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ON THE BEHAVIOUR OF INCLUSIVE HADRON CROSS SECTIONS AT LARGE TRANSFERRED MOMENTA AND CERN ISR AND SPS COLLIDER ENERGIES

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As is known, the perturbatuve quantum chromodynamics predicts the cross sections of the processes in which the main contribution comes from the quark-gluon interaction at small distances. The study of the inclusive processes with the production of an emitted particle with a large momentum perpendicular to the collision axis is most convenient. The data of the experiments /1-6/ on measurement of the cross sections*

$$E\frac{d^3\sigma}{dp^3}(AB \to CX)\Big|_{\theta=90^\circ}$$

at the CERN ISR and SPS Collider are collected in table 1.

Table 1

Expt. N	Process AB→CX	$\sqrt{s}$ GeV	p_{min} GeV	p_{max}	$x_{T\,min}$	$x_{T\,max}$	Number of points
Ia	$p\bar{p} \to \pi^0$	540	I.5I	4.42	0.0056	0.0I60	I4
IB	$p\bar{p} \to \pi^0$	540	4.20	II.60	0.0I60	0.0430	4
Io	$pp \to \frac{p+\bar{p}}{2}$	540	0.55	I.35	0.0020	0.0050	9
2	$pp \to \pi^0$	52.7	3.05	II.00	0.II0	0.4200	23
	$pp \to \pi^0$	62.8	3.05	I3.50	0.097	0.43	I6
3	$pp \to p$	53.0	I.00	4.70	0.038	0.I80	II
	$pp \to p$	63.0	I.00	2.30	0.032	0.073	8
4	$pp \to \pi^0$	53.I	3.7I	I2.70	0.I4	0.48	I6
	$pp \to \pi^0$	62.4	3.72	I3.70	0.I2	0.44	2I
5	$pp \to \pi^0$	45.I	I.08	8.02	0.048	0.36	34
	$pp \to \pi^0$	53.2	I.28	7.8I	0.048	0.29	33
	$pp \to \pi^0$	62.7	I.08	6.42	0.034	0.2I	27
6	$pp \to \pi^0$	53.0	5.25	I4.30	0.20	0.50	I5
	$pp \to \pi^0$	63.0	5.25	I4.60	0.I7	0.46	I5
		$45.I \le \sqrt{s} \le 540$	0.55	I4.60	0.06	0.05	256

The aim of this paper is the description of the cross sections of these reactions within the quark counting rules of anomalous dimensions in QCD /7,8/.

The inclusive cross section of hadron production with large p_T in the hard collision /9/ has the form

* A,B.C are hadrons.

$$E\frac{d^3\sigma}{dp^3}(AB \to CX) = \sum_{a,b,c} \int_{x_a^{min}}^{1} dx_a \int_{x_b^{min}}^{1} dx_b\, F_{a/A}(x_a) \times$$

$$\times F_{b/B}(x_b) \int dx_c\, D_{c/C}(x_c)/x_c^2\ \hat{s}/\pi \times$$

$$\times \delta(\hat{s}+\hat{t}+\hat{u})\left(\frac{d\sigma}{dt}\right)_{ab},$$

where $\left(\frac{d\sigma}{dt}\right)_{ab}$ is the cross section of elementary parton subprocesses $a, b = q, \bar{q}, G$, $F_{a/A}(x_a)$ is the distribution function of partons a (b) in the hadron A(B) with momentum $x_{a,b} = \frac{2p_{a,b}}{\sqrt{s}}$, The variables $\hat{s}$, $\hat{t}$ and $\hat{u}$ are related with the Mandelstam variables

$$s = (p_A + p_c)^2,$$

$$t = (p_A - p_C)^2,$$

$$u = (p_B - p_C)^2,$$

in the following way in the case of jet production have the form

$$\hat{s} \simeq x_a x_b s,$$

$$\hat{t} \simeq x_a t/z,$$

$$\hat{u} \simeq x_b u/z.$$

where

$$z = \frac{x_1}{x_a} + \frac{x_2}{x_b}, \quad x_1 = \frac{t}{s}, \quad x_2 = \frac{u}{s},$$

