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M.Bordag, D.Robaschik, E.Wieczorek

QUANTUM FIELD THEORETIC TREATMENT OF THE CASIMIR EFFECT.

Quantization Procedure and Perturbation Theory in Covariant Gauge

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1. INTRODUCTION

An interesting feature of QCD is the nontrivial vacuum structure which is however very difficult to investigate [1]. Phenomenologically the vacuum structure is described by nonvanishing vacuum expectation values of local operators, a situation which contradicts the usual normal ordering applied in perturbation theory. On the other hand, such a situation is already known in physics and has experimental consequences. An example of this type is the well-known Casimir effect [2]. Two conducting neutral plates attract each other because the vacuum energy of the electromagnetic field of this system is different from zero and depends on the distance between the two plates. This effect tests in principle the ordering of the energy operator and rules out the usually applied normal ordering. In this sense this effect is one of the basic effects of QED.

The treatment of this effect on a consequent field-theoretical level has several interesting theoretical aspects:

- The quantum field theory of gauge fields with boundary conditions has to be formulated. This is a nontrivial task if we apply a covariant gauge fixing term.
- The physical quantity to be calculated primarily is the energy of the vacuum state as a function of the distance between the two plates. This means one has to calculate vacuum diagrams with operator insertions (energy operator). The calculation of such bubble diagrams lies beyond the usual applicability of renormalization procedure, and it is also interesting in itself. An important point thereby is the finiteness of physical quantities. As will be shown in a subsequent paper, the Casimir force as an observable quantity is finite without any renormalization up to second order of perturbation theory.

Besides these theoretical aspects, of course, there are practical ones. How large are radiative corrections to the Casimir force, could these be experimentally verified?

To calculate corrections we must fix our model. In the free field theory we have to consider the electromagnetic field only, the plates are realized by boundary conditions for the electromagnetic field strength at the places of the plates. For the calculation of radiative corrections the electron field interacting with the photon field has to be taken into account. So, we must have some imaginations about the influence of the plates on the electron field. Therefore we propose the following physical simplifications: The physically superconducting plates are infinitely thin, do not influence the electron field, interact with the electromagnetic field via the boundary conditions only. So, we postulate no boundary conditions for the electron field (note: boundary conditions for the electron field would lead to an additional not observed Casimir effect. Moreover the question appears: "Which kind of boundary conditions could be realized experimentally?"). On the basis of this model we calculate the radiative corrections to the Casimir force.

The present paper is organized in three sections. After the introduction we derive in the second part the quantization of the spinor electrodynamics with boundary conditions in a covariant gauge. As a technical tool we use the path integral that here also allows a simple derivation of modified Feynman rules. In the third section we derive a closed but formal expression for the energy and for the Casimir force up to second order of perturbation theory. A further discussion of these expressions including the proof of their finiteness in postponed to a second paper/5/.

2. QUANTIZATION OF SPINOR ELECTRODYNAMICS WITH BOUNDARY CONDITIONS

Here we will formulate the quantization procedure for the photon and electron fields with boundary conditions. We assume that the space is divided into three regions by two infinite large and infinite thin superconducting plates perpendicular to the x_3-axis at $x_3 = a_0$ and $x_3 = a_1$. As boundary conditions for the electromagnetic field we have $n^\mu F^*_{\mu\nu}(x)_{x_3=a_i} = 0$, where $F^*_{\mu\nu}$ is the dual electromagnetic field strength tensor related to the usual one by $F^*_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta} F^{\alpha\beta}$; $n_\mu = (0,0,0,1)$ is the normal vector of the plates. For the spinor field ψ we propose no boundary condition. The classical Lagrangian of spinor electrodynamics including the gauge breaking term with the gauge parameter α reads

$$\mathfrak{L}(x) = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2\alpha}(\partial_\mu A^\mu)^2 + \bar{\psi}(i\hat{\partial} - m + e\hat{A})\psi . \tag{2.1}$$

A_μ denotes the electromagnetic potential, scalar products with γ-matrices, e.g., $A_\mu \gamma^\mu$ are abbreviated by $\hat{A}$. The corresponding canonical energy momentum tensor $T_{\mu\nu}$ has the form

$$T_{\mu\nu} = -\frac{\partial A^\lambda}{\partial x^\nu} F_{\lambda\mu} - \frac{1}{\alpha}\frac{\partial A_\mu}{\partial x^\nu}\frac{\partial A^\rho}{\partial x^\rho} - g_{\mu\nu}\{-\frac{1}{4}F_{\rho\lambda}F^{\rho\lambda} - \frac{1}{2\alpha}\frac{\partial A^\rho}{\partial x^\rho}\frac{\partial A^\lambda}{\partial x^\lambda}\} +$$

$$+ \frac{i}{2}[\bar{\psi}\gamma_\mu \frac{\partial \psi}{\partial x^\nu} - \frac{\partial \bar{\psi}}{\partial x^\nu}\gamma_\mu \psi] . \tag{2.2}$$

