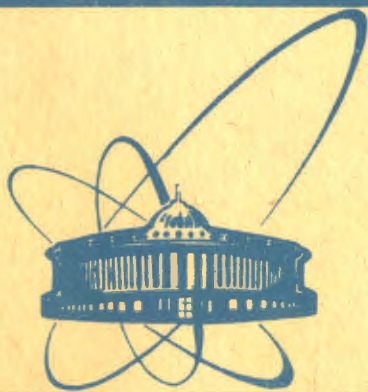


31/2-83



СООБЩЕНИЯ
ОБЪЕДИНЕННОГО
ИНСТИТУТА
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ
ДУБНА

5756/83

9/11-83

E2-83-533

S.G.Gorishny, A.L.Kataev,¹ S.A.Larin²

**NEXT-NEXT-TO-LEADING
PERTURBATIVE QCD CORRECTIONS
AND LIGHT QUARK MASSES**

¹ Institute for Nuclear Research
of the Academy of Sciences of the USSR,
Moscow.

² Moscow State University.

1. Motivations of Calculations. The continuing improvement of the QCD methods and, in particular, the sum rule technique made it possible to obtain a new information on light quark masses^{/1/}. Thus, in refs.^{/2,3/} their values have been estimated by means of finite energy sum rules (FESR)^{/4/}, analogous to the strong interaction dual sum rules^{/5/}, while in^{/1,6-8/} the same aim was achieved with the help of Borel transformed sum rules (for a review see^{/9/}).

These sum rules allow both perturbative and nonperturbative effects to be taken into account. However, in the sum rule analysis of the spin zero quark current correlators power terms do not play a decisive role^{/9/}. Furthermore, direct instanton contributions to two-point functions at low momenta (see, e.g.,^{/9/}) do suffer from serious uncertainties and are not calculable from the first principles within QCD. At the same time the two-loop perturbative contribution to the spin zero quark correlators is about 45% of the leading term (at $\bar{\alpha}_s = 0.25$)^{/8/} and turns out to be the most significant unambiguously calculated correction.

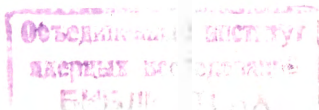
These facts make the calculation of the $O(\alpha_s^2)$ perturbative corrections to this correlator very desirable. Indeed, taking them into account would affect the QCD predictions of light quark masses and smooth over the existing discrepancy between the FESR ($m_u + m_d$ (1 GeV) = 20 MeV^{/2,3/}) and the current algebra ($m_u + m_d = 11$ MeV^{/1/}) results.

Thus, evaluating the three-loop correction to the (pseudo) scalar quark current correlator and taking them into account in the FESR procedure is the aim of this paper.

At first we present our result for $O(\alpha_s^2)$ correction, which was obtained with the help of the SCHOONSCHIP program, implementing the algorithm of ref.^{/10/} for computing the massless Feynman integrals. Then, we estimate the absolute values of light quark masses, saturating the FESR version^{/3/} for the correlator considered by spin zero intermediate states. Following^{/11/}, we also propose "linear dual models" for the radial excitations spectra of the (pseudo) scalar mesons (K , δ and k) and discuss their phenomenological consequences.

2. Results of Calculations. Current methods of determining light quark masses are based on sum rules for the two-point functions of the (axial) vector current divergences*

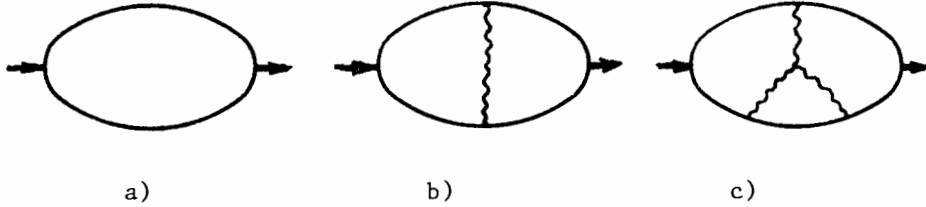
*We analyze only the leading correction in quark masses.