and

$$x_a^{min} = \frac{x_1}{1-x_2}, \quad x_b^{min} = \frac{x_a x_2}{x_a - x_1}.$$

By using the quark counting rules the asymptotic formulae have been calculated /8/ for the cross sections in the leading logarithmic approximation of QCD at large x_T and $\theta = 90^\circ$

$$E\frac{d^2\sigma}{dp^3}(AB\to CX)\Big|_{\theta=90^\circ}=\left(\frac{\alpha_s}{p_T^2}\right)^2\left(\frac{(1-x)^{n-1}}{(n-1)!}\,\alpha_s^{-2r\ln 2x+hd(n,x)}\right), \qquad (1)$$

where $\alpha_s=\dfrac{12\pi}{(33-2n_f)\ln(Q^2/\Lambda^2)}$, n_f is the number of quark flavours, Λ is the quantumchromodynamic scale, $r=\dfrac{16}{33-2n_f}$ and h is the number of hadrons.

The function $d(n,x)$ is

$$d(n,x)=-r\left[\frac{3}{4}+\frac{2}{n(n+1)}-\sum_{i=1}^{n}\frac{1}{i}+\left(\frac{1}{n}+\ln(1-x)\right)\right],$$

where n is the twice of the number of noninteracting quarks $-n=2\sum_i(n_i-1)$ (n_i is the number of quarks in the hadron i).

The approach[7,8] uses essentially the solutions of the evolution equations[10] for the distribution functions and for the quark and gluon fragmentation with boundary conditions imposed by the quark counting rules[11]

$$F(x)=F(x,\ Q^2=Q_0^2)\ .$$

By representing $\sigma(p_T,s)\equiv E\dfrac{d^3\sigma}{dp^3}$ in the form of $\sigma(p_T,s)-\sigma\left(\dfrac{p_T}{p_0}\right)^{-n_{eff}}$

and using the identity

$$\left(\frac{2p_{T_1}}{\sqrt{s_1}}\right)^{-n_{eff}}\equiv\left(\frac{2p_T}{\sqrt{s_2}}\right)^{-n_{eff}}\equiv x_T^{-n_{eff}}\ ,$$

one can define n_{eff} by

$$n_{eff}(x_T,s)=\lim_{s_2\to s_1\to s}\frac{\ln[\sigma(x,s_1)/\sigma(x,s_2)]}{\ln(s_2/s_1)}.$$

Substituting into this formula the cross section (1), we get

$$n_{eff}(x_T,s)=4-2[2-2r\ln 2x_T+hd(n,x_T)]\ \ln(Q^2/\Lambda^2)\ .$$

For the range $x_T\geq 2$ we have

$$n_{eff}(x_T,s)=4-2[2-2r\ln 2x_T+hd(n,x_T)+a\ln x_T+b]\,\frac{1}{\ln(Q^2/\Lambda^2)}\ ,$$

where a and b are numerical parameters which can be estimated. It what follows we shall find their values solving the overdetermined algebraic system

$$\sigma^{expt}(x_i,s_j)=\sigma^{th}(x_i,s_j,A)\ , \qquad (3)$$

where

$$\sigma^{th}(x_i,s_j,A)=cp^{-4}(p/p_0)^{-(n_{eff}-4)}\ , \qquad (4)$$

by the method of Gauss-Newton autoregularized iteration processes[12] (the COMPIL program in the JINR library of standard programs for the CDC 6500 computer -C401,F421). In this case the expression

