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R.P.Zaikov

SUPERSYMMETRIC QUASIPOTENTIAL EQUATIONS. SUPERSYMMETRIC GENERALIZATION OF THE TODOROV-TYPE EQUATIONS

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1. INTRODUCTION

In papers [1,2] (referred to as I and II) the possibility of extension to the supersymmetric case of the quasipotential equations[3,4] has been discussed. In paper I the transition to the three-dimensional equations may be carried out by the relative-time [3] vanishing in some (fixed in the fermionic sector) centre-of-mass system. The corresponding "equal-time" operation is performed in II by vanishing one of the variables in the light-front [4]. As is well known, for the simplest scalar chiral superfields one multiplet contains the fields with spin 0 and 1/2 (ref. [15]). Therefore, in the supersymmetric case two-particle equations of the quasipotential-type for particles with spin 0 and 1/2 are written down in a common way. Moreover, in the supersymmetric theories there are less divergences than in the ordinry case [5].

In the present paper the two-particle relativistic equations, for which the transition to the three-dimensional equations is achieved by vanishing the relative energy in the center-of-mass system [6-8], are generalized to the supersymmetric case. As has been shown in ref. [7], vanishing of the relative energy (in the centre-of-mass system) is equivalent to the relativistic invariant condition of Markov-Yukawa $(pq = 0)$ in the two-particle wave function, where p is a total momentum and q is a relative momentum. It is evident (see (A.1)) that the Markov-Yukawa condition is invariant with respect to the supertransformations. There are found the supersymmetric equations for the two-particle wave function, where the quasipotential is found in a perturbative way from the quantum field theory. The interaction between one scalar massless and one massive superfield as well as the case with self-interaction are considered. In the lowest order in the coupling constant the quasipotential and the corresponding equations are local as in the ordinary case [8].

In the case when the supersymmetry-breaking terms which give a suitable mass-splitting between bosons and fermions are contained in the Lagrangian they are included in the quasipotential function. Then after eliminating the nondynamical component of the wave function by using the equations of motion, we get the necessary mass-splitting in two-particle quasipotential equations.

Note that as has been shown in ref. [9], the operation of vanishing the relative energy (in the c.m. frame) is equivalent to the gauge fixing procedure for the B-S amplitude with respect to the special type of gauge transformations. The origin of these gauge transformations is the nondefiniteness of the position of the center-of-mass in the case of two interacting particles [9]. Then, the operation of vanishing the relative time or one of coordinates on the light-front is equivalent to the transition from the B-S amplitude to quantities invariant with respect to these transformations.

2. SUPERSYMMETRIC MARKOV-YUKAWA CONDITION

In papers [6-8] the nonphysical relative energy of the two-particle system is removed by the Markov-Yukawa condition

$$p^\mu q_\mu \psi(p,q) = \frac{1}{2}(p_1^2 - p_2^2 - m_1^2 + m_2^2)\psi(p_1, p_2) = 0 \tag{2.1}$$

in the two-particle wave function. Here

$$p = p_1 + p_2, \quad q = \mu_2 p_1 - \mu_1 p_2, \tag{2.2}$$

$$\mu_1 = (p^2 + m_1^2 - m_2^2)/2p^2, \quad \mu_2 = (p^2 - m_1^2 + m_2^2)/2p^2$$

are the total momentum and the relative momentum of the two-particle system, and $\psi(p,q)$ is the Fourier transform of the two-particle Bethe-Salpeter (B-S) amplitude

$$\psi(x_1, x_2) = \langle 0|T(\phi^*(x_1)\phi(x_2))|p\rangle .$$

In the centre-of-mass frame ($p = (E, \underline{0})$) the condition (2.1) is equivalent to the vanishing of the relative energy, i.e., $q_0 = 0$.