$$F^{(\pm)}(Q^2 = -p^2) = i \int d^D x e^{ipx} \langle 0 | T \{ \partial_\mu J_\mu^{(5)}(x), \partial_\nu J_\nu^{(5)}(0) \} | 0 \rangle \quad (2.1)$$

$$J_\mu^{(5)} = i \bar{q} \gamma_\mu (\gamma_5) u; \quad \partial_\mu J_\mu^{(5)} = i m_q^{(\pm)} \bar{q} (\gamma_5) u,$$

where $m_q^\pm = m_q \pm m_u$; $q = d, s$; $D = 4 - 2\epsilon$ is the space-time dimension in the dimensional regularization; γ_5 is the fully anti-commuting analogue of the 4-dimensional matrix γ_5 . To find the three-loop approximation of (2.1), one needs the one-loop approximation for the bare charge a_B , the mass renormalization constant up to the two-loop level (see, e.g., /13/) and the relevant terms in the ϵ -expansion of the corresponding diagrams (some typical diagrams are shown in the figure). In particular, only the pole parts of the fourteen three-loop diagrams are required.



Typical representatives of: a) 1-loop, b) 2-loop and c) 3-loop contributions to the spin zero quark current correlator.

The diagrams were evaluated analytically with the help of the SCHOONSCHIP program mentioned above. Leaving all the details for a subsequent publication, we present here only the final result in the \overline{MS} -scheme

$$F_{\overline{MS}}^{(\pm)}(Q^2) = \frac{3}{8\pi^2} (m_q^{(\pm)})^2 Q^2 \ln \frac{Q^2}{\mu^2} \left\{ 1 + \frac{a_s}{\pi} \left(\frac{17}{3} - \ln \frac{Q^2}{\mu^2} \right) + \left(\frac{a_s}{\pi} \right)^2 \left[\left(\frac{11089}{144} - \frac{611}{24} \zeta(3) \right) + \left(\frac{2}{3} \zeta(3) - \frac{65}{24} \right) f \right] + \left(-\frac{53}{3} + \frac{11}{18} f \right) \ln \frac{Q^2}{\mu^2} + \left(-\frac{19}{12} - \frac{1}{18} f \right) \ln \frac{Q^2}{\mu^2} \right\}, \quad (2.2)$$

where f is the number of quarks; $\zeta(3) = 1.202\dots$. Notice, that the cancellation of divergences in the renormalized quantity relates the coefficients of the non-leading log terms to the pole part of a_B and the mass renormalization constant Z_m , providing us with a useful check of the calculations.

However, only the absorptive part $R(s) = \frac{1}{\pi} \text{Im} F(-s + i\epsilon)$ of the correlator (2.1) is of physical interest. To find it, one should continue eq. (2.2) to the timelike region. Doing so, we get for $f = 3$:

$$R_{\overline{MS}}^{(\pm)}(s) = \frac{3}{8\pi^2} (m_q^{(\pm)})^2 s \left\{ 1 + \frac{a_s}{\pi} (5.667 - 2 \ln \frac{s}{\mu^2}) + \left(\frac{a_s}{\pi} \right)^2 (40.684 - 1.417 \pi^2 - 31.667 \ln \frac{s}{\mu^2} + 4.51 \ln^2 \frac{s}{\mu^2}) \right\}. \quad (2.3)$$

This result demonstrates two new features typical of physical quantities calculated in the timelike region up to the next-to-leading order, namely, the scheme dependence of two perturbative terms and the manifestation of the $O(a_s^2 \pi^2)$ correction associated with the analytical continuation of the log terms. In our case the contribution diminishes the total three-loop coefficient, so we need not worry about the redefinition of the coupling constant in the spacelike region proposed in /14/ to absorb the corrections of a similar nature. As to the total magnitude of the $O(a_s^2)$ term, it amounts to about 18% of the leading one (at $\bar{a}_s = 0.25$) and is three times as less as the two-loop one.

