bar H^{\dot\mu}$. It has a clear geometrical meaning. In the case $n=0$ (and only in it) the Berezinian of changing variables from the left- to right-handed parametrization of $C^{4/4}$ (see (2.1), (2.5))

$$
U = Ber\left(\frac{\partial z_L}{\partial z_R}\right) \tag{2.9}
$$

is invariant. Indeed, $d^8 z_L$ and $d^8 z_R$ are invariant (see (2.8)), and on the hypersurface U relates them to each other, $d^8 z_L = U d^8 z_R$. The constraint on U is

$$
U = 1. \tag{2.10}
$$

It reduces the number of fields to 12+12 again and leads to the so-called new minimal supergravity/4/. Notice that the local U(1) invariance remains in this version of the $n=0$ theory.

As shown in Ref./5/, there is an alternative approach to the case $n=0$. The local U(1) invariance can be compensated by introducing a real pseudoscalar compensating superfield $\varphi(x,\theta,\bar\theta)$. It transforms as follows:

$$
\varphi' = \varphi + \frac{i}{2}(\ell - \tau). \tag{2.11}
$$

(The chiral superfunctions-parameters ℓ, τ are defined in (2.18); the first component of $\frac{i}{2}(\ell-\tau)$ is just the U(1) parameter). Thus, the number of fields becomes 28+28 (20+20 in $H^m, H^\mu, \bar H^{\dot\mu}$, 8+8 in φ), no constraints are imposed, no local U(1) invariance is present. This is a new, "flexible" version of the "new minimal" supergravity (or rather, the latter is a truncated version of the former). As we shall see in what follows, this version is more versatile in matter couplings than the new minimal one. In other words, the flexible formulation restores the equal rights of the new minimal version as a member of the family of non-minimal supergravities.

iii) $n \neq -\frac{1}{3}, 0$. In this case we have the 20+20 fields of the so-called non-minimal supergravities/2,3/.

Here we would like to make the following comment. Another widely used scheme of classification of N=1 supergravities is the one based on compensating conformal supergravity with various multiplets/17,3/. Thus, the minimal supergravity corresponds to a chiral compensator, the non-minimal version - to a complex linear one, the new minimal version - to a real linear one. As is shown in/18/, the flexible supergravity also belongs to this scheme with a relaxed linear multiplet/19/ as a compensator.

II.B. Building blocks

The standard way of describing the invariant properties of the curved superspace $R^{4/4}$ is to develop the formalism of differential geometry, i.e. to introduce supervierbeins, connections, covariant derivatives, torsion, and curvature. All this can be done in the scheme described in section II.A./10/. However, for a number of practical purposes, such as writing down actions, one can avoid using the whole machinery. Instead, one can introduce a few objects with simple transformation properties ("blocks") and then construct various quantities out of them.

We begin with defining the basic differential operator in the framework of sect. II.A. It is the spinor derivative [5]

$$\nabla_\alpha \Phi(z) = (1+\Delta H)^{-1}{}_\alpha{}^\beta \Delta_\beta \Phi(z) = \frac{\partial}{\partial \varphi_R^\alpha} \Phi(z) \qquad (2.12)$$

of a scalar superfield $\Phi(z)$ with respect to φ_R^α (in the right-handed parametrization $x_R^m = x^m - iH^m$, $\bar\theta_R^{\dot\mu} = \bar\theta^{\dot\mu}$, $\varphi_R^\mu = \theta^\mu + H^\mu$, see (2.5)). Here

$$\Delta_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\frac{\partial H^m}{\partial\theta^\alpha}(1+i\partial H)^{-1}{}_m{}^n \frac{\partial}{\partial x^n}\,, \qquad (2.13)$$

$$(1+i\partial H)_m{}^n = \delta_m^n + i\partial_m H^n, \quad (1+\Delta H)_\alpha{}^\beta = \delta_\alpha^\beta + \Delta_\alpha H^\beta.$$

It is easy to check that $\nabla_\alpha \Phi$ transforms homogeneously under the group (2.2), (from there on we consider only infinitesimal parameters):

$$\delta(\nabla_\alpha \Phi) = (\nabla_\alpha \Phi)' - \nabla_\alpha \Phi = -(\nabla_\alpha \beta^\beta)\nabla_\beta \Phi. \qquad (2.14)$$

In the case $n = -\frac{1}{3}$ (minimal supergravity), where H^μ and ρ^μ are absent, the role of ∇_α is played by Δ_α (2.13):

$$\delta(\Delta_\alpha \Phi) = -(\Delta_\alpha \lambda^\beta)\Delta_\beta \Phi.$$

Notice the following algebraic properties (most easily proved in the right-handed basis in $R^{4|4}$):

$$\{\nabla_\alpha, \nabla_\beta\} = 0, \quad \{\Delta_\alpha, \Delta_\beta\} = 0. \qquad (2.15)$$

Now we come to the definition of our basic building blocks. Consider the quantities

$$A = \det\left(\tfrac{1}{4}\tilde\sigma_a^{\dot\alpha\alpha}[\Delta_\alpha, \bar\Delta_{\dot\alpha}]H^m\right), \quad A^+ = A \qquad (2.16)$$

$$B = \det(\delta_m^n + i\partial_m H^n), \quad C = \det(\delta_\alpha^\beta + \Delta_\alpha H^\beta).$$

They transform infinitesimally as follows

$$\delta A = (\omega + \bar\omega + \gamma)A,$$

$$\delta B = (\bar\omega + \ell - \gamma)B, \qquad (2.17)$$

$$\delta C = (\omega + \tau - R)C.$$

Here

$$\omega = \Delta_\alpha \lambda^\alpha = \det{}^{-1}(\Delta_\alpha \theta'^\beta) - 1 + O(\lambda^2),$$

$$\gamma = \tfrac{1}{2}\partial_m(\lambda^m + \bar\lambda^m) - \partial_\alpha \lambda^\alpha - \bar\partial_{\dot\alpha}\bar\lambda^{\dot\alpha} = \mathrm{Ber}\left(\frac{\partial z'}{\partial z}\right) - 1 + O(\lambda^2),$$

$$\ell = \partial_m^L \lambda^m - \partial_\mu^L \lambda^\mu = \mathrm{Ber}\left(\frac{\partial z_L'}{\partial z_L}\right) - 1 + O(\lambda^2); \quad \tau = \ell^+ \qquad (2.18)$$

$$L = \partial_m^L \lambda^m - \partial_\mu^L \lambda^\mu - \bar\partial_{\dot\mu}^L \bar\rho^{\dot\mu} = \mathrm{Ber}\left(\frac{\partial z'}{\partial z}\right) - 1 + O(\lambda^2); \quad R = L^+.$$