The general solution of eq. (2.1) is given by

$$\psi(p,q) = \delta(pq)\psi_p(q).$$

In the case of spinor equal masses, using the identity

$$p_1^2 - p_2^2 = (\not p_1 + \not p_2)(\not p_1 - \not p_2),$$

where

$$\not p_1 = (\gamma_1^\mu \otimes I)p_\mu^1, \quad \not p_2 = (I \otimes \gamma_2^\mu)p_\mu^2,$$

γ_μ^a ($a = 1, 2$) are the Dirac matrices acting on an a-th particle, we have the following condition on the two-particle B-S amplitude

$$(\not p_1 - \not p_2)\psi(p_1, p_2) = 0. \tag{2.3}$$

It may be pointed out that when $m_1 \neq m_2$ the condition (2.3) is more complicated (see ref. [7]).

Then, the following relativistic three-dimensional equation [6-8] (for scalar equal-mass particles)

$$(\frac{1}{4}p^2 + q^2 - m^2)\psi_p(q) = \int d^4k\,\delta(pk)V(p,q,k)\psi_p(k),$$

where V is a quasipotential, is proposed.

Consider the supersymmetric case. In the simplest case when we are dealing with the scalar (real or complex) chiral superfields [5], it is convenient (because of the invariant with respect to spatial reflections) to write the corresponding B-S amplitude in the following form (see I, II):

$$\psi(x_1, x_2; \theta_1, \theta_2) = \begin{pmatrix} \psi^{++}(x_1, x_2; \theta_1, \theta_2) \\ \psi^{-+}(x_1, x_2; \bar\theta_1, \theta_2) \\ \psi^{+-}(x_1, x_2; \theta_1, \bar\theta_2) \\ \psi^{--}(x_1, x_2; \bar\theta_1, \bar\theta_2) \end{pmatrix}, \tag{2.4}$$

where

$$\psi^{\alpha\beta} = \langle 0|T(\Phi^\alpha(x_1, \theta_1)\Phi^\beta(x_2, \theta_2))|p\rangle \qquad (\alpha, \beta = +, -)$$

and $\Phi^+(\Phi^-)$ are left (right) chiral superfields. Here, as in papers I and II, the two-component spinor formalism is used.

In the supersymmetric case the nonphysical relative energy can be removed applying to the B-S amplitude (2.4) the constraint (2.1), which is supersymmetric (see (A.1)). As will be shown below, the constraint (2.1) in the equal-mass case can be found from the following supersymmetric equation:

$$\int K_1(p_1, \theta_1, \theta_3)[d^2\theta_3]\psi(p_1, p_2, \theta_3, \theta_2) - \tag{2.5}$$

$$- \int K_2(p_2, \theta_2, \theta_4)[d^2\theta_4]\psi(p_1, p_2, \theta_1, \theta_4) = 0,$$

where $K_{1(2)}$ are the operator parts of the free field equations for the first and second particles, respectively, $[d^2\theta]$ is given by (A.3). The explicit form of the operators K is given by the formula (A.2). Substituting (A.2) into (2.5), we find the following constraints on the components of the two-particle B-S amplitude

$$
\begin{aligned}
&A\psi(2,0) - B\psi(0,2) = 0,\\
&p_1^2 A\psi(0,0) - B\psi(2,2) = 0,\\
&A\psi(2,2) - p_2^2 B\psi(0,0) = 0,\\
&p_2^2 A\psi(2,0) - p_2^2 B\psi(0,2) = 0,\\
&\not{p}_1^2 A\psi(2,0)\psi(1,1) = 0, \qquad (2.6)\\
&\not{p}_1 \psi(1,0) + B\psi(1,2) = 0,\\
&A\psi(2,1) + \not{p}_2\psi(0,1) = 0,\\
&\not{p}_1 \psi(1,2) + p_2^2 B\psi(1,0) = 0,\\
&p_1^2 A\psi(0,1) + \not{p}_2^2 \psi(2,1) = 0.
\end{aligned}
$$