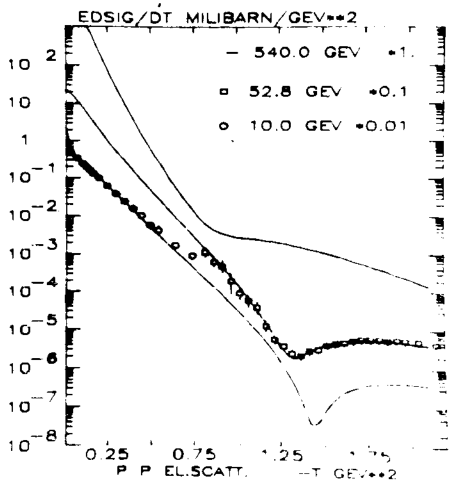
$$\chi^2=\sum_i\frac{(\sigma^{expt}-\sigma^{th})^2}{\Delta^2}$$

is minimized, where Δ is the measurement error of σ^{expt}. It has been found that (4) describes well the data at $x_T>0{,}2$ and $\sqrt{s}\geq 40$ GeV. If $\Delta=\Delta^{stat.}+\Delta^{syst.}$, $\Delta^{syst.}=\sigma^{expt}/10$, then χ^2/DF 1.04. This can be seen from table 2 and fig.1, and

Table 2

$x\geq 0{,}2$

Expt. №		Number of points M_i	χ^2/M_i	Normalization coefficients
2	52.7	7	3.34/7	I.27 ± 0.I2
	62.8	8	5.08/8	I.32 ± 0.0II
4	53.I	I2	2.I5/2	I.II ±0.09
	62.5	I5	I2.4 /I5	0.95 ± 0.08
5	53.2	I3	20.6/I3	I.II ± 0.09
	45.I	I7	29.3/I7	0.72 ± 0.08
6	53.0	I4	I2.2/I4	0.94 ± 0.09
	63.0	I2	I3.3/I2	I.I8 ± 0.I0



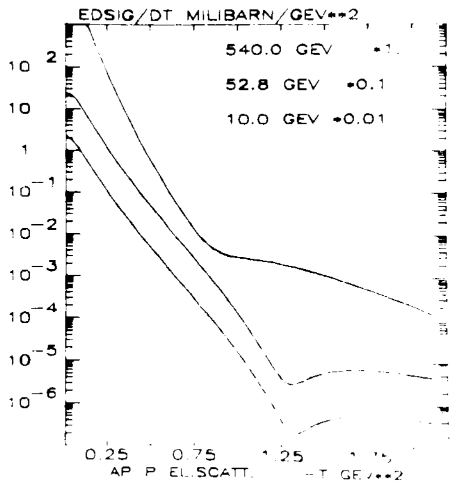


Fig.3a. Description[15] of the differential cross sections of the elastic pp-scattering at $\sqrt{s} = 10.0$, and 52.8 GeV. The upper curve is the prediction for the behaviour of $\frac{d\sigma}{dt}(s,t)$ of the elastic pp scattering at $\sqrt{s} = 540$ GeV. The values of $\frac{d\sigma}{dt}(s,t)$ for each curve (beginning with the upper one) are multipplied by $10^{(-n+1)}$, $n = 1,2,3$.

Fig.3b. Prediction made in ref.[15] for the behaviour of the elastic $p\bar{p}$-scattering at $\sqrt{s} = 10$, 52.8 and 540 GeV. The values of $\frac{d\sigma}{dt}(s,t)$ for each curve (beginning with the upper one) are multiplied by $10^{(-n+1)}$, $n=1,2,3$.

Let us illustrate the aforesaid by a simple example. Let us consider the function

$$f(p) = e^{-bm\chi} \equiv e^{-bm\ln\left(\sqrt{1+\frac{p^2}{m^2}}+\frac{p}{m}\right)} \equiv$$

$$\equiv \frac{1}{\left(\sqrt{1+\frac{p^2}{m^2}}+\frac{p}{m}\right)^{bm}} .$$

We evidently have

$$f(p) = \begin{cases} e^{-bp} & \text{at} \quad \frac{p}{m} \ll 1 , \\ \left(\frac{p}{m}\right)^{-bm} & \text{at} \quad \frac{p}{m} \gg 1 . \end{cases}$$