The Einstein subgroup condition (2.6) becomes in these terms

$$(3n+1)L = 2n\ell. \qquad (2.19)$$

The Berezinians of changes of variables between different bases in $R^{4|4}$ are expressed in terms of the blocks in the following way

$$\mathrm{Ber}\left(\frac{\partial z_L}{\partial z}\right) = B\bar C^{-1}; \quad \mathrm{Ber}\left(\frac{\partial z_R}{\partial z}\right) = \bar B C^{-1}; \qquad (2.20)$$

$$U = \mathrm{Ber}\left(\frac{\partial z_L}{\partial z_R}\right) = B\bar B^{-1} C \bar C^{-1}.$$

We are going to make heavy use of the blocks defined above for constructing various quantities given their transformation properties.

II.C. Action formulas

Now it is very easy to find the general action formula for all n (except $n=0$). It is simply the supervolume [20,13]

$$S = \frac{1}{n\kappa^2}\int d^8 z\, E \qquad (2.21)$$

and it says that our hypersurface is the minimal one. The density must compensate the transformations of the volume element:

$$\delta E = -\gamma E. \qquad (2.22)$$

Such a quantity can easily be built from the blocks (2.17) (taking into account (2.19))

$$E = A^n (B\bar B)^{\frac{n+1}{2}} (C\bar C)^{-\frac{3n+1}{2}}. \qquad (2.23)$$

As can be expected, at $n = -\frac{1}{3}$ the blocks $C, \bar C$ containing disappear.

The formula (2.23) is valid for $n=0$ as well but (2.21) is not an action any more. An indication of this is the dropping out of the block A. The latter is the only one which contains the scalar curvature as a component field. In the case $n=0$ the supergravity action can be written down in one of the following two ways:

i) $n=0$: new minimal supergravity. Taking into account the constraint (2.10), (2.20) and (2.23) one finds

$$E = \mathrm{Ber}\left(\frac{\partial Z_L}{\partial Z}\right) = \mathrm{Ber}\left(\frac{\partial Z_R}{\partial Z}\right). \tag{2.24}$$

Therefore

$$\int d^8 z\, E = \int d^8 z_L \cdot 1 = 0 \quad (\text{if } U=1), \tag{2.25}$$

i.e. the supervolume of $R^{4/4}$ vanishes in this case[15,11], and we cannot use it to write down the action. However, there is an action of the form[11] *)

$$S^{new\,min} = \frac{1}{\varkappa^2}\int d^8 z\, E \ln f. \tag{2.26}$$

It will be invariant if the quantity f transforms according to

$$\delta f = -\frac{1}{2}(\ell+\tau) f \qquad \text{or} \qquad \delta \ln f = -\frac{1}{2}(\ell+\tau). \tag{2.27}$$

Indeed, taking into account $U=1$ we have

$$\delta \int d^8 z\, E \ln f = -\int d^8 z\, E \frac{1}{2}(\ell+\tau) =$$

$$= -\frac{1}{2}\int d^8 z_L\, \ell(\zeta_L) - \frac{1}{2}\int d^8 z_R\, \tau(\zeta_R) \equiv 0.$$

Such a quantity can uniquely be built out of the blocks,

$$f = A^{1/2}(B\bar{B})^{1/4}(C\bar{C})^{-3/4}. \tag{2.28}$$

Notice that the important "kinetic" block A is again present in (2.26).

ii) $n=0$: the flexible supergravity. As we have explained in sect. II.A, in the flexible supergravity we lift the constraint $U=1$. Then the action (2.27) ceases to be invariant. The U(1) gauge compensator (2.11) introduced above is used to restore the invariance. The integral

$$I = \int d^8 z_L (\ln f + i\varphi) \tag{2.29}$$

*) It is worthwhile to note that the action (2.26) can be obtained from Eq. (2.21) by taking the limit $n \to 0$ [21] and imposing the constraint U=1.

is invariant $(\delta(\ln f + i\varphi) = -\ell(\zeta_L))$ and the action becomes[5]

$$S^{flex} = \frac{1}{2\varkappa^2}\int d^8 z\, E\left[U^{1/2}(\ln f + i\varphi) + U^{-1/2}(\ln f - i\varphi)\right]. \tag{2.30}$$

It is important to realize that the compensator φ appears in (2.30) as a Lagrange multiplier. The variation of φ produces the equation of motion

$$U^{1/2} = U^{-1/2} \tag{2.31}$$

which coincides with the constraint (2.10). So, on-shell and in the absence of matter the flexible supergravity is equivalent to the new minimal one. The substantial difference between the new minimal and flexible supergravity becomes clear in the context of matter coupling which is the subject of the next section.

III. Matter Coupling in N=1 Supergravities

There are several kinds of globally supersymmetric matter action terms which are of interest for possible phenomenological applications[22]. They are:

1) The kinetic term for chiral matter:

$$S_{kin} = \int d^8 z\, \Phi\bar{\Phi} \tag{3.1}$$

$$\mathcal{D}_\alpha \bar{\Phi} = \bar{\mathcal{D}}_{\dot\alpha}\Phi = 0, \quad \text{i.e.} \quad \Phi = \Phi(\zeta_L).$$

It can be generalized to the non-linear σ-model-type action[22]

$$S_{kin} = \int d^8 z\, K(\Phi,\bar{\Phi}). \tag{3.2}$$

The latter has the Kähler invariance[23]

$$\delta K(\Phi,\bar{\Phi}) = \tau(\Phi) + \bar{\tau}(\bar{\Phi}) \tag{3.3}$$

owing to the property of the flat superspace integral

$$\int d^8 z\, \tau(\Phi) = \int d^8 z_L\, \tau(\Phi(\zeta_L)) \equiv 0. \tag{3.4}$$

2) The potential term for chiral matter:

$$S_{pot} = \int d^6 \zeta_L\, P(\Phi(\zeta_L)) + \text{H.C.} \tag{3.5}$$

3) The Yang-Mills term

$$S_{Y.M} = \int d^6 \zeta_L\, f^{ij}(\Phi)\, W_i^{\alpha} W_{\alpha j} + \text{H.C.} \tag{3.6}$$

Here W_i^{α} is the (chiral) field-strength of some Yang-Mills gauge prepotential V_i, f^{ij} is some function of the chiral matter superfields Φ, and i, j are indices of the adjoint representation of gauge group.