Here by $\psi(a,b)$ $(a, b = 0,1,2)$ (momentum variables p, q are omitted) we denote the components of the supersymmetric B-S amplitude (2.4), i.e.,

$$
\psi(p_1, p_2; \theta_1, \theta_2) = \psi(p_1, p_2; 0, 0) + (\theta_1)_a \psi^a(p_1, p_2; 1, 0) +
$$
$$
+ (\theta_2)_b \psi^b(p_1, p_2; 0, 1) + \dots + (\theta_1 \epsilon \theta_1)(\theta_2 \epsilon \theta_2)\psi(p_1, p_2; 2, 2),
$$

where it should be taken into account that only chiral representations of the supergroup are considered. For the compactness the constraints (2.6) are written down in terms of bispinor formalism and the following notation is used:

$$
\not{p}_1 = p_1^\mu \Gamma^1_\mu, \quad \not{p}_2 = p_2^\mu \Gamma^2_\mu,
$$

$$
\Gamma^{(1)}_\mu = \begin{pmatrix} 0 & \underset{\sim}{\sigma}_\mu & 0 & 0 \\ \tilde{\sigma}_\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \underset{\sim}{\sigma}_\mu \\ 0 & 0 & \tilde{\sigma}_\mu & 0 \end{pmatrix}, \quad
\Gamma^{(2)}_\mu = \begin{pmatrix} 0 & 0 & \underset{\sim}{\sigma}_\mu & 0 \\ 0 & 0 & 0 & \underset{\sim}{\sigma}_\mu \\ \tilde{\sigma}_\mu & 0 & 0 & 0 \\ 0 & \tilde{\sigma}_\mu & 0 & 0 \end{pmatrix}, \qquad (2.7)
$$

$$
A = \begin{pmatrix} \tau_1 & 0 \\ 0 & \tau_1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}.
$$

Here $\sigma_0 = I - 2 \times 2$ is the identity matrix, $\sigma_j\ (\tau_j)$ $(j = 1,2,3)$ are the Pauli matrices, $\tilde{\sigma}_\mu = \epsilon \sigma_\mu^T \epsilon^{-1}$, $\epsilon = i\sigma_2$. It can be checked that the following identities for the matrices (2.7) are satisfied:

$$
\begin{aligned}
&A^2 = B^2 = I,\\
&[A, B] = 0, \quad [A, \Gamma^j_\mu] = [B, \Gamma^j_\mu] = 0, \qquad (2.8)\\
&\{\Gamma^j_\mu, \Gamma^j_\nu\} = 2g_{\mu\nu} I,
\end{aligned}
$$

where II is the identity matrix and $[\,,\,]$, $\{\,,\,\}$ denote the matrix commutator and anticommutator, respectively. From (2.6) it follows (after some algebraic operation where (2.8) are used) that for any of the components of $\psi(x_1, x_2, \theta_1, \theta_2)$ the constraints (2.1), i.e.,

$$
(p_1^2 - p_2^2)\psi(p_1, p_2, a, b) = (pq)\psi(p, q, a, b) = 0, \qquad (2.9)
$$
$$
(a, b = 0, 1, 2)
$$

are satisfied. It should be pointed out, that as in the ordinary case [7], for the spin 1/2 particles $m_1 \neq m_2$, because of the supersymmetry in the constraints (2.6), the quasipotential must be present.

The general solution of the constraint, eq. (2.9) or (2.6), can be written in the form

$$
\psi(p, q, a, b) = \delta(pq)\psi_p(q, a, b), \qquad (2.10)
$$

where p and q are the total and relative momenta of the two-particle system defined by (2.2).

It should be pointed out that there is a difference as compared to the Logunov-Tavkhelidze approach (I, II) in which the corresponding constraints are not invariant with respect to the supertransformations. Consequently, the corresponding reference frame in the superspace in which the constraints are applied should be fixed (as well as in the fermionic sector). Here, the constraints are invariant with respect to the supertransformations, and consequently, they can be applied in an arbitrary reference frame.

