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# QCD SUM RULE ANALYSIS OF $J/\psi \to \eta_c \gamma$ DECAY

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## 1. INTRODUCTION

QCD sume rules [1,2] are a very effective tool to study the hadronic structure within the QCD framework. One of the most important advantages of the QCD sum rule approach is the possibility of taking into account the nonperturbative effects and their influence on physical quantities characterizing the dynamical properties of hadrons. One of the basic assumptions of the method concerns the nontrivial structure of the QCD vacuum, and the quantitative predictions are based on the analysis of the power series resulting from the Wilson expansion valid up to some critical dimension of the local composite operators [2]. The nonzero vacuum expectation values like $\langle 0|G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle$, $\langle 0|\bar{q}q|0\rangle$, etc., are treated as some universal parameters describing the dynamics of quark-gluon interactions on distances of an order of the confinement radius, i.e., the resonance formation zone. Fixing these parameters, e.g., from the charmonium sum rules [2], it is possible, in principle, to get parameter-free predictions concerning the static properties of hadrons. This program was partially accomplished in refs. [2-8], and the method proved out to be a very fruitful tool to study the hadrons composed both of light and of heavy quarks.

In the present paper we use the QCD sum rules to study the radiative transitions between the charmonium states. Earlier this problem was investigated in refs. [9-12]. However, the corrections due to the gluon condensate $\langle 0|(\alpha_s/\pi)G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle$ were not taken into account in these papers.

The dispersion theory of the charmonium [1] revealed a rather high degree of duality between the physical charmonium spectrum and the theoretical one calculated up to $O(\alpha_s^2)$ terms. For the radiative decays the duality was demonstrated in ref. [12]. However, a more precise fixing of characteristics of the $\bar{c}c$-system requires to include into consideration also power corrections to the corresponding amplitudes. A detailed study of the nonperturbative $\langle GG\rangle$ contributions into the polarization operator (i.e., 2-point function) for the $\bar{c}\Gamma c$-currents allowed one to calculate masses of all $s$- and $p$-levels of the charmonium [2,3] in good agreement with experiment.

Recently [8,13,14] the QCD charmonium sum rule approach was applied to a 3-point function $T_{5\mu\nu}(q,q_1,q_2)$ in the simplest kinematics $q_1^2 = q_2^2 = 0$ to extract information concerning the $\eta_c \to 2\gamma$ decay widths. We generalize the analysis of refs. [8,13] onto a more complicated kinematics ($q_2^2 = 0$, but $q_1^2 - q^2 \neq 0$) relevant to the study of $J/\psi \to \eta_c\gamma$ radiative transitions.



## II. DERIVATION OF THE SUM RULE

Consider a 3-point function

$$T_{5\mu\nu}(q,q_1,q_2) = \int d^4x\, d^4y \exp\{-iq_1x - iq_2y\} \times$$
$$\times \langle 0|T\{j_\mu(0)\, j_\nu(y)\, J_5(x)\}|0\rangle = \qquad (1)$$
$$= 3eQ\epsilon_{\mu\nu\alpha\beta}\, q^\alpha q^\beta A(q^2, q_1^2, q_2^2),$$

where $Q = 2/3$ is the electric charge of the $c$-quark, $q^2 \neq 0, q_1^2 \neq 0$, $q_2^2 = 0$,

$$j_\mu(x) = \bar{c}(x)\gamma_\mu c(x),$$
$$J_5(x) = i\bar{c}(x)\gamma_5 c(x).$$

In the nonrelativistic approximation the amplitude $A(q^2, q_1^2, 0)$ describes the dipole M1 transitions between the charmonium states. Diagrams contributing to A are presented in fig.1. The nonperturbative corrections proportional to $\langle 0|G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle$ are due to diagrams 1b-g.