3. FESR and Light Quark Masses. Consider now the FESR in application to eq. (2.3):

$$M_k^{\text{th}} = \int_0^{s_0} R^{\text{th}}(s) s^k ds = \int_0^{s_0} R^{\text{exp}}(s) s^k ds \equiv M_k^{\text{exp}} \quad (3.1)$$

$$R^{\text{th}}(s) = \frac{1}{\pi} \text{Im} [F_{P.T.} + F_{N.P.}](-s + i\epsilon),$$

where $F_{P.T.}$ is defined via eq. (2.2), while $F_{N.P.}$ denotes the non-perturbative power contributions (see /9/). As has been proposed in ref. /3/, we apply the renormalization group method not to the spectral density itself, but to its moments M_k :

$$\left(\mu^2 \frac{\partial}{\partial \mu^2} + \beta(a_s) \frac{\partial}{\partial a_s} + \gamma_m(a_s) \frac{\partial}{\partial m} \right) M_k^{\text{th}}(s_0, \mu^2, m, a_s) = 0 \quad (3.2)$$

$$M_k^{\text{th}}(s_0, \mu^2, m, a_s) = M_k^{\text{th}}(s_0 = -\mu^2, \bar{m}(s_0), \bar{a}_s(s_0)).$$

Here the running coupling constant $\bar{a}_s(s)$ and the running mass $\bar{m}(s)$ are determined by solving the differential equations

$$\frac{1}{4\pi} \frac{d\bar{\alpha}_s}{d \ln|s|} = \frac{1}{4\pi} \beta(\bar{\alpha}_s); \quad \frac{d\bar{m}}{d \ln|s|} = \bar{m} \gamma_m(\bar{\alpha}_s) \quad (3.3)$$

with the three-loop approximations for the QCD β -function^{/15/} and mass anomalous dimension γ_m ^{/16/}:

$$\frac{1}{4\pi} \beta(\alpha_s) = -\sum_{i=0}^2 \beta_i \left(\frac{\alpha_s}{4\pi}\right)^{i+2}, \quad \gamma_m(\alpha_s) = \sum_{i=0}^2 \gamma_i \left(\frac{\alpha_s}{4\pi}\right)^{i+1},$$

$$\beta_0 = 11 - \frac{2}{3}f, \quad \beta_1 = 102 - \frac{38}{3}f, \quad \beta_2 = \frac{2857}{2} - \frac{5033}{18}f + \frac{325}{54}f^2, \quad (3.4)$$

$$\gamma_0 = 4, \quad \gamma_1 = \frac{202}{3} - \frac{20}{9}f, \quad \gamma_2 = \frac{374}{3} + \left(\frac{160}{3}\zeta(3) - \frac{2216}{27}\right)f - \frac{140}{81}f^2.$$

$\bar{\alpha}_s$ and \bar{m} can be parametrized in terms of the scale parameter Λ and the invariant mass \hat{m} as (see, e.g.,^{/13/}):

$$\frac{\bar{\alpha}_s(s)}{4\pi} = \frac{1}{\beta_0 L} - \frac{\beta_1 \ln L}{\beta_0^3 L^2} + \frac{1}{\beta_0^5 L^3} (\beta_1^2 \ln^2 L - \beta_1 \ln L + \beta_2 \beta_0 - \beta_1^2)$$

$$\bar{m}(s) = \hat{m} \exp\left(\int \frac{\bar{\alpha}_s}{\beta(x)} dx + \frac{\gamma_0}{\beta_0} \ln \frac{\beta_0}{2\pi}\right) = \{f=3\}$$

$$= \hat{m} \left(\frac{9\bar{\alpha}_s}{2\pi}\right)^{4/9} \left[1 + 0,895 \left(\frac{\bar{\alpha}_s}{\pi}\right) + 2,707 \left(\frac{\bar{\alpha}_s}{\pi}\right)^2\right], \quad (3.5)$$

where $L = \ln s/\Lambda^2$.