4) The Fayet-Illiopoulos term

$$S_{FI} = \xi \int d^8 z V + S_{YM}(V), \tag{3.7}$$

where V is the prepotential for an additional $\widetilde{U}(1)$ gauge invariance and ξ measures the scale of supersymmetry breaking. Equation (3.7) is invariant under $\widetilde{U}(1)$ gauge transformations

$$\delta V = i(\Lambda - \bar{\Lambda}), \quad \bar{D}_{\dot{\alpha}} \Lambda = 0 \tag{3.8}$$

as the flat space integral over $\mathbb{R}^{4|4}$ vanishes if the integrand is a chiral superfield (3.4). We would like to emphasize a deep analogy between the Kähler (3.3) and U(1) transformations. This analogy remains valid in supergravity and it will be used below in Sect. III.C.

In the present section we shall generalize the above matter couplings to the case of local supersymmetry (i.e. the curved superspace).

III.A. Kinetic terms and Kähler invariance

We shall begin with an extension of the kinetic term (3.2) for supergravity versions with $n \neq 0$. By analogy with the minimal case $n = -\frac{1}{3}$ /3,14,24/ it can be written down as (the supergravity action itself is also included)

$$S^{cov}_{kin} = \frac{1}{n \varkappa^2} \int d^8 z\, E \exp(n \varkappa^2 K(\Phi, \bar{\Phi})). \tag{3.9}$$

Here Φ, $\bar{\Phi}$ are covariantly chiral superfields

$$\bar{\nabla}_{\dot{\alpha}} \Phi = \nabla_{\alpha} \bar{\Phi} = 0. \tag{3.10}$$

The remarkable peculiarity of the action (3.9) consists in its invariance under Kähler transformations of $K(\Phi, \bar{\Phi})$ keeping exactly the same form (3.3) which they have in the flat superspace. The Kähler transformations

$$\delta K(\Phi, \bar{\Phi}) = \tau(\Phi) + \bar{\tau}(\bar{\Phi}) \tag{3.3'}$$

are accompanied now by the super-Weyl ones. For the latter one has to relax the group condition (2.19). In the presence of matter it has to be replaced by

$$(3n+1) L = 2n \ell - 2n \varkappa^2 \tau(\Phi). \tag{3.11}$$

Taking into account (2.17), (2.23) and (3.11), it is easy to check that

$$\delta E = -[\gamma + n \varkappa^2 \tau(\Phi)] E \tag{3.12}$$

and that the action (3.9) is invariant under transformations (3.3') and (3.12).

However, the kinetic term (3.2) can be generalized also in other ways. One can, e.g., use a straightforward generalization (we include again the supergravity action)

$$\delta S^{cov}_{kin} = \frac{1}{n \varkappa^2} \int d^8 z\, E\, [1 + n \varkappa^2 K(\Phi, \bar{\Phi})].$$

Evidently this action has also the combined Kähler-Weyl invariance. However, the latter comes now in a slightly modified form

$$\delta \left[\frac{1}{n \varkappa^2} \ln \left(1 + n \varkappa^2 K(\Phi, \bar{\Phi})\right) \right] = \tau(\Phi) + \bar{\tau}(\bar{\Phi}). \tag{3.13}$$

A comparison of (3.3') and (3.13) shows that in the first case the Kähler metrics $g = \partial^2 K / \partial\Phi \partial\bar{\Phi}$ coincides with the one in the flat superspace limit while in the second case $g' = \frac{\partial^2}{\partial\Phi\partial\bar{\Phi}} \left[\frac{1}{n \varkappa^2} \ln \left(1 + n \varkappa^2 K(\Phi, \bar{\Phi})\right) \right]$ and this coincidence is absent. Therefore, it is just the action (3.9) that corresponds to the simplest interaction of supergravity with matter fields. (For the minimal version, $n = -\frac{1}{3}$, this fact was mentioned in /14,24/).

Now we shall proceed to the $n = 0$ supergravity versions. The following generalizations of the kinetic term (3.2) preserve the Kähler invariance of the flat case

1) The new minimal version:

$$\delta S^{cov}_{kin} = \int d^8 z\, E\, K(\Phi, \bar{\Phi}). \tag{3.14}$$

There is no need here to combine the Kähler transformations with the Weyl ones. Indeed, in this version $\mathcal{V} = 1$, so

$$\delta S^{cov}_{kin} = \int d^8 z\, E \tau(\Phi) + H.C. = \int d^8 z_L\, \tau(\Phi(z_L)) + H.C. \equiv 0.$$

ii) The flexible supergravity (the supergravity self-interaction is included)

$$S = \frac{1}{2\varkappa^2} \int d^8 z\, E\, \mathcal{V}^{\frac{1}{2}} [\ln f + i\varphi + \varkappa^2 K(\Phi, \bar{\Phi})] + H.C. \tag{3.15}$$

Here the Kähler transformations (3.3) have to be accompanied by the transformations of the Lagrange multiplier

$$\delta_{K\ddot{a}hle}\,\varphi = -i\varkappa^2\left[\tau(\Phi)-\bar\tau(\bar\Phi)\right]. \qquad (3.16)$$

Then the action (3.15) is invariant under (3.3') together with (3.16):

$$\delta S' = \frac{1}{2\varkappa^2}\int d^8 z\, E\, U^{\frac12}\,\ell\,\varkappa^2\,\tau(\Phi) + H.C. =$$

$$= \int d^8 z_L\, \tau\left(\Phi(z_L)\right) + H.C. \equiv 0.$$

Therefore for all the supergravity versions one can arrange such an interaction with non-linear σ-models (3.2) which preserves the Kähler invariance of the latter.