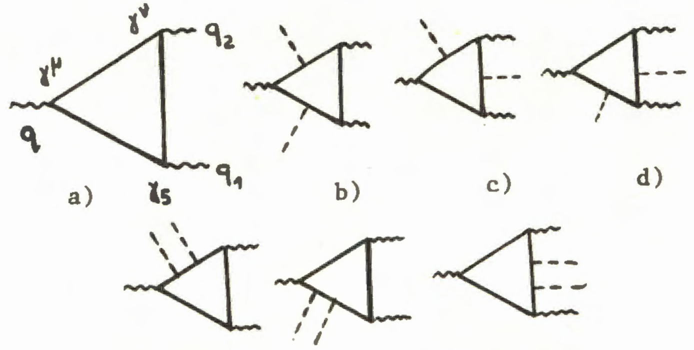


Fig.1. Diagrams contributing to $T_{5\mu\nu}(q,q_1,q_2)$. Momenta $q$, $q_1$, $q_2$ correspond to $J/\psi$, $\eta$ and $\gamma$, respectively. Dashed lines are related to the external gluonic field.

The simplest diagram 1a is most conveniently calculated by using a double dispersion relation. The result looks as follows:

$$A^{(0)}(q^2,q_1^2,0)=\frac{1}{2\pi^2 m_c}\int_0^1 dz\int_0^1 dx\, x\left[1-\frac{q^2}{m_c^2}x\bar{x}z-\frac{q_1^2}{m_c^2}x\bar{x}\bar{z}\right]^{-1}= \tag{2}$$
$$=\frac{m_c}{2\pi^2}\int_{4m_c^2}^{\infty}ds_1\int_{4m_c^2}^{\infty}ds\,\frac{\delta(s-s_1)}{(s-q^2)(s_1-q_1^2)}\ln\left(\frac{1+v}{1-v}\right),$$

where

$$\bar{x}=1-x,\quad \bar{z}=1-z,\quad v=(1-m_c^2/s)^{1/2}.$$

The amplitude $A(q^2,0,0)$ analysed in refs.[8,13] can be extracted from eq. (2) in a straightforward way.

To calculate the $O(\langle GG\rangle)$ power corrections we incorporated the method described in ref.[7] and obtained the following result for the sum of fig.1b-g contributions:

$$\begin{aligned}A^{(GG)}(q_1^2,q_1^2,0)=&-\frac{3}{2\pi^2 m_c}\cdot\phi\cdot\{-2I(0,0,2,0,3)+\\&+4[I(1,0,2,0,3)-I(1,0,3,0,3)-I(2,0,3,0,3)+\\&+I(1,0,3,1,3)-I(1,2,4,0,3)+I(1,0,2,2,3)-\\&-I(0,2,3,1,3)-I(0,3,4,0,3)]+\\&+6[I(3,0,4,0,4)+I(0,0,3,1,4)+I(0,3,4,0,4)-\\&-I(0,0,2,2,3)-I(0,0,3,1,3)]+\\&+18[I(2,1,4,0,4)+I(1,2,4,0,4)],\end{aligned} \tag{3}$$

where

$$\phi=4\pi^2\langle 0|\frac{\alpha_s}{\pi}G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle\cdot\frac{1}{9(4m_c^2)^2}, \tag{4}$$

$$I(a,b,c,d,p)=\int_0^1 dx\int_0^1 dz\,\frac{z^a\bar{z}^b x^c\bar{x}^d}{\left(1-\frac{q^2}{m_c^2}x\bar{x}z-\frac{q_1^2}{m_c^2}x\bar{x}\bar{z}\right)^p}, \tag{5}$$

It makes sense not to perform integrations over the Feynman parameters in eq. (5), since in what follows we will be interested only in the moments of the amplitide $A(q^2,q_1^2,0)$, and the basic integrals have just the form best suited for their computation.