The application of the renormalization group to M_k in the time-like region has several attractive features. Firstly, it allows one to integrate explicitly all log terms in eq. (2.3) before their final summing up. Secondly, using this approach one can single out the contribution to M_k due to power terms suppressed by $(Q^2)^k$ only. Indeed, since absorptive parts of power corrections are proportional to $\delta^{(n)}(s)$, integration over s automatically nullifies them, if $n \neq k$.

To determine the absolute values of m_d^+ and m_s^+ from (3.1), we will consider the one- and two-resonance approximations of the physical spectral density (in full analogy with the two-loop calculations of $m_d^+/3/$), saturating the FESR (3.1) by π, π' and K, K' pseudoscalar mesons, the ground states of which (π and K) are the Goldstone bosons responsible for chiral symmetry breaking.

First we model the physical spectra by $R_1^{\text{exp}} = f_p^2 m_p^4 \delta(s - m_p^2)$, where $p = \pi, K$; $f_\pi = 132$ MeV; $f_K = 153$ MeV, and obtain for the zero moment:

$$\frac{f_p^2 m_p^4}{(\hat{m}_q^+)^2} = \frac{3}{8\pi^2} \frac{(s_0)_p^2}{2} \left[1 + 8,457 \left(\frac{\bar{\alpha}_s}{\pi}\right) + 58,737 \left(\frac{\bar{\alpha}_s}{\pi}\right)^2\right] + \frac{1}{8} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \left[1 + 1,790 \left(\frac{\bar{\alpha}_s}{\pi}\right) + 6,215 \left(\frac{\bar{\alpha}_s}{\pi}\right)^2\right] \left(\frac{9\bar{\alpha}_s}{2\pi}\right)^{8/9}. \quad (3.6)$$

$(p,q) = (\pi, d), (K, s)$; $\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle = 0.012 \text{ GeV}^4$. Choosing the duality interval as $(s_0)_p = (m_p^2 + m_p'^2)/2 \approx m_p^2/2$ (for details see, e.g.,^{/3/} and references therein) with $m_{\pi'} = 1.24 \text{ GeV}/17/$, $m_{K'} = 1.45 \text{ GeV}/18/$ and solving eq. (3.5), we get:

$$\hat{m}_d^+ \approx 26-21 \text{ (MeV)}, \quad \bar{m}_d^+ (1 \text{ GeV}) \approx 16 \text{ MeV};$$

$$\hat{m}_s^+ \approx 320-260 \text{ (MeV)}, \quad \bar{m}_s^+ (1 \text{ GeV}) \approx 200 \text{ MeV}; \quad (3.7)$$

$$0.1 \leq \Lambda \overline{M_S} \leq 0.2 \text{ (GeV)}.$$

While obtaining these estimates the validity of the duality relations (3.1) in a rather small interval of energies was taken for granted. To verify the stability of (3.6) with respect to the choice of s_0 and to check the reliability of the FESR approach we adopt now the two-resonance model of the spectral density:

$$R_2^{\text{exp}} = f_p^2 m_p^4 \delta(s - m_p^2) + f_{p'}^2 m_{p'}^4 \delta(s - m_{p'}^2),$$

where the last term corresponds to the π' and K' excitations. In this case we do not use any additional suppositions to fix s_0 , but extract it together with $f_{p'}$ and \hat{m}_q^+ from the system of three equations obtained from (3.1) at $k = 0, 1, 2$.*

$$\frac{f_p^2 m_p^4 + f_{p'}^2 m_{p'}^4}{(\hat{m}_q^+)^2} = \frac{3}{8\pi^2} \frac{(s_0)_p^2}{2} \left[1 + 8,457 \left(\frac{\bar{\alpha}_s}{\pi}\right) + 58,737 \left(\frac{\bar{\alpha}_s}{\pi}\right)^2\right] \left(\frac{9\bar{\alpha}_s}{2\pi}\right)^{8/9},$$