3. SUPERSYMMETRIC THREE-DIMENSIONAL EQUATION FOR THE TWO-PARTICLE WAVE FUNCTION

For the two-particle wave function (2.7), the following supersymmetric three-dimensional equation is proposed

$$\int K_1(p, q, \theta_1, \theta_3)[d^2\theta_3]\psi_p(q, \theta_3, \theta_2) + \\ + \int K_2(p, q, \theta_2, \theta_4)[d^2\theta_4]\psi_p(q, \theta_1, \theta_4) = \\ = \int V(p, q, k; \theta_1, \ldots, \theta_4)\, d\Omega\, \psi_p(k, \theta_3, \theta_4), \tag{3.1}$$

where

$$d\Omega = d^4k\,[d^2\theta_3] \otimes [d^2\theta_4]\,\delta(pk) \tag{3.2}$$

and $[d^2\theta]$ is given by (A.3).Eq. (3.1) is the supersymmetric generalization of the equations proposed in papers /6-8/. The supersymmetric quasipotential $V(p, q, k, \theta_1 \ldots \theta_4)$ can be determined from the series in the coupling constant

$$V = V_1 + V_2 + \ldots, \tag{3.3}$$

where

$$V_1 = -T_1, \quad V_2 = -T_2 + \underset{\smile\smile}{T_1 G T_2}. \tag{3.4}$$

T_k is the k-th term of the supersymmetric transition amplitude, $\smile$ denote integration over the intermediate spinor variables θ, and integration over intermediate momentum variables is implied.

Substituting (A.2) into (3.1) and taking into account (2.6) after integration over the intermediate spinor variables, we find the following equations for the componenst of the superwave function:

$$A\psi(2, 0) + B\psi(0, 2) - 2m\psi(0, 0) = -W(0, 0),$$

$$p_1^2 A\psi(0, 0) + B\psi(2, 2) - 2m\psi(2, 0) = -W(2, 0),$$

$$A\psi(2, 2) + p_2^2 B\psi(0, 0) - 2m\psi(0, 2) = -W(0, 2),$$

$$p_1^2 A\psi(2, 0) + p_2^2 B\psi(0, 2) - 2m\psi(2, 2) = -W(2, 2),$$

$$\not{p}_1 \psi(1, 0) - B\psi(1, 2) + 2m\psi(1, 0) = W(1, 0),$$

$$\not{p}_2 \psi(0, 1) - A\psi(2, 1) + 2m\psi(0, 1) = W(0, 1), \tag{3.5}$$

$$(\not{p}_1 + \not{p}_2 - 2m)\psi(1, 1) = W(1, 1),$$

$$\not{p}_1 \psi(1, 2) - p_2^2 B\psi(1, 0) + 2m\psi(1, 2) = W(1, 2),$$

$$\not{p}_2 \psi(2, 1) - p_1^2 A\psi(0, 1) + 2m\psi(2, 1) = W(2, 1),$$

where the following notation

$$W(a, b) = \int V(p, q, k; a, b, \theta_3, \theta_4)\, d\Omega\, \psi_p(k; \theta_3, \theta_4) \tag{3.6}$$

is introduced, and $d\Omega$ is given by (3.2).

Eliminating the nondynamical components of the wave function from the system of equations (3.5) we get

$$(p_1^2 + p_2^2 + 2m^2)\psi(0, 0) = -mW(0, 0) - \frac{1}{4}W(2, 0) - \frac{1}{4}W(0, 2),$$

$$(\not{p}_1 + m)\psi(1, 0) = \frac{1}{2}W(1, 0),$$

$$(\not{p}_2 + m)\psi(0, 1) = \frac{1}{2}W(0, 1),$$

$$(\not{p}_1 + \not{p}_2 + 2m)\psi(1, 1) = W(1, 1),$$

where the constraints (2.6) are used. We note that according to (3.6) in the r.h.s. of eqs. (3.7) the nondynamical components are present. The last components can be eliminated from (3.7) for any quasipotential.