III.B. Chiral densities and superpotentials

The superpotential term (3.5) requires special care in a supergravity background. In general, one has to insert a density $\mathcal{D}$ in (3.5)*):

$$S^{L.cov}_{pot} = \int d^6 z_L\, \mathcal{D}\, P(\Phi) + H.C. \qquad (3.17)$$

It must transform as follows

$$\delta\mathcal{D} = -\ell\mathcal{D} \qquad (3.18)$$

to compensate for the transformation of the volume element of $\mathbb{C}^{4/2}$ (see (2.18)). Further, $\mathcal{D}$ must be chiral, $\bar\nabla_{\dot\alpha}\mathcal{D}=0$, and must have flat limit 1 (because (3.5) has to be recovered at $\varkappa\to 0$)

$$\lim_{\varkappa\to 0}\mathcal{D} = 1. \qquad (3.19)$$

If such a density is not available, the potential $P(\Phi)$ and, respectively, Φ must be densities themselves. This means that only monomials in Φ are allowed:

$$P(\Phi)\sim\Phi^q, \quad \delta\Phi = -\frac{1}{q}\,\ell\,\Phi. \qquad (3.20)$$

This in turn implies that $K(\Phi,\bar\Phi)$ in (3.10) cannot be an arbitrary function of $\Phi,\bar\Phi$. So, the lack of chiral densities leads to severe restrictions in the matter sector /6,7,12,14/.

One of the main purposes of this paper is to show that such densities exist for most of N=1 supergravities (except the new minimal one). Now we proceed to a separate discussion of each case.

*) To make contact with the language used in other papers we would say that the necessity to have a density in (3.17) corresponds to the assignment of the "Weyl weight 3 in an F - type formula" (see /6/ and references therein).

i) The minimal supergravity. According to (2.7) $d^6 z_L$ is invariant in this case, so no density is in fact required. Therefore the superpotential is given by

$$S^{cov}_{pot} = \int d^6 z_L\, P(\Phi) + H.C. \qquad (3.21)$$

It is instructive to rewrite Eq. (3.22) as an integral over $R^{4|4}$:

$$S^{cov}_{pot} = \frac12\int d^8 z_L\, \bar\varphi_L^2\, P(\Phi) + H.C. =$$

$$= \frac12\int d^8 z\, \bar\theta^2\, \mathrm{Ber}\left(\frac{\partial z_L}{\partial z}\right) P(\Phi) + H.C. \qquad (3.22)$$

($\bar\varphi^{\dot\alpha}_L = \bar\theta^{\dot\alpha}_L$ in the minimal case). Using (2.20) one finds ($C=1$ here)

$$S^{cov}_{pot} = \int d^8 z\, \delta(\bar\theta^2)\det\left(\delta_m^n + i\partial_m H^n\right) P(\Phi) + H.C. \qquad (3.23)$$

Here $\delta(\bar\theta)=\frac12\bar\theta^2$ is the Grassmann δ-function.

Owing to the properties of $\delta(\bar\theta)$ such a representation of the potential term is useful in the supergraph technique. We would like to mention that the variation of (3.23) with respect to the gravitational superfield H^m is easily shown to vanish (the chiral superfield P depends on $x_L^m = x^m + iH^m$). This means that in the minimal case the superpotential does not contribute to the supercurrent defined by the Hilbert prescription:

$$\mathcal{J}_m = \frac{\delta S}{\delta H^m}.$$

This generalizes the analogous statement for the flat superspace /25/.

ii) The non-minimal supergravity. In this case the density in (3.16) is indispensable. It is not hard to check that the following expression can play this role:

$$\mathcal{D} = \frac{\bar\nabla\bar\nabla F}{\bar\nabla\bar\nabla \ln F}. \qquad (3.24)$$

Here F is a quantity with the transformation law

$$\delta F = -\ell F. \qquad (3.25)$$

Correspondingly it can be constructed out of building blocks as follows

$$F = \left[A^{2n} B^{n-1} \bar B^{n+1} C^{-3n-1} \bar C^{-3n+1}\right]^{\frac{3n+1}{4n}}. \qquad (3.26)$$

Taking into account that ℓ is chiral ($\bar\nabla_{\dot\alpha}\ell=0$), and also (2.14) and (2.19) one sees that (3.17) is fulfilled. Obviously, $\mathcal{D}$ is chiral (see (2.15)). Finally, $F=1+O(\varkappa)$, so

$$\lim_{\varkappa\to 0}\mathcal{D} = \lim_{\varkappa\to 0}\frac{\bar\nabla\bar\nabla e^{\ln F}}{\bar\nabla\bar\nabla \ln F} = \lim_{\varkappa\to 0}\frac{\bar\nabla\bar\nabla\left(1+\ln F + O(\varkappa^2)\right)}{\bar\nabla\bar\nabla \ln F} = 1.$$

The integral (3.16) with $\mathcal{D}$ (3.24) can be rewritten in $R^{4|4}$:

$$
\begin{aligned}
S^{cov}_{pot} &= \int d^6 z_L \frac{\bar\nabla\bar\nabla F}{\bar\nabla\bar\nabla \ln F} P + H.C. = \\
&= \int d^8 z_L \frac{F P}{\bar\nabla\bar\nabla \ln F} + H.C. = \int d^8 z \frac{\mathrm{Ber}\left(\frac{\partial z_L}{\partial z}\right) F P}{\bar\nabla\bar\nabla \ln F} + H.C. = \qquad (3.27) \\
&= \int d^8 z \frac{E P}{\bar\nabla\bar\nabla \ln f} F^{\frac{n+1}{3n+1}} + H.C.
\end{aligned}
$$

Here (2.12), (2.20), (2.23), (3.26) are used. With the help of (2.14), (2.19), (3.25) one can see that the quantity

$$
R = F^{-\frac{n+1}{3n+1}} \bar\nabla\bar\nabla \ln f \qquad (3.28)
$$

is a scalar satisfying the modified chirality condition

$$
\left(\bar\nabla_{\dot\alpha} + \frac{n+1}{3n+1} \bar\nabla_{\dot\alpha} \ln f\right) R = 0 . \qquad (3.29)
$$

Then (3.27) becomes

$$
S^{cov}_{pot} = \int d^8 z \frac{E}{R} P . \qquad (3.30)
$$

A similar expression exists in the minimal case, too.