$$\frac{f_p^2 m_p^6 + f_{p'}^2 m_{p'}^6}{(\hat{m}_q^+)^2} = \frac{3}{8\pi^2} \frac{(s_0)_p^3}{3} \left[1 + 8,124 \left(\frac{\bar{\alpha}_s}{\pi}\right) + 55,757 \left(\frac{\bar{\alpha}_s}{\pi}\right)^2\right] \left(\frac{9\bar{\alpha}_s}{2\pi}\right)^{8/9} (3.8)$$

$$\frac{f_p^2 m_p^8 + f_{p'}^2 m_{p'}^8}{(\hat{m}_q^+)^2} = \frac{3}{8\pi^2} \frac{(s_0)_p^4}{4} \left[1 + 7,957 \left(\frac{\bar{\alpha}_s}{\pi}\right) + 52,406 \left(\frac{\bar{\alpha}_s}{\pi}\right)^2\right] \left(\frac{9\bar{\alpha}_s}{2\pi}\right)^{8/9},$$

*We omitted the non-perturbative terms, which are negligible at $(s_0)_p > 1 \text{ GeV}$ as compared to the perturbative ones.

The solution of (3.8) is:

$$\begin{aligned} (s_0)_\pi &= 2 \text{ GeV}, & (s_0)_K &= 2.8 \text{ GeV}, \\ f_{\pi'} &= 4.7 \text{ MeV}, & f_{K'} &= 51 \text{ MeV}, & (3.9) \\ \hat{m}_d^+ &= 27 \pm 22 \text{ (MeV)}, & \hat{m}_s^+ &= 325 \pm 264 \text{ (MeV)}, \\ & & 0.1 \leq \Lambda_{\overline{MS}} &\leq 0.2 \text{ (GeV)}. \end{aligned}$$

The corresponding running mass values $\bar{m}_d^+(1 \text{ GeV}) \approx 17 \text{ MeV}$ and $\bar{m}_s^+(1 \text{ GeV}) \approx 205 \text{ MeV}$ are in agreement with (3.7). Thus, we conclude that taking into account the $O(\alpha_s^2)$ correction makes the difference between the FESR mass estimates and those of the current algebra ($m_d^+ \approx 11 \text{ MeV}$) smaller. (Note, however, that the connection between \bar{m} , \hat{m} and current algebra mass definitions is not yet clearly understood).

Our results for the ratios $f_{\pi'}/f_\pi = 0.036$ and $f_{K'}/f_K = 0.335$ are consistent with those obtained by other methods, in particular, with $f_{p'}/f_p = \sqrt{r} m_p^2/m_{p'}^2$, $r = 6 \pm 2$ ^{/7/}. It is also interesting to note, that the use of the FESR version considered in^{/11/} leads to the similar analytic expression for $f_{p'}$. Indeed, following the ideas of ref.^{/11/}, one can make the $1/N$ -expansion inspired conjecture that radial excitation spectra consist of an infinite number of the infinitely narrow resonances, and combine it with the FESR to derive "linear dual models" for such spectra*:

$$f_p(n) = 2\sqrt{2} \frac{m_p^2}{m_p m_p(n)} f_p; \quad (3.10)$$

$$m_p^2(n) = n \cdot m_p^2; \quad n = 1, 2, \dots; \quad p = \pi, K.$$

Notice, that the agreement of $f_{p'} \approx f_p(1)$ with the above estimates is an argument in favour of both the results (3.9) and the "linear dual models". Moreover, as has been emphasized in^{/3/}, the prediction (3.10) of the mass of the second π -meson excitation $m_\pi(2) = 1.75 \text{ GeV}$ agrees with the observation of the pseudoscalar resonance $m_{\pi''} = 1.77 \pm 0.03 \text{ GeV}$ ^{/17/} considered to be the candidate for this role. Encouraged by these facts we propose to search for the second radial excitation of the K -meson near $m_K(2) = 2 \text{ GeV}$ **.

* Analogous "linear dual spectra" are also predicted by the Veneziano model.

**After completing this paper we became aware of the recent experimental data, which suggest the existence of the second excitation of the K -meson around 1.83 GeV ^{/19/}.