Note, that in ref.[15] it has been found that $m = \text{const}\,R(s)$, where

$$\sigma_t(s) = 2\pi R^2(s) .$$

Thus, we shall consider

$$\sigma(p_T, s) = a\exp(-bm\chi(p,m))\, n_{eff}(x_T, s) ,$$
$$n_{eff}(x_T, s) = 4 - 2[2 - 2r\ln 2x + hd(n,x) + c\ln x]\,/\ln(Q^2/\Lambda^2), \qquad (6)$$

where

$$m = m_0\, n/\ln(s/\Lambda^2) .$$

The unknown parameters are estimated by solving the system (5). It has been found that the solutions describing well the experimental data of table 1 -

$$\chi^2/DF = \frac{187.2}{256-5} = 0.75$$

are valid at*

Λ = 0.05 GeV,

m_0 = 2.95±0.06 GeV,

b = 0.96±0.02 GeV^{-1},

c = 3.22±0.07

a = 4.45±0.65 mb/GeV^2,

that is seen fron table 3 and figs.4-9.

*See footnote in page 4.

Table 3

Expt. №	s GeV	Number of points μ_r	χ^2/μ.	Normalization coefficients
Ia	540	I4	5.I6/I4	0.95±0.07
IB	540	4	3.32/4	0.42 ± 0.I4
Ic	540	9	4.08/9	I.82 ± 0.09
2	52.7	23	23.7/23	0.87 ± 0.06
	62.8	I6	26.2/I6	0.99 ± 0.05
3	53.0	II	8.8/II	0.90 ± 0.08
	54.0	8	I9.0/8	0.9I ± 0.I0
4	53.I	I6	6.3/I6	I.07 ± 0.07
	62.4	2I	I0.3/2I	0.95 ± 0.06
5	45.I	34	I9.I/34	0.84 ± 0.05
	53.2	33	I8.0/33	I.I0 ± 0.08
	62.7	27	I0.0/27	I.30 ± 0.05
6	53.0	I5	I3.I/I5	0.95 ± 0.07
	63.0	I5	I9.0/I5	I.06 ± 0.08

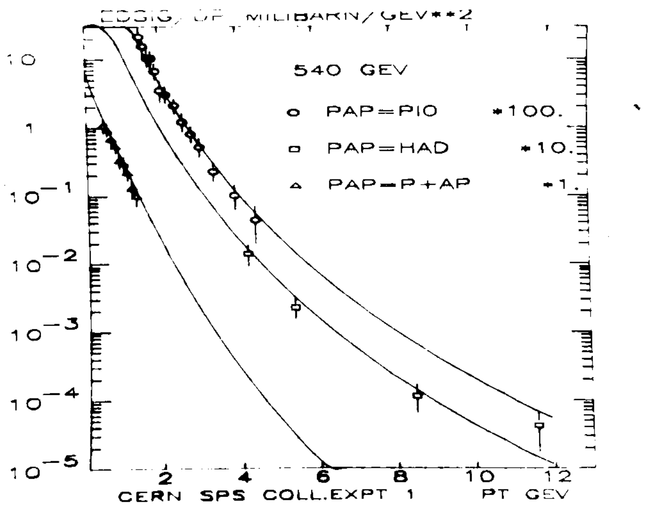


Fig.4. Description of the experimental data[1] for $E\frac{d^3\sigma}{dp^3}$ obtained by formula (6). The value of $E\frac{d^3\sigma}{dp^3}$ for each curve (beginning with the lower one) are multiplied by $10^{(n-1)}$, $n = 1,2,3$.