The quantum field theory is fixed by knowing the set of all complete Green functions. Within the path-integral formalism the generating functional of all these Green functions is represented by the functional integral

$$Z(j, \bar{\eta}, \eta) = C \int DAD\bar{\psi}D\psi \exp i \int d^4x [\mathfrak{L}(x) + j_\mu A^\mu + \bar{\eta}\psi + \bar{\psi}\eta] . \tag{2.3}$$

C is a normalization factor, j_μ, $\bar{\eta}$, η denote the sources of the electromagnetic potential and the spinor fields. The integration runs over all fields satisfying the usual asymptotic conditions*. In this way QED is defined formally. If we want to set up a quantum field theory with boundary conditions, then the functional integral has additionally to respect these boundary conditions; the integration has to be performed only over fields that satisfy the boundary conditions. This restriction is most simply guaranteed by insertion of δ-functions directly into the functional integral

$$\delta(n^\mu F^*_{\mu\nu}(x)|_{x_3=a_0}) \; \delta(n^\mu F^*_{\mu\nu}(x)|_{x_3=a_1}) .$$

The integration over A_μ than runs over all fields as above. It is important that the introduction of such gauge invariant conditions does not change the usual integration over the volume of the gauge group, so that the original Faddeev-Popov procedure/6/ is not changed. For further calculations it is convenient to represent the δ-functions by functional integrals

$$\delta(n^\mu F^*_{\mu\nu}(x)|_{x_3=a_i}) = C\int DB_i \exp i \int dS_i(x)\, B^{i\alpha}(x)\, H_{\alpha\mu}(x,\partial_x)\, A^\mu(x) \tag{2.4}$$

with

$$dS_i(x) = d^4x\, \delta(x_3 - a_i), \tag{2.5}$$

$$H_{\alpha\mu}(x,\partial_x) = -n^\lambda \epsilon_{\lambda\alpha\mu\gamma}\frac{\partial}{\partial x_\gamma}$$

and C a normalization factor. $B^{i\alpha}(x)$ are auxiliary fields which exist on the plate i only, i.e., they depend on the variables x^α $(\alpha = 0,1,2)$.

Now the integrand $B^{i\alpha}(x)\, H_{\alpha\mu}(x,\partial_x)\, A^\mu(x)$ is invariant under the gauge transformation

* For the asymptotic conditions see, for example, refs./3,4/.

$$B^{i\alpha}(x) \to B^{i\alpha}(x) + \frac{\partial}{\partial x^\alpha}\phi(x^\beta) \qquad (\alpha, \beta = 0, 1, 2).$$

In some sense this reflects the fact that there are only two independent boundary conditions. In solving this problem in a standard manner we introduce an additional gauge fixing term, integrate over the volume of the gauge group, and rewrite (2.4) in the form

$$\delta(n^\mu F^*_{\mu\nu}(x)|_{x_3=a_i}) = C\int DB^i \exp i\Big[\int dS_i(x)\, B^{i\alpha}(x)\, H_{\alpha\mu}(x, \partial_x)\, A^\mu(x) + \frac{1}{2\beta}\int dS_i(x)\, dS_i(y)\, B^{i\alpha}(x)\frac{\partial}{\partial x^\alpha}\frac{\partial}{\partial y^\beta} D^c(x_\alpha - y_\alpha, 0)\, B^{i\beta}(y)\Big], \tag{2.6}$$

where $D^c(x_\alpha - y_\alpha, 0)$ is the usual Green function of the massless scalar field

$$D^c(x-y) \equiv D^c(x_\alpha - y_\alpha, x_3 - y_3) = \int \frac{d^4p}{(2\pi)^4}\frac{e^{ip(x-y)}}{-p^2 - i\epsilon}, \tag{2.7}$$

taken at $x_3 - y_3 = 0$.