For the moments $M_{nr}$ defined by

$$M^A_{nr}=\frac{1}{n!\,r!}\left(\frac{d}{dq^2}\right)^r\left(\frac{d}{dq_1^2}\right)^n A(q^2,q_1^2,0)\Big|_{q^2=0,\ q_1^2=0}, \tag{6}$$

after rather cumbersome calculations we obtained from eqs. (2), (3):

$$\begin{aligned}M^A_{n,r}=&\frac{1}{2\pi^2 m_c}\cdot\left(\frac{1}{m_c^2}\right)^{n+r}\cdot\frac{[(n+r)!]^2}{(2n+2r+2)!}\{1+O(\alpha_s)-\\&-\phi\,\frac{n+r+1}{2n+2r+3}[(n+r)^3+(n+r)^2+6(n+r)-4n+4nr+4r(n+r)]\}.\end{aligned} \tag{7}$$

From eq. (7) it is not difficult to extract the results of ref.[8]: one should just take $r=0$. From eq. (7) it is also clear that the nonperturbative corrections rapidly (like $(n+r)^3$) increase with $n$ and $r$. To maintain the power corrections at the level of $\lesssim$20-25%, one should not consider large values of $n$ and $r$.

To incorporate the sum rule technology, we represent the absorptive part of A in the narrow resonance approximation

$$\mathrm{Im}A=\pi^2\sum_{i,j}g_i h_j F_{ij} M_i^{a_i} M_j^{b_j}\delta(s-M_i^2)\delta(s_1-M_j^2)\,. \tag{8}$$

where $g_i(h_j)$ is the coupling constant characterizing the projection of the corresponding current onto an $i$-th ($j$-th) resonance state, $F_{ij}$ are dimensionless transition matrix elements, and the constants $a_i$, $b_j$ are determined by quantum numbers of the resonances.

The sum rule can be written now as

$$M^A_{nr}=\frac{1}{\pi^2}\int_{\text{thr.}}^{\infty}\frac{ds}{s^{r+1}}\int_{\text{thr.}}^{\infty}\frac{ds_1}{s_1^{n+1}}\,\mathrm{Im}A\,. \tag{9}$$

Using eqs. (7), (8) and extracting from eq. (8) only the lowest transitions shown in fig.2 (cf.[2]) we get

$$F_{\psi\eta_c}+\frac{h_{\eta'_c}}{h_{\eta_c}}\left(\frac{M_{\eta_c}}{M_{\eta'_c}}\right)^{2r+1}F_{\eta'_c\psi}+\frac{g_{\psi'}}{g_\psi}\left(\frac{M_{J/\psi}}{M_{\psi'}}\right)^{2n}F_{\psi'\eta_c}+$$

$$+\frac{h_{\eta'_c}}{h_{\eta_c}}\,\frac{g_{\psi'}}{g_\psi}\left(\frac{M_{J/\psi}}{M_{\psi'}}\right)^{2n}\left(\frac{M_{\eta_c}}{M_{\eta'_c}}\right)^{2r+1} F_{\psi'\eta'_c} + \text{"continuum"} = \tag{10}$$

$$= \frac{1}{g_\psi h_{\eta_c}}\left(\frac{M_{J/\psi}}{m_c}\right)^{2n}\left(\frac{M_{\eta_c}}{m_c}\right)^{2r+1} M^A_{nr}\ .$$

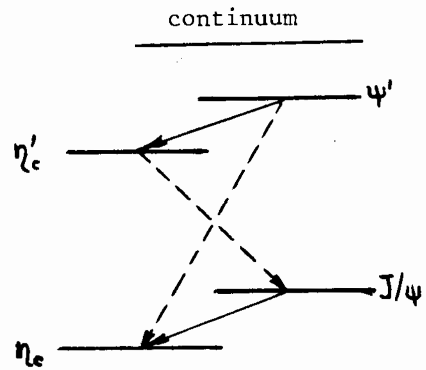


Fig.2. Radiative transitions between charmonium states. Solid lines correspond to the diagonal transitions $J/\psi \to \eta_c\gamma$ and $\psi' \to \eta'_c\gamma$, dashed lines to the nondiagonal ones: $\eta'_c \to J/\psi\,\gamma$, $\psi' \to \eta_c\gamma$ .