To be complete, let us improve the FESR predictions for the scalar meson channel^{/2/} by taking into account the $O(\alpha_s^2)$ correction. Following the standard procedure, we saturate (3.1) by the lowest mesonic state $\delta(980)$ considered here as a quark-antiquark system (for discussions of other possibility see, e.g.,^{/20/}). The existence of the corresponding excitations has not yet been experimentally established. Thus, to calculate $m_d^- = m_d - m_u$ we use only the one-resonance model of the spectral density, the value $f_\delta = 1.2 \text{ MeV}$ ^{/2,6/} for the decay constant being taken as an input parameter. Solving the system of the first two equations (3.8), we get $s_0 = 3/2 m_\delta^2$ and

$$\begin{aligned} \hat{m}_d^- &= 10 \pm 8 \text{ (MeV)}; & m_d^-(1 \text{ GeV}) &= 6 \text{ MeV}; & (3.11) \\ & & 0.1 \leq \Lambda_{\overline{MS}} &\leq 0.2 \text{ (GeV)}. \end{aligned}$$

Eqs. (3.7) and (3.11) lead to the following absolute values of the light quark masses:

$$\begin{aligned} \hat{m}_u &= 8 \pm 6.5 \text{ (MeV)}, & m_u(1 \text{ GeV}) &= 5 \text{ MeV}, \\ \hat{m}_d &= 18 \pm 14.5 \text{ (MeV)}, & m_d(1 \text{ GeV}) &= 11 \text{ MeV}, & (3.12) \\ \hat{m}_s &= 312 \pm 253.5 \text{ (MeV)}, & m_s(1 \text{ GeV}) &= 195 \text{ MeV}, \\ & & 0.1 < \Lambda_{\overline{MS}} < 0.2 \text{ (GeV)} \end{aligned}$$

(the accuracy is about 25%). These results are consistent with those of the previous works on the subject^{/1,6,7/}.

As to the "linear dual models" predictions for this channel, the corresponding model spectra must be different from eqs. (3.10) due to the essentially nonzero masses of the ground scalar hadron states (i.e., the $\delta(980)$ and $k(1350)$ mesons). Indeed, changing the value of the duality interval from $(s_0)_p = m_p^2/2$ to $(s_0)_s = 3m_s^2/2$, we obtain

$$\begin{aligned} m_s^2(n) &= (n+1)m_s^2, \\ f_s^2(n) &= \frac{1}{n+1} f_s^2, \quad n = 0, 1, 2, \dots; \quad s = \delta, k. \end{aligned} \quad (3.13)$$

Thus, on assuming the two-quark structure of the δ -meson, one may expect its radial excitation to have the mass $m_\delta(1) \approx 1.4 \text{ GeV}$. Eqs. (3.13) lead also to $m_k(1) \approx 1.9 \text{ GeV}$ for the mass of the $k(1350)$ excitation, and this estimate does not contradict the experimental indications for this excitation to exist near $m_{k'} = 1.85 \text{ GeV}$ ^{/21/}. Of course, the "linear dual models" are no more than model equations, and thus, one should not overestimate their prediction ability. However, it is worth noting, that in all previously considered cases (see, e.g.,^{/11,3/}) the local duality ideas worked well enough.

4. Acknowledgements. We are grateful to Professors V.A.Matveev and A.N.Tavkhelidze for continuing support and to O.V.Tarasov for informing us of his result^{/16/}. It is also a pleasure to thank K.G.Chetyrkin, A.A.Pivovarov, F.V.Tkachov and especially N.V.Krasnikov for valuable comments. We also wish to express our gratitude to the JINR directorate (Dubna), and especially to Professor I.S.Zlatev for the support in this work and allowing access to the JINR computer center.