The appearance of the denominator in (3.24) or (3.30) may lead to singularities. To avoid this one has to always use a potential P starting with a constant term (the supercosmological constant *)). Then R will not cause singularities (see (4.6) below).

Although the form (3.30) of the superpotential term has been proposed a long time ago /3/, in the literature doubts have been cast on the existence of arbitrary chiral potentials in the non-minimal supergravity /11,12/. However, the corresponding density does exist. At the same time the expression (3.30) (or (3.24)) is rather complicated and

*) This is compatible with the vanishing of the x-space cosmological constant because the latter obtains also a contribution of the opposite sign from the spontaneous supersymmetry breaking term in the matter sector /13/.

a careful examination is needed to make sure that it leads to meaningful and trouble-free Lagrangians. We are going to do this in section IV using the superfield technique.

iii) The new minimal supergravity. In this case a density of the type (3.24) cannot be constructed. Indeed, playing with the blocks (2.16) one can only build the quantity f (2.26) but not F with the transformation law (3.25). The reason is that if an F (3.25) existed, then fF^{-1} could be used to compensate the local U(1) invariance in the theory: $\delta fF^{-1} = (\tau - \ell) fF^{-1}$, cf. (2.11)). Therefore the new minimal version is the only one not allowing general couplings to chiral matter. The remedy for this is provided by:

iv) The flexible supergravity. In this case, as explained in section II, the necessary U(1) gauge compensator φ (2.11) is introduced. Combining it with the quantity f (2.26) one can construct a quantity f_0 (the naught stands for $n=0$) of the type (3.26):

$$
f_0 = f e^{i\varphi} , \quad \delta f_0 = -\ell f_0 . \qquad (3.31)
$$

Then a density like $\mathcal{D}$ (3.24) can be written down

$$
\mathcal{D}_0 = \frac{\bar\nabla\bar\nabla (f e^{i\varphi})}{\bar\nabla\bar\nabla (\ln f + i\varphi)} . \qquad (3.32)
$$

Putting together the supergravity action, the chiral matter kinetic term (3.15), and the potential term (3.16) one finds the following full action

$$
\begin{aligned}
S &= \frac{1}{2\varkappa^2} \int d^8 z\, E \left[U^{\frac{1}{2}} \left(\ln f + i\varphi + \varkappa^2 K(\Phi, \bar\Phi) \right) + H.C. \right] + \\
&\quad + \left[\int d^6 z_L \frac{\bar\nabla\bar\nabla (f e^{i\varphi})}{\bar\nabla\bar\nabla (\ln f + i\varphi)} P(\Phi) + H.C. \right] . \qquad (3.33)
\end{aligned}
$$

We stress the following important feature of the flexible supergravity. The variation of (3.33) with respect to the compensator φ gives the following equation of motion (we omit the straightforward calculation) :

$$
\frac{1}{2\varkappa^2} \left(U^{\frac{1}{2}} - U^{-\frac{1}{2}} \right) = U^{1/2} f e^{i\varphi} \frac{\bar\nabla_{\dot\alpha} (\ln f + i\varphi) \bar\nabla^{\dot\alpha} (\ln f + i\varphi)}{[\bar\nabla\bar\nabla (\ln f + i\varphi)]^2} P(\Phi) . \qquad (3.34)
$$

Now, we compare this equation with (2.31) obtained in the absence of matter. If the latter leads to the constraint U=1 of the new minimal supergravity, the new one produces a modified constraint in which the matter fields take part. Here is the crucial difference between the new minimal and flexible supergravity. The former employs a rigid constraint regardless of the presence of matter whereas in the latter the constraint is "self-adjusting" to the matter sector (with the help of the Lagrange multiplier φ). This explains the greater versatility of the flexible version in matter couplings.

III.C. Fayet-Iliopoulos term

The covariantization of the Yang-Mills term (3.6) causes no problems, because $W^{\alpha}_{i} W_{\alpha j}$ is the density needed itself.

The FI-term (3.7), however, is not so easy to generalize. In the flat case it possesses the U(1) gauge invariance (3.8). If one insists on preserving this invariance in the supergravity background, one can apply the trick described in section III.A. There Kähler transformations were compensated by super Weyl transformations. We have mentioned above the analogy between Kähler and $\widetilde{U}(1)$ invariances. Consequently, to generalize the FI-term it is sufficient to substitute $K(\Phi,\bar{\Phi})\to \xi V,\ \tau(\Phi)\to i\xi\Lambda(z_L)$ in formulas sect. III.A. E.g. for $n\neq 0$ versions:

$$S_{FI}^{cov}=\frac{1}{n\varkappa^2}\int d^8z\, E\, e^{n\varkappa^2\xi V}+S_{kin}(V) \tag{3.35}$$

while the group restriction (2.19) becomes

$$(3n+1)L=2n\ell-2ni\varkappa^2\xi\Lambda. \tag{3.36}$$

However, in this approach, the problem appears in the superpotential. There V is not present, and the Weyl-transformations of the gravitational superfields remain non-compensated. This implies that the superpotential must be R-invariant /14/.

As is explained in section III.A, in the new minimal supergravity the form of Kähler (the same applies to $\widetilde{U}(1)$) invariance remains unchanged. There, however, the R-invariance of the superpotential is a consequence of the coupling to supergravity itself /4,15/.

Here we are going to propose a completely different mechanism of accomodating the FI-term together with its U(1) invariance but without the requirement of R-invariance in the matter sector. This can be done in the flexible and non-minimal supergravity only. The idea is to use modified $\widetilde{U}(1)$ transformations which do not imply compensating super Weyl transformations. Let us consider the simply covariantized FI term:

$$S_{FI}=\xi\int d^8z\, E\,\vartheta. \tag{3.37}$$

In the non-minimal versions it is invariant under the following U(1) transformations:

$$\delta\vartheta=iE^{-1}\mathrm{Ber}\left(\frac{\partial z_L}{\partial z}\right)\mathcal{D}^{\frac{2n}{3n+1}}\Lambda(z_L)+H.C. \tag{3.38}$$

Really,

$$\delta S_{FI}=\xi\int d^8z_L\,\mathcal{D}^{\frac{2n}{3n+1}}(z_L)\Lambda(z_L)+H.C.=0.$$

The density $\mathcal{D}^{\frac{2n}{3n+1}}$ serves to ensure invariance of the integral (then $\Lambda(z_L)$ has no weight).