## III. ANALYSIS OF THE SUM RULE

For the resonance masses we take their experimental values $M_{J/\psi}$ = 3.095 GeV, $M_{\eta_c}$ = 2.981 GeV, $M_{\psi'}$ = 3.685 GeV, $M_{\eta'_c}$ = = 3.592 GeV. For the c-quark mass we take the value $m_c$ = = 1.28 GeV [3]. For the g, h constants we take the estimates obtained in refs.[1,8,12,13]:

$$g_\psi = 0.125;\quad g_{\psi'} = 0.0755;\quad h_{\eta_c} = 0.12;\quad h_{\eta'_c} = 0.072.$$

Most popular value for $\langle 0|(\alpha_s/\pi)\, G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle$ is 0.012 GeV$^4$ obtained in ref.[2]. Recently, however, there were claims that $\langle 0|(\alpha_s/\pi)\, GG|0\rangle$ is higher: $\langle 0|(\alpha_s/\pi)\, GG|0\rangle$ = 0.017-0.025 GeV$^4$ (see, e.g., refs. [15,16]).

Now all the parameters entering into eq. (10) are fixed, and we can try to extract information about the matrix elements of the radiative M1 transitions. The values of n, r will be chosen in the region, where

a) power corrections are smaller than 25%,
b) sensitivity to the resonance contribution is high enough,
c) all $F_{ij}$ simultaneously are most stable with respect to small changes of n, r allowing for 10% deviations for the leading contribution into the l.h.s. of eq. (10).



Table 1

Transition matrix elements for three values of the $\langle 0|(\alpha_s/\pi)\, G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle$ - parameter: 0.012; 0.017 and 0.025 GeV$^4$

| | |
|---|---|
| $F_{\psi\eta_c}$ | 2.5 |
| | 2.4 |
| | 2.3 |
| $F_{\eta'_c\psi}$ ($F_{\psi'\eta_c}$) | 0.42 |
| | 0.36 |
| | 0.32 |
| $F_{\psi'\eta'_c}$ | 5.3 |
| | 4.8 |
| | 4.6 |

There exists also one very important constraint on the admissible values of n, r, which is due to the fact that we have not included $O(\alpha_s)$ corrections into our sum rules. The calculation of these corrections is a separate and a rather complicated problem. Note, however, that for moments of the 2-point function analysed in refs.[2,3], the $O(\alpha_s)$ correction changes its sign for $4 < n < 5$ and as a consequence, it is rather small for n = 3-6. That is why we restrict our analysis of eq. (10) to the region $3 \le n + r \le 6$.

The continuum contribution in the region considered is ≲10% (cf. refs.[1,13]). As a result, we obtain from eq. (10) the values collected in Table 1. Note that the most broad stability region r = 3-6, n + r = 4-6 was observed for $\phi$ = 0.0013 while for higher $\phi$ values the stability region is smaller: r = 3-5, n + r = 3-5.

For simplicity we used the estimate $F_{\eta'_c\psi} \sim F_{\psi'\eta_c}$. Indeed, incorporating the potential analogy, one should expect that these matrix elements are given by similar (and small) integrals. Due to their smallness, the specific values of $F_{\eta'_c\psi}$ and $F_{\psi'_c\eta_c}$ as extracted from eq. (10) are very sensitive to a particular choice of parameters in eq. (10), and the results have large errors. In fact, what we obtained is only a very rough estimate of $\eta'_c \to J/\psi\gamma$ and $\psi' \to \eta_c\gamma$ decay widths. Hence, our sum rule, just like it was observed in ref.[12], does not allow one to get accurate results concerning the nondiagonal transitions.