REFERENCES

1. Gasser J., Leutwyler H. Phys.Rep., 1982, 87, p.79; de Rafael E. Preprint CTP-81/P.1344, 1981.
2. Truong T. Phys.Lett., 1982, 117B, p.109.
3. Kataev A.L., Krasnikov N.V., Pivovarov A.A. Preprint CERN-TH.3413, Geneva, 1982; Phys.Lett., 1983, 123B, p.93.
4. Gerasimov S.B. Proc. of the Int. Seminar on Vector Mesons and E.M.Int., Dubna, 1969, p.367; Sakurai J.J. Phys.Lett., 1973, 46B, p.207; Chetyrkin K.G., Krasnikov N.V., Tavkhelidze A.N. Phys.Lett., 1978, 76B, p.83.
5. Logunov A.A., Soloviev L.D., Tavkhelidze A.N. Phys.Lett., 1967, 24B, p.181.
6. Narison S. et al. Nucl.Phys., 1983, B121, p.365.
7. Mallik S. Nucl.Phys., 1982, B206, p.90.
8. Becchi C. et al. Z.Phys., 1981, C8, p.335.
9. Novikov V.A. et al. Nucl.Phys., 1981, B191, p.301.
10. Tkachov F.V. Phys.Lett., 1981, 100B, p.65; Chetyrkin K.G., Tkachov F.V. Nucl.Phys., 1981, B192, p.159.
11. Krasnikov N.V., Pivovarov A.A. Phys.Lett., 1982, 112B, p.397.
12. Chanowitz M., Furman M., Hinchliffe I. Nucl.Phys., 1979, B159, p.225.
13. Narison S. Phys.Rep., 1982, 84, p.263.
14. Pennington M.R., Ross G.G. Phys.Lett., 1981, 102B, p.167; Radyushkin A.V. JINR, E2-82-159, Dubna, 1982.
15. Tarasov O.V., Vladimirov A.A., Zharkov A.Yu. Phys.Lett., 1980, 93B, p.429.
16. Tarasov O.V. JINR, P2-82-900, Dubna, 1982.
17. Bellini G. et al. Dubna - Milano - Bologna Collab. Phys. Rev.Lett., 1982, 48, p.1697.
18. Daum C. et al. Nucl.Phys., 1981, B187, p.1.
19. Armstrong T. et al. Preprint CERN/EP 82-95, Geneva, 1982.
20. Achasov N.N., Devyanin S.A., Shestakov G.N. Z.Phys., 1982, C16, p.55.
21. Aston D. et al. Phys.Lett., 1981, 106B, p.235.

Received by Publishing Department
on July 22, 1983.

Горинский С.Г., Катаев А.Л., Ларин С.А. E2-83-533
Высшие пертурбативные КХД-поправки
и массы легких кварков

Вычислены трехпетлевые поправки к двухточечной функции псевдоскалярных кварковых токов в квантовой хромодинамике. Они составляют около 20% от ведущего члена ряда теории возмущений при значениях констант связи $\bar{\alpha}_s = 0,25$. С помощью метода конечно-энергетических правил сумм определены значения масс легких кварков $m_u / 1 \text{ ГэВ} \approx 5 \text{ МэВ}$, $m_d / 1 \text{ ГэВ} \approx 11 \text{ МэВ}$, $m_s / 1 \text{ ГэВ} \approx 195 \text{ МэВ}$. Обсуждаются предсказания линейных дуальных моделей для масс вычетов радиальных возбуждений K , δ и k мезонов.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1983

Gorishny S.G., Kataev A.L., Larin S.A. E2-83-533
Next-Next-to-Leading Perturbative QCD Corrections
and Light Quark Masses

We present the three-loop perturbative corrections to the (pseudo) scalar quark current correlator. They amount to about 20% of the leading term at $\bar{\alpha}_s = 0.25$. Taking them into account and exploiting finite energy sum rules we find the following values of light quark masses: $m_u (1 \text{ GeV}) \approx 5 \text{ MeV}$, $m_d (1 \text{ GeV}) \approx 11 \text{ MeV}$, $m_s (1 \text{ GeV}) \approx 195 \text{ MeV}$. We also discuss the phenomenological predictions of "linear dual models" for radial excitations of the K , δ , and k mesons.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1983