Having in mind these considerations the generating functional reads now

$$\begin{aligned} Z_B(j, \bar\eta, \eta) = G\int DA\, DB\, D\bar\psi\, D\psi \exp i\Big\{\int d^4x\,[\mathcal{L}(x) &+ j_\mu A^\mu + \bar\eta\psi + \bar\psi\eta] + \\ &+ \int dS_i(x)\, B^{i\alpha}(x)\, H_{\alpha\mu}(x, \partial_x)\, A^\mu(x) - \\ &- \frac{1}{2\beta}\int dS_i(x)\, dS_i(y)\, B^{i\alpha}(x)\frac{\partial}{\partial x^\alpha}\frac{\partial}{\partial y^\beta} D^c(x_\alpha - y_\alpha, 0) B^{i\beta}(y)\Big\} \end{aligned} \tag{2.8}$$

for the case of QED with boundary conditions. A further simplification of this expression is desirable, however. Let us proceed in the following way. First we eliminate the term linear in the B -field by the follows shift of the integration variable A_μ

$$A_\mu(x) \to A_\mu(x) - \int dS_i(z)\, B^{i\alpha}(z)\, H_{\alpha\nu}(z, \partial_z)\, D^c_{\nu\mu}(z - x)$$

and receive a term quadratic in the B -field. This term together with the gauge fixing term can be written as

$$-\frac{i}{2}\int dS_j(x)\, dS_i(y)\, B^{i\alpha}(y)\, H_{\alpha\mu}(y, \partial_y)\, D^{c\,\mu\nu}(y-x)\, H_{\nu\beta}(x, \partial_x)\, B^{j\beta}(x) -$$

$$-\frac{i}{2\beta}\int dS_i(x)\, dS_j(y)\, B^{i\alpha}(x)\frac{\partial}{\partial x^\alpha}\frac{\partial}{\partial y^\beta} D^c(x_\alpha - y_\alpha; 0)\, B^{i\beta}(y) = \tag{2.9}$$

$$\equiv \frac{i}{2}\int dS_i(x)\, dS_j(y)\, B^{i\alpha}(x)\, \tilde{k}^{ij}_{\alpha\beta}(x-y)\, B^{j\beta}(y).$$

It is clear from the construction that the just defined function (2.9) $\tilde{k}^{ij}_{\alpha\beta}(x-y)$ is defined on the three-dimensional subspace x^α $(\alpha = 0, 1, 2)$ only. The momentum space representation is

$$\tilde{k}^{ij}_{\alpha\beta}(x-y) = \int \frac{d^3p}{(2\pi)^3} e^{-ip_\alpha (x-y)^\alpha} \frac{-i\Gamma}{2}\Big\{\Big(g_{\alpha\beta} - \frac{p_\alpha p_\beta}{\Gamma^2}\Big)h_{ij} + \frac{1}{\beta}\frac{p_\alpha p_\beta}{\Gamma^2}\delta_{ij}\Big\} \tag{2.10}$$

with

$$\Gamma = \sqrt{p_\alpha p^\alpha + i\epsilon}, \qquad \mathrm{Im}\,\Gamma > 0$$

and $h_{ij} = \exp i\Gamma|a_i - a_j|$ as a 2x2 matrix. This leads us to a final closed expression for the functional Z_B

$$\begin{aligned} Z_B(j, \bar\eta, \eta) = G\int DA\, DB\, D\bar\psi\, D\psi \exp i\Big\{\int d^4x\Big[\frac{1}{2}A_\mu(x)K^{\mu\nu}(\partial_x)A_\nu(x) &+ \\ + \bar\psi(i\hat\partial - m)\psi\Big] + \frac{1}{2}\int dS_i(x)\, dS_j(y)\, B^{i\alpha}(x)\, \tilde{k}^{ij}_{\alpha\beta}(x-y)\, B^{j\beta}(y) &+ \\ + \int d^4x\,(j^\mu + \bar\psi\gamma^\mu\psi)\Big(A_\mu(x) - \int dS_i(z)\, B^{i\alpha}(z)\, H_{\alpha\nu}(z, \partial_z)\, D^{c\,\nu\mu}(z-x)\Big) &+ \\ + \int d^4x\,(\bar\psi\eta + \bar\eta\psi). & \end{aligned} \tag{2.11}$$

For concrete calculations, however, we have to do with perturbative calculations which usually work remarkably good in QED. A formulation suited for perturbation theory can be developed along standard methods. We perform the Gauss integrations in (2.11) and obtain

$$\begin{aligned} Z_B(j, \bar\eta, \eta) = \exp\Big(\frac{1}{2}\frac{\delta}{\delta A_\mu} iD^c_{\mu\nu}\frac{\delta}{\delta A_\nu} + \frac{1}{2}\frac{\delta}{\delta B^{i\alpha}} i\tilde{k}^{-1\,ij}_{\alpha\beta}\frac{\delta}{\delta B^{j\beta}} + \frac{\delta}{\delta\bar\psi}\frac{1}{i}\hat{S}^c\frac{\delta}{\delta\psi}\Big) \\ \times \exp i\{(j_\mu + e\bar\psi\gamma_\mu\psi)(A^\mu - D^{c\,\mu\nu}H_{\nu\alpha}B^{i\alpha}) + \\ + \bar\eta\psi + \bar\psi\eta\}|_{A=B=\bar\psi=\psi=0}. \end{aligned} \tag{2.12}$$