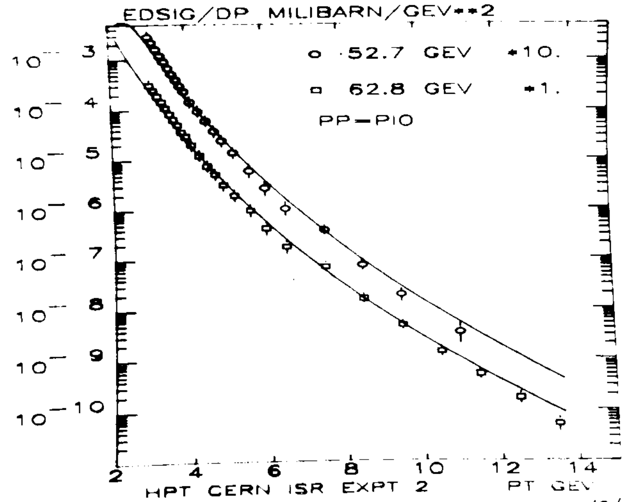


Fig.5. Description of the experimental data[2] for $E\frac{d^3\sigma}{dp^3}(pp \to \pi^0 X)$ obtained by formula (6). The values of $E\frac{d^3\sigma}{dp^3}$ for each curve (beginning with the lower one) are multiplied by $10^{(n-1)}$, $n = 1,2$.

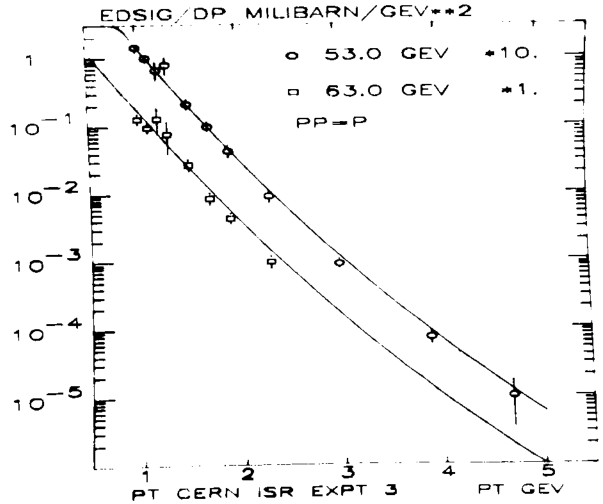


Fig.6. Description of the experimental data[3] for $E\frac{d^3\sigma}{dp^3}(pp \to pX)$, obtained by formula (6). The values of $E\frac{d^3\sigma}{dp^3}$ for each curve (beginning with the lower one) are multiplied by $10^{(n-1)}$, $n = 1,2$.

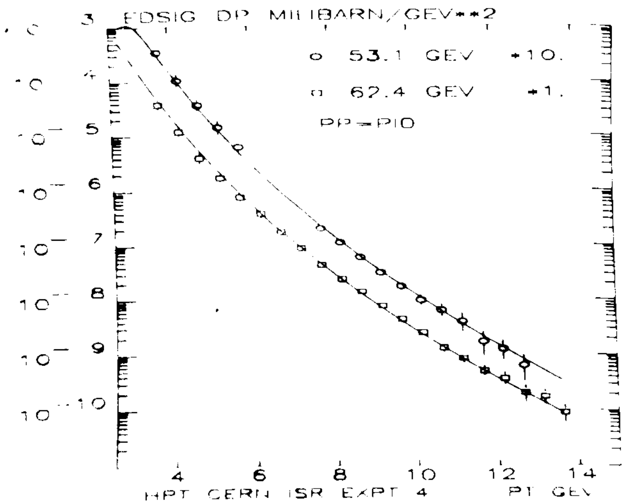


Fig.7. Description of the experimental data [4] for $E\frac{d^3\sigma}{dp^3}(pp \to \pi^o X)$, obtained ny (6). The values of $E\frac{d^3\sigma}{dp^3}$ for eacg curve are multiplied by $10^{(n-1)}$, $n = 1,2$.