It should be pointed out that eqs. (3.5) as well as (3.7) have a more simple form in the same fixed reference frame. As such systems there can be considered the c.m.s. ($\underline{p} = 0$) in which case eqs. (3.7) have the form

$$E\left(\frac{1}{4}E^2 - \underline{q}^2 - m^2\right)\psi_E(\underline{q}; 0, 0) = \\ = -\frac{m}{2}W(E, \underline{q}; 0, 0) - \frac{1}{4}W(E, \underline{q}; 2, 0) - \frac{1}{4}W(E, \underline{q}; 0, 2),$$

$$E\left(\frac{1}{2}E\gamma_0^{(1)} - \underline{q}\cdot\gamma^{(1)} + m\right)\psi_E(\underline{q}; 1, 0) = \frac{1}{2}W(E, \underline{q}; 1, 0),$$

$$E\left(\frac{1}{2}E\gamma_0^{(2)} + \underline{q}\cdot\underline{\gamma}^{(2)} + m\right)\psi_E(\underline{q};0,1) = \frac{1}{2}W(E,\underline{q};0,1)\,,$$

$$E\left[\frac{1}{2}E(\gamma_0^{(1)} + \gamma_0^{(2)}) + \underline{q}\cdot(-\underline{\gamma}^{(1)} + \underline{\gamma}^{(2)}) + 2m\right]\psi_E(\underline{q};1,1) = W(E,\underline{q},1,1)\,, \tag{3.7}$$

where $E = p_0$ and

$$W(E,\underline{q};a,b) =$$

$$= \int d^3k\, V(E,\underline{q},\underline{k};a,b,\theta_3,\theta_4)[d^2\theta_3]\,[d^2\theta_4]\psi_E(\underline{k};\theta_3,\theta_4)\,.$$

The l.h.s. of the first eq. (3.7) coincides with the corresponding equation for scalar particles. It may be pointed out that, as follows from (3.6), in the r.h.s. of any of the eqs. (3.5) all the components of $\psi(a,b)$ are present. The contents of $\psi(a,b)$ in $W(a,b)$ depends on the explicit form of the quasipotential V, which can be found as a perturbative series (see (3.3) and (3.4)). To find this dependence, we consider two particular cases for equal-mass particles interacting via the Lagrangians:

1. A massive scalar chiral superfield $\Phi(x,\theta)$, which interacts with a massless chiral field $\chi(x,\theta)$. In this case the interaction Lagrangian has the following form:

$$\mathcal{L}_{int} = g\int d^4x\{\int d^2\theta\,(\Phi^+(x,\theta))^2\chi^+(x,\theta) + $$

$$+ \int d^2\bar\theta\,(\Phi^-(x,\bar\theta))^2\chi^-(x,\bar\theta)\}\,. \tag{3.8}$$

From (3.4) and (3.7), it follows that the nonzero elements of the Born term of the quasipotential are given by

$$V^{+-,+-} = g^2\frac{\exp[2\theta_1(p_1-p_3)\bar\theta_2]}{(p_1-p_3)^2+i\epsilon}\,\delta^{(2)}(\theta_1-\theta_3)\,\delta^{(2)}(\bar\theta_2-\bar\theta_4)\times$$

$$\times\,\delta^{(4)}(p_1+p_2-p_3-p_4)\,, \tag{3.9}$$

$$V^{-+,-+} = g^2\frac{\exp[2\bar\theta_1(\bar p_1-\bar p_3)\theta_2]}{(p_1-p_3)^2+i\epsilon}\,\delta^{(2)}(\bar\theta_1-\bar\theta_3)\,\delta^{(2)}(\theta_2-\theta_4)\times$$

$$\times\,\delta^{(4)}(p_1+p_2-p_3-p_4)\,.$$

Here $\delta^{(2)}(\theta)$ is the Grassmann δ-function [5]. Substituting (3.9) into (3.6), we get the r.h.s. of eqs. (3.5) in the Born approximation. After integration over θ_3 and θ_4, we obtain

$$W_1^{+-}(0,0) = g^2\int\frac{d^4k\,\delta(pk)}{(q-k)^2+i\epsilon}\psi_p^{+-}(k;0,0)\,,$$