Analogously, in the flexible version modified U(1) transformations are

$$\delta\vartheta=i\left[\mathcal{V}^{\frac{1}{2}}\Lambda(z_L)-\mathcal{V}^{-\frac{1}{2}}\bar{\Lambda}(z_R)\right]. \tag{3.39}$$

The question arises: how to construct a gauge invariant kinetic term for ϑ ? Our suggestion is as follows. One should try to define another $\widetilde{U}(1)$ gauge prepotential $V=V(\vartheta)$ having the standard transformation law (3.35) and then construct the standard kinetic term for it. Here we shall give an idea how this can be done in principle (the flexible version (3.39) will be considered).

According to (2.9) $\mathcal{V}\mathcal{V}^{+}=1$, i.e. $\mathcal{V}=e^{2iu}$, $u=u^{+}$. Putting this into (3.39) one gets

$$\delta\left(\vartheta/\cos u\right)=i(\Lambda-\bar{\Lambda})-(\Lambda+\bar{\Lambda})\,\mathrm{tg}\,u. \tag{3.40}$$

From this one obtains

$$\delta\hat{\mathcal{D}}_\alpha\vartheta=\delta\left[\frac{1}{1+i\,\mathrm{tg}\,u}\mathcal{D}_\alpha\left(\frac{\vartheta}{\cos u}\right)\right]=i\mathcal{D}_\alpha\Lambda-(\Lambda+\bar{\Lambda})\frac{\mathcal{D}_\alpha\mathrm{tg}\,u}{1+i\,\mathrm{tg}\,u}. \tag{3.41}$$

Thus one derives a modified derivative $\hat{\mathcal{D}}_\alpha\vartheta$ which transforms with the standard inhomogeneous term $i\mathcal{D}_\alpha\Lambda$. In a similar way one can modify $\bar{\mathcal{D}}_{\dot\alpha}u$.

Further, one applies $\bar{\mathcal{D}}_{\dot\alpha}$ to (3.41) and after some simple algebra obtains

$$\begin{aligned}\delta\left[\widehat{\bar{\mathcal{D}}_{\dot\alpha}\mathcal{D}_\alpha}\vartheta\right]&=\delta\left[\bar{\mathcal{D}}_{\dot\alpha}\hat{\mathcal{D}}_\alpha\vartheta+i\bar{\mathcal{D}}_{\dot\alpha}\vartheta\frac{\mathcal{D}_\alpha\mathrm{tg}\,u}{1+i\,\mathrm{tg}\,u}\right]=\\&=i\bar{\mathcal{D}}_{\dot\alpha}\mathcal{D}_\alpha\Lambda-(\Lambda+\bar{\Lambda})\left[\bar{\mathcal{D}}_{\dot\alpha}\left(\frac{\mathcal{D}_\alpha\mathrm{tg}\,u}{1+i\,\mathrm{tg}\,u}\right)+\frac{i\bar{\mathcal{D}}_{\dot\alpha}\mathrm{tg}\,u\,\mathcal{D}_\alpha\mathrm{tg}\,u}{(1-i\,\mathrm{tg}\,u)(1+i\,\mathrm{tg}\,u)}\right].\end{aligned} \tag{3.42}$$

The same pattern is observed: $\widehat{\bar{\mathcal{D}}_{\dot\alpha}\mathcal{D}_\alpha}\vartheta$ transforms with the usual (for a standard gauge prepotential) term $i\bar{\mathcal{D}}_{\dot\alpha}\mathcal{D}_\alpha\Lambda$. Proceeding in the same way, one finally arrives at the expression $\mathcal{D}^\alpha\bar{\mathcal{D}}\bar{\mathcal{D}}\mathcal{D}_\alpha$. It is a modification of the l.h.s. of the equation of motion for a $\widetilde{U}(1)$ prepotential (the necessary curvature terms are omitted for simplicity).

It transforms as follows

$$\delta(\widehat{\mathcal{D}\bar{\mathcal{D}}\bar{\mathcal{D}}\mathcal{D}\,\vartheta}) = i\mathcal{D}\bar{\mathcal{D}}\bar{\mathcal{D}}\mathcal{D}(\Lambda-\bar{\Lambda}) - (\Lambda+\bar{\Lambda})\,\Sigma(u). \tag{3.43}$$

Here the first term vanishes (the standard equation of motion is gauge invariant), and $\Sigma(u)$ is some complicated function of u. Finally, one is now able to define

$$V = \frac{\vartheta}{\cos u} - \frac{\widehat{\mathcal{D}\bar{\mathcal{D}}\bar{\mathcal{D}}\mathcal{D}\,\vartheta}}{\Sigma(u)}\, tg\, u \tag{3.44}$$

which has the standard transformation law (3.8).

The potentially dangerous point in this construction is the division by $\Sigma(u)$ in (3.44). To avoid singularities one has to make sure that $\Sigma(u)$ does not vanish. This means that some auxiliary field (the first component of Σ) must have non-zero vacuum expectation value. In a theory with spontaneously broken supersymmetry it is plausible, but careful explicit checks are still required.

IV. Consistency Checks for Non-Minimal Supergravity Coupled to Chiral Matter

In this section we check the consistency of the equations of motion for the non-minimal supergravity coupled to matter in the presence of a general R-non-invariant superpotential (3.30) using the chiral density (3.24). No difficulties are found. Our analysis does not cover the density (3.32) which we propose for the flexible supergravity. However, it is rather similar to the one for the non-minimal supergravity (3.24).

We shall restrict ourselves to the case $n=-1$ where the algebra is most simple. For instance, there $\mathcal{D}_\alpha\varphi = \nabla_\alpha\varphi$ (for $n \neq -1$ the factor f (3.26) appears in the r.h.s.). The connection $\omega_{\alpha\beta\gamma}=0$, so $\nabla^\alpha\nabla_\alpha\varphi$ is a covariantly chiral superfield (not a density, as it is for $n\neq -1$), etc.

We are going to examine the full action consisting of the kinetic term (3.9) and the potential term (3.16), (3.24). We begin with varying this action with respect to the gravitational spinor prepotential H^α and the matter superfield Φ. The first equation of motion allows us to establish the values of the supergravity auxiliary fields, the second one- to look at the self-interaction of the chiral matter superfields.