Finally, using the formula

$$\Gamma(V_j \to P_i\,\gamma) = \frac{\alpha Q^2}{24}(F_{ij})^2\, m_{V_j}\left(\frac{m_{V_j}}{m_{P_i}}\right)^2\left(1 - \frac{m_{P_i}}{m_{V_j}}\right)^3$$

we find $\Gamma(\psi \to \eta_c\gamma)$ and $\Gamma(\psi' \to \eta'_c\gamma)$. The results of our calculations are presented in Table 2.

Table 2

Decay widths for charmonium transitions. The three values displayed in the rightmost column correspond to the three values of $\langle 0|(\alpha_s/\pi)GG|0\rangle$ parameter: 0.012, 0.017, and 0.025 $GeV^4$

| Decays | Experiment (keV) [17] | Predictions of refs.[9-11] (keV) | Sum rules without GG [12] (keV) | Potential model [18] (keV) | Sum rules including <GG> (keV) |
|---|---|---|---|---|---|
| $J/\psi \to \eta_c \gamma$ | $0.76^{+0.25}_{-0.22}$ | $2.21 \pm 0.54$ | $1.51 \pm 0.57$ | $1.64 \pm 0.6$ | $1.08 \pm 0.38$<br>$0.98 \pm 0.35$<br>$0.91 \pm 0.33$ |
| $\psi' \to \eta_c \gamma$ | $0.62 \pm 0.17$ | 0.75 | < 8 | $0.97 \pm 0.19$ | < 7<br>< 5<br>< 4 |
| $\psi' \to \eta'_c \gamma$ | 0.43–2.8 | – | $0.47 \pm 0.2$ | $0.97 \pm 0.24$ | $1.8 \pm 0.8$<br>$1.5 \pm 0.7$<br>$1.4 \pm 0.7$ |
| $\eta'_c \to \gamma J/\psi$ | – | – | < 7 | – | < 4.4<br>< 3.2<br>< 2.5 |



## IV. CONCLUSIONS

Now we summarize main observations:

a) The inclusion of $O(\langle GG\rangle)$ nonperturbative corrections into the amplitude describing the radiative M1 transitions between the charmonium states improves the accuracy of estimates for the transition matrix elements.

b) Taking into account the $O(\alpha_s)$ -contributions should not change the final result drastically: in the considered region of $n, r$ ($n, r \lesssim 3-5$) we expect only 20-30% correction (cf. /12,14/).

c) The nondiagonal transition matrix elements cannot be extracted reliably from the sum rules. This is clear, in particular, from the results of ref. /10/, where these matrix elements are shown to be small compared to the diagonal ones.

d) It is desirable to calculate $O(G^3)$ and $O(G^4)$ contributions to the 3-point amplitude, at least to compare the results with the $O(\alpha_s)$ terms which also are worth calculating.

e) Our analysis shows that assuming higher values for the $\langle 0|(\alpha_s/\pi)\, G^a_{\mu\nu} G^a_{\mu\nu}|0\rangle$ -parameter one obtains a better agreement with experimental data. However, any definite statement concerning this point cannot be justified without taking into account the higher-dimension contributions.

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Анализ распада $J/\psi \to \eta_c \gamma$ методом КХД правил сумм

Проведен анализ радиационных M1-переходов в чармонии с учетом непертурбативных поправок $O(\langle GG \rangle)$ методом КХД правил сумм. Вычисленные ширины $\Gamma(J/\psi \to \eta_c \gamma)$ и $\Gamma(\psi' \to \eta'_c \gamma)$ близки к экспериментальным значениям. Прослежена зависимость результатов от выбора параметра $\langle GG \rangle$.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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QCD Sum Rule Analysis of $J/\psi \to \eta_c \gamma$ Decay

On the basis of the QCD sum rule approach an analysis is performed of the radiative M1 transitions between charmonium states including the nonperturbative corrections $O(\langle GG \rangle)$. The calculated widths $\Gamma(J/\psi \to \eta_c \gamma)$ and $\Gamma(\psi' \to \eta'_c \gamma)$ are close to their experimental values. Dependence of the results on the magnitude of the $\langle GG \rangle$ parameter is investigated.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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