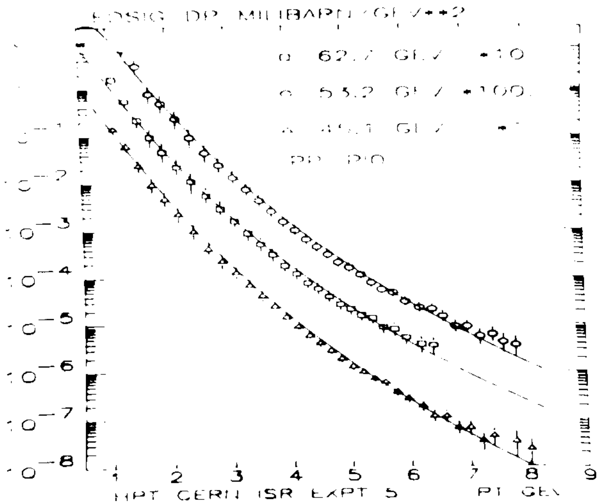


Fig.8. Description of the experimental data [5] for $E\frac{d^3\sigma}{dp^3}(pp \to \pi^o X)$, obtained by (6). The values of $E\frac{d^3\sigma}{dp^3}$ for each curve are multiplied by $10^{(n-1)}$, $n = 1,2,3$.

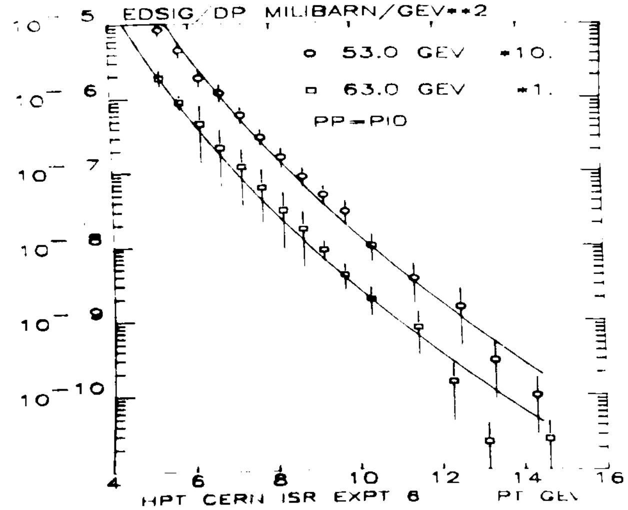


Fig.9. Description of the experimental data [6] for $E\frac{d^3\sigma}{dp^3}$ obtained ny (6). The values of $E\frac{d^3\sigma}{dp^3}$ for each curve are multiplied by $10^{(n-1)}$, $n = 1,2$.

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О поведении сечений инклюзивных адронных процессов
при больших переданных импульсах и энергиях CERN ISR и SPS Collider

В рамках правил кваркового счета аномальных размерностей для инклюзивных сечений в ведущем логарифмическом приближении квантовой хромодинамики получено описание экспериментальных данных CERN ISR и SPS Collider для сечения $E\frac{d^3\sigma}{dp^3}(AB \to CX)$ при $x_T = \frac{2p_T}{\sqrt{s}} > 0{,}2$ и $\sqrt{s} \geq 40$ ГэВ. При замене

$$p_T \to m\chi_T = m \ln\left(\sqrt{1 + \frac{p_T^2}{m^2}} + \frac{p_T}{m}\right)$$

приводящей к экспоненциальному поведению сечений с малыми p_T, сечения описываются во всем экспериментально доступном интервале x_T.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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On the Behaviour of Inclusive Hadron Cross Sections
at Large Transferred Momenta and CERN ISR and SPS Collider Energies

The CERN ISR and SPS Collider experimental data for the $E\frac{d^3\sigma}{dp^3}(AB \to CX)$ cross section at $x_T = \frac{2p_T}{\sqrt{s}} \geq 0{,}2$ and $\sqrt{s} \geq 40$ GeV have been described in the framework of quark counting rules of anomalous dimensions in the leading logarithmic approximation of quantum chromodynamics. A good description of the behaviour of the cross section in the whole exnrimentally possible interval is obtained under the change

$$p_T \to m\chi_{p_T} = m \ln\left(\sqrt{1 + \frac{p_T^2}{m^2}} + \frac{p_T}{m}\right)$$

leading to the exponential behaviour of the cross section in the region of small p_T.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1983

Перевод авторов.