At the beginning we shall drop temporarily the matter kinetic term ($K(\bar{\Phi}\Phi)=0$) and reinstate it later on. Recalling the explicit expressions for E (2.23) and f (3.26) we find the following action

$$S = -\frac{1}{\varkappa^2}\int d^8z\, A^{-1} C\bar{C} +$$

$$+\left[\int d^6 z_L \frac{\bar{\nabla}\bar{\nabla}(A^{-1}B^{-1}C\bar{C}^2)}{\bar{\nabla}\bar{\nabla}\ln(A^{-1}B^{-1}C\bar{C}^2)} P(\Phi) + H.c.\right]. \tag{4.1}$$

The variation of H^α in the first term is very easy. There only C (2.16) depends on H^α:

$$\delta C = C(1+\Delta H)^{-1}{}_\alpha{}^\beta \Delta_\beta\, \delta H^\alpha = C\nabla_\alpha\, \delta H^\alpha;$$

$$\delta \frac{1}{\varkappa^2}\int d^8z\, A^{-1}C\bar{C} = \frac{1}{\varkappa^2}\int d^8z\, A^{-1}C\bar{C}\,\nabla_\alpha\,\delta H^\alpha = \tag{4.2}$$

$$= \frac{1}{\varkappa^2}\int d^8 z_R\, f\,\nabla_\alpha\,\delta H^\alpha = \int d^8 z_R \frac{1}{\varkappa^2}\,\delta H^\alpha\,\nabla_\alpha f = \frac{1}{\varkappa^2}\int d^8z\, E\,\delta H^\alpha \nabla_\alpha \ln f.$$

Here (2.20) was used; the change of basis $z \to z_R$ permits easy integration by parts with $\nabla_\alpha (= \frac{\partial}{\partial\varphi^\alpha_R})$. So, this term gives a contribution $\frac{1}{\varkappa^2}\nabla_\alpha \ln f$ to the equation of motion. The variation of the other two terms in (4.1) proceeds along the same lines. The new element is the restoration of the full superspace integral $d^6 z_L \bar{\nabla}\bar{\nabla} = d^8 z_L \to d^8 z$. After some algebra one arrives at the following equation of motion

$$(\varkappa^{-2}\bar{R} + 2\bar{P} - \bar{R}P\bar{f}^2)t_\alpha = \nabla_\alpha(P\bar{f}^2), \tag{4.3}$$

where

$$\bar{R} = \bar{\nabla}\bar{\nabla}\ln f \tag{4.4}$$

$$t_\alpha = \frac{\nabla_\alpha \ln F}{\nabla\nabla \ln f}; \qquad \nabla_\alpha t_\beta = \frac{1}{2}\varepsilon_{\alpha\beta}. \tag{4.5}$$

Differentiating (4.3) by ∇^α once and then a second time one obtains two equations

$$\varkappa^{-2}\bar{R} + 2\bar{P} - \bar{R}P\bar{f}^2 - \nabla^2(P\bar{f}^2)(1+\bar{R}t^2) = 0 \tag{4.6}$$

$$\nabla_\alpha(P\bar{f}^2) - \nabla^2(P\bar{f}^2)t_\alpha = 0, \tag{4.7}$$

which are equivalent to (4.3). The chiral scalar R contains the scalar curvature, the γ-trace of the gravitino field-strength and some complex scalar auxiliary field. Equation (4.6) expresses R in terms of the (chiral) matter potential (a similar equation exists in the minimal case[20,3]). The spinor t_α contains multiplet of auxiliary fields only. The proof is as follows. From the property (4.5) of t_α it follows that the only non-trivial components of t_α are

$$\varphi_\alpha = t_\alpha|_{\theta=\bar\theta=0}, \quad A_{\alpha\dot\alpha} = \bar{\mathcal{D}}_{\dot\alpha} t_\alpha|_{\theta=\bar\theta=0}, \quad \varkappa_\alpha = \bar{\mathcal{D}}^2 t_\alpha|_{\theta=\bar\theta=0} \tag{4.8}$$

We are interested in a solution which does not break the Lorentz invariance, i.e. in which

$$\varphi_\alpha = A_{\alpha\dot\alpha} = \mathcal{H}_\alpha = 0 \tag{4.9}$$

(only scalars can have a non-vanishing vacuum expectation value). So, we have to decompose eq. (4.7) in terms of components and check the compatibility with the solution (4.9). Taking (4.7) at $\theta=\bar\theta=0$ we get

$$(\nabla_\alpha P)_0 \bar\varphi^2 + 2(P\nabla_\alpha \bar t_{\dot\alpha})_0 \bar\varphi^{\dot\alpha} - (\nabla^2(P\bar t^2))_0 \varphi_\alpha = 0 \tag{4.10}$$

($(\ )_0$ means the value at $\theta=\bar\theta=0$). This equation is algebraic in $\varphi, \bar\varphi$ and is obviously compatible with (4.9). The remaining components of (4.7) are obtained by $\bar{\mathcal{D}}$ -differentiation (the $\mathcal{D}$ -derivative of the l.h.s. of (4.7) vanishes identically, see (4.5)). E.g., applying $\bar{\mathcal{D}}_{\dot\alpha}$ to (4.7) and using $\varphi_\alpha = 0$ we get the equation for an auxiliary complex vector

$$P[\bar A_{\alpha\dot\alpha} - 2\bar A^{\beta\dot\beta}\bar A_{\beta\dot\beta} A_{\alpha\dot\alpha}] = 0. \tag{4.11}$$

This algebraic equation has the Lorentz invariant solution $A_{\alpha\dot\alpha} = 0$ again. So (4.9) is indeed an appropriate solution of (4.7).

Taking (4.9) into account we find that eq. (4.6) is greatly simplified. Its components are

$$\frac{1}{\varkappa^2}(\bar R)_0 + 2(\bar P)_0 = 0,$$

$$\frac{1}{\varkappa^2}(\bar\nabla_{\dot\alpha} \bar R)_0 + 2(\bar\nabla_{\dot\alpha} \bar P)_0 = 0, \tag{4.12}$$

$$\frac{1}{\varkappa^2}(\bar\nabla^2 \bar R)_0 + 2(\bar\nabla^2 \bar P)_0 + (RP)_0 + (\nabla^2 P)_0 = 0$$

and one can see that no potentially dangerous (e.g., higher derivative) terms appear.