$$W_1^{+-}(1,0) = g^2\int\frac{d^4k\,\delta(pk)}{(q-k)^2+i\epsilon}\psi_p^{+-}(k;1,0)\,,$$

$$W_1^{+-}(0,1) = g^2\int\frac{d^4k\,\delta(pk)}{(q-k)^2+i\epsilon}\psi_p^{+-}(k;0,1)\,,$$

$$W_1^{+-}(2,0) = g^2\int\frac{d^4k\,\delta(pk)}{(q-k)^2+i\epsilon}\psi_p^{+-}(k;2,0)\,,$$

$$W_1^{+-}(0,2) = g^2\int\frac{d^4k\,\delta(pk)}{(q-k)^2+i\epsilon}\psi_p^{+-}(k;0,2)\,,$$

$$W_1^{+-}(1,1) = g^2\int\frac{d^4k\,\delta(pk)}{(q-k)^2+i\epsilon}[2(\underset{\sim}{q}-\underset{\sim}{k})\psi_p^{+-}(k;0,0) + \psi_p^{+-}(k,1,1)]\,,$$

$$W_1^{+-}(2,1) = g^2\int\frac{d^4k\,\delta(pk)}{(q-k)^2+i\epsilon}[\psi_p^{+-}(k;2,1) + (\tilde q-\tilde k)\psi_p^{+-}(k;0,1)]\,,$$

$$W_1^{+-}(1,2) = g^2\int\frac{d^4k\,\delta(pk)}{(q-k)^2+i\epsilon}[\psi_p^{+-}(k;1,2) + (\underset{\sim}{q}-\underset{\sim}{k})\psi_p^{+-}(k;1,2)]\,,$$

$$W_1^{+-}(2,2) = g^2\int\frac{d^4k\,\delta(pk)}{(q-k)^2+i\epsilon}[\psi_p^{+-}(k;2,2) + \tag{3.10}$$

$$+ (\epsilon(\underline{q}-\underset{\sim}{k}))^{a}{}_{\dot b}(\psi^{+-})_a{}^{\dot b}(k;1,1) + (q-k)^2\psi_p^{+-}(k;0,0)]\,.$$

Analogously one can find W_1^{+-}. From (3.10) it follows that the first term of the quasipotential and eqs. (3.5) are local.

Solutions to eqs. (3.5) can be found iteratively. From (3.5) and (3.10) it follows that in any order in the coupling constant the nondynamical auxiliary components of the wave functions $\psi(2,a)$ $(\psi(a,2))$ $(a = 0,1,2)$ can be expressed only in terms of the dynamical components $\psi(j,k)$ $(j,k = 0,1)$.

2. Self-interaction of massive chiral superfield. In this case the interaction Lagrangian is given by

$$\mathcal{L}_{int} = g\int d^4x\{\int d^2\theta\,(\Phi^+(x;\theta))^3 + \int d^2\bar\theta\,(\Phi^-(x;\bar\theta))^3\}\,. \tag{3.11}$$

Then the Born term of the quasipotential is given by

$$V_1^{++,++} = \frac{mg^2}{(q-k)^2 - m^2 + i\epsilon}\delta^2(\theta_1 - \theta_2)\delta^2(\theta_1 - \theta_3)\delta^2(\theta_1 - \theta_4), \tag{3.12}$$

$$V_1^{+-,+-} = \frac{g^2}{2}\frac{\exp[\theta_1(q-k)\bar{\theta}_2]}{(q-k)^2 - m^2 + i\epsilon}\delta^2(\theta_1 - \theta_3)\delta^2(\bar{\theta}_2 - \bar{\theta}_4).$$

Inserting (3.12) into (3.6), we get the r.h.s. of eqs. (3.5). Because of a combersome structure of W_1, they are not written here. It should be pointed out that in the second case, too, the quasipotential in the lowest order in the coupling constant and the corresponding equations are local and that the auxiliary components of the wave function can be expressed only in terms of its dynamical components.