Now let us go back to the kinetic term $e^{-K(\varphi,\bar\varphi)}$ (3.12). If we just insert e^{-K} into the first integral in (4.1) we run into trouble. The point is that an inhomogeneous term $\varkappa^2 e^{-K}\nabla_\alpha K$ will appear in the l.h.s. of eq. (4.3). It will enormously complicate the simple analysis we have just carried out. The consistency of the new equation will become obscure if not doubtful. Therefore we propose the following trick. Instead of introducting e^{-K} in the first term in (4.1) only, we substitute

$$A^{-1} \to A^{-1} e^{-K} \tag{4.13}$$

in all the three terms. The modified block A has the same transformation properties as the initial one. Then (4.1) becomes

$$S = -\frac{1}{\varkappa^2}\int d^8z\, E e^{-K} + \left[\int d^6 z_L \frac{\bar\nabla\bar\nabla F e^{-K}}{\bar\nabla\bar\nabla(\ln F - K)} + h.c.\right]. \tag{4.14}$$

It is not hard to see that the block A^{-1} does not participate actively in the derivation of the equation of motion (4.3). Therefore we obtain again the same equation with just one modification: the quantity F in (4.4), (4.5) should be replaced by Fe^{-K}. This will not affect the subsequent analysis. In particular, the same solution (4.9) to eq. (4.7) will be obtained but its meaning will change. Before we had, e.g.,

$$(\nabla_\alpha \ln F)_0 = 0 \tag{4.15}$$

and now it will read

$$(\nabla_\alpha \ln F)_0 - (\nabla_\alpha K)_0 = 0. \tag{4.15'}$$

In other words, (4.15) means that some auxiliary supergravity field vanishes, and (4.15') means that it is expressed in terms of some matter fields. Again, we do not see anything inconsistent in the new equations.

This concludes our discussion of the equation of motion obtained by variation of H^α. Let us now turn to the matter equation. Varying the matter superfield Φ in (4.14) we find the following equation:

$$P' + \bar\nabla^2[K'(\varkappa^{-2} - \bar P t^2 - P\bar t^2)(1 - R\bar t^2)] = 0, \tag{4.16}$$

where

$$P' = \frac{\partial P}{\partial \Phi}, \quad K' = \frac{\partial K}{\partial \Phi}.$$

Taking into account (4.6), (4.7), (4.9) we can write down the components of eq. (4.16):

$$(P')_0 - (PK')_0 + \varkappa^{-2}(\bar\nabla^2 K')_0 = 0,$$

$$(\nabla_\alpha P')_0 - (\nabla_\alpha(PK'))_0 + \varkappa^{-2}(\nabla_\alpha \bar\nabla^2 K')_0 = 0, \tag{4.17}$$

$$(\nabla^2 P')_0 - (\nabla^2(PK'))_0 + \varkappa^{-2}(\nabla^2\bar\nabla^2 K')_0 + (P\bar\nabla^2 K')_0 = 0.$$

Comparing (4.16), (4.17) with the corresponding equation in the minimal case and its components:

$$(P' + \varkappa^{-2}\bar{\mathcal{D}}^2 K')_0 = 0;$$

$$(P')_0 + \varkappa^{-2}(\bar{\mathcal{D}}^2 K')_0 = 0;$$
$$(\mathcal{D}_\alpha P')_0 + \varkappa^{-2}(\mathcal{D}_\alpha \bar{\mathcal{D}}^2 K')_0 = 0; \qquad (4.18)$$
$$(\mathcal{D}^2 P')_0 + \varkappa^{-2}(\mathcal{D}^2 \bar{\mathcal{D}}^2 K')_0 = 0.$$

We can see only insignificant (from the consistency point of view) changes.

We do not examine there the equation of motion for H^{m}. It contains equations for the graviton, gravitino and the second of two axial auxiliary fields.

V. Conclusion

The flexible N=1 supergravity version proposed is an improved modification of the new minimal one. In contrast to the latter it does not imply local U(1) symmetry subjected to anomalies and it admits a wide class of matter couplings.

The first main result of the paper is the derivation of chiral densities appropriate for the integration over chiral $\mathbb{C}^{4/2}$-superspace. Using these densities one can arrange at the classical level R-non-invariant matter couplings in all N=1 supergravity versions except the new minimal one. This fact seems to be rather important because the R-invariance has to be apparently broken at the quantum level by anomalies (like the local U(1) symmetry above).

The second main result is the suggestion of a new kind of the Fayet-Iliopoulos mechanism for spontaneous supersymmetry breaking. This construction does not imply an accompanying super Weyl transformation. It works only in the non-minimal and flexible versions. So these versions will not be equivalent to the minimal one if a future careful analysis will confirm a consistency of the construction conjectured.

It is worthwhile also to reanalyse the auxiliary field anomalies/8/ taking into account chiral densities discussed in the present paper.

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Материальные поля в 1-супергравитациях

Предлагается гибкая версия N=1 супергравитации с 28+28 полями, являющаяся обобщением новой минимальной версии. Обсуждаются взаимодействия материальных полей в различных версиях N=1 супергравитации. Построены киральные плотности для неминимальной и гибкой версий, вследствие чего они допускают общие R-неинвариантные материальные связи аналогично минимальной супергравитации. В отличие от минимальных старой и новой версий в неминимальной и гибкой версиях, по-видимому, возможен видоизмененный механизм Файе-Илиопулоса, не требующий R-инвариантности суперпотенциала.

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On Matter Couplings in N=1 Supergravities

A flexible version of N=1 supergravity is proposed. It contains 28+28 fields and is an extension of the new minimal supergravity version. Matter couplings in various N=1 supergravity versions are discussed. The chiral densities are constructed for non-minimal and flexible versions. Therefore these versions admit a general R-non-invariant matter coupling as the minimal supergravity does. A modified Fayet-Iliopoulos type mechanism is conjectured which apparently can work in the non-mimimal and flexible versions without R-symmetry of the superpotential unlike the minimal and new minimal ones.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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