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APPENDIX

Generators of the supertransformations for the left (right) chiral superfields $\Phi^+(x,\theta)$ $(\Phi^-(x,\bar{\theta}))$ in the two-component spinor formalism are given by

$$S_a^+ = -i\frac{\partial}{\partial\theta^a}, \quad S_a^- = -i\frac{\partial}{\partial\theta^a} + 2(\partial\bar{\theta})_a, \tag{A.1}$$

$$\bar{S}_{\dot{a}}^+ = i\frac{\partial}{\partial\bar{\theta}^{\dot{a}}} - 2(\theta\partial)_{\dot{a}}, \quad \bar{S}_{\dot{a}}^- = i\frac{\partial}{\partial\bar{\theta}^{\dot{a}}}.$$

Then, it can be checked that the supersymmetric propagator is

$$D = \begin{pmatrix} a\delta^2(\theta_1 - \theta_2) & b\exp(2\theta_1 p\bar{\theta}_2) \\ c\exp(2\bar{\theta}_1 \bar{p}\theta_2) & d\delta^2(\bar{\theta}_1 - \bar{\theta}_2) \end{pmatrix}(p^2 - m^2 + i\epsilon)^{-1},$$

where a, b, c, d are the normalization constants and

$$D^{\alpha\beta}(x_1 - x_2; \theta_1, \theta_2) = \langle 0|T(\Phi^\alpha(x_1,\theta_1)\Phi^\beta(x_2,\theta_2))|0\rangle.$$

If we put

$$a = d = m, \quad b = c = \frac{1}{2}$$

we have

$$\int D(p,\theta_1,\theta_2)[d^2\theta]K(p,\theta_2,\theta_3) = -\delta(\theta_1 - \theta_3),$$

where

$$K = -D^{-1} = \begin{pmatrix} m\delta^2(\theta_1 - \theta_2) & -\frac{1}{2}\exp(2\theta_1 p\bar{\theta}_2) \\ -\frac{1}{2}\exp(2\bar{\theta}_1\bar{p}\theta_2) & m\delta^2(\bar{\theta}_1 - \bar{\theta}_2) \end{pmatrix} \tag{A.2}$$

and

$$[d^2\theta] = \begin{pmatrix} d^2\theta & 0 \\ 0 & d^2\bar{\theta} \end{pmatrix} \quad [\delta(\theta)] = \begin{pmatrix} \delta^2(\theta) & 0 \\ 0 & \delta^2(\bar{\theta}) \end{pmatrix}. \tag{A.3}$$

Consequently, G is the time-ordered Green function for the equation

$$\int K[d^2\theta]\Phi = R(x,\theta). \tag{A.4}$$

It can be checked that the l.h.s. of eq. (A.4) is equivalent to the corresponding equations written down by the spinor covariant derivatives /6/.

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Зайков Р.П. E2-83-49

Суперсимметричные квазипотенциальные уравнения. Суперсимметричное обобщение уравнения Тодорова

Рассмотрено суперсимметричное обобщение квазипотенциального уравнения в калибровке Маркова-Юкавы. Накладывая на двухчастичную амплитуду Б-С суперсимметричное условие Маркова-Юкавы получаем условия, дающие переход к одновременной волновой функции. Для этой волновой функции записано суперсимметричное трехмерное уравнение. Найден борновский член квазипотенциала для самодействующего кирального суперполя, а также для взаимодействия с безмассовым киральным суперполем.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Zaikov R.P. E2-83-49

Supersymmetric Quasipotential Equations. Supersymmetric Generalization of the Todorov-Type Equations

A supersymmetric extension of the Todorov-type quasipotential equations is considered. Applying to the two-particle Bethe-Salpeter amplitude the supersymmetric Markov-Yukawa condition, the corresponding constraints are found which give the transition to the one-time wave function. The corresponding supersymmetric three-dimensional equations are written down. The Born terms of the quasipotential in the case of a self-interacting chiral superfield as well as of interaction with one massless chiral superfield are given.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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