Here the corresponding integrals over all variables are dropped, furthermore we have used the following notation:
The electron propagator

$$\hat{S}^c(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{\hat{p}+m}{-p^2+m^2-i\epsilon} e^{-ip(x-y)} . \qquad (2.13)$$

The inversed kernel $\tilde{k}^{-1ij}_{\alpha\beta}$ defined by

$$\int dS_j(y) \tilde{k}^{ij}_{\alpha\beta}(x-y) \tilde{k}^{-1jk}_{\beta\gamma}(y-z) = g_{\alpha\gamma}\delta_{ik}\delta(x_\alpha - z_\alpha)$$

reads

$$\tilde{k}^{-1ij}_{\alpha\beta}(x-y) = \int \frac{d^3p}{(2\pi)^3} \frac{2i}{\Gamma} \{ (g_{\alpha\beta} - \frac{p_\alpha p_\beta}{\Gamma^2}) h^{-1}_{ij} + \beta \frac{p_\alpha p_\beta}{\Gamma^2} \delta_{ij} \} e^{-ip_\alpha(x-y)^\alpha} . \qquad (2.14)$$

The inverse matrix

$$h^{-1}_{ij} = \frac{-1}{2i \sin \Gamma|a_0 - a_1|} \begin{pmatrix} e^{-i\Gamma|a_0-a_1|}, & -1 \\ -1, & e^{-i\Gamma|a_0-a_1|} \end{pmatrix} \qquad (2.15)$$

depends on the distance of the two plates only. Because the variables A_μ and B^i_α enter into the expression (2.12) in the combination $A_\mu - D_{\mu\nu}H_{\nu\alpha}B^i_\alpha$ only and by means of

$$\frac{\delta}{\delta B^i_\alpha} F(A_\mu - D_{\mu\nu}H_{\nu\alpha}B^i_\alpha)|_{B=0} = H^i_{\alpha\nu} D_{\nu\mu} \frac{\delta}{\delta A_\mu} F(A) , \qquad (2.16)$$

in the whole expression appears the combination of propagators

$$^sD^c_{\mu\nu}(x,y) = D^c_{\mu\nu}(x-y) + \tilde{D}^c_{\mu\nu}(x,y) \qquad (2.17)$$

only, and the generating functional reads

$$Z_B(j,\bar{\eta},\eta) =$$

$$= \exp\left(\frac{1}{2} \frac{\delta}{\delta A_\mu} i\, ^sD^c_{\mu\nu} \frac{\delta}{\delta A_\nu} + \frac{\delta}{\delta\psi} \frac{1}{i} \hat{S}^c \frac{\delta}{\delta\bar{\psi}}\right) \times \qquad (2.18)$$

$$\times \exp i(e\bar{\psi}\hat{A}\psi + j_\mu A^\mu + \bar{\psi}\eta + \bar{\eta}\psi)|_{A=\bar{\psi}=\psi=0} .$$

In (2.17) $D^c_{\mu\nu}$ is the usual free field propagator and $\tilde{D}^c_{\mu\nu}$ is constructed out of the kernel $\tilde{k}^{-1ij}_{\alpha\beta}$ by taking into account (2.16). It has the form

$$\tilde{D}^c_{\mu\nu}(x,y) = \qquad (2.19)$$

$$= \int dS_i(z) dS_j(z') D^c_{\mu\rho}(x-z) H_{\rho\alpha}(z,\partial_z) \tilde{k}^{-1ij}_{\alpha\beta}(z,z') H_{\beta\rho'}(z',\partial_{z'}) D^c_{\rho'\nu}(z'-y) .$$

Its momentum space representation reads

$$\tilde{D}^c_{\alpha\beta}(x,y) = \int \frac{d^3p}{(2\pi)^3} \frac{-i}{2\Gamma} (g_{\alpha\beta} - \frac{p_\alpha p_\beta}{\Gamma^2}) e^{ip_\alpha(x-y)^\alpha} \qquad (2.20)$$

$$\times e^{i\Gamma|x_3-a_i|} h^{-1}_{ij} e^{i\Gamma|y_3-a_j|}$$

$$\tilde{D}^c_{\alpha 3} = \tilde{D}^c_{3\beta} = \tilde{D}^c_{33} = 0 .$$

The obtained result is quite remarkable: We have recovered formally the standard perturbation theory of QED. There is only one essential change: the photon propagator $D^c_{\mu\nu}$ has to be substituted by $^sD^c_{\mu\nu}$. So the usual Feynman diagram technique remains. This has to be expected for general reasons. The advantage of our treatment is that it can be applied in principle to boundary-value problems on arbitrary manifolds and for non Abelian gauge theories, too.

For further applications we list some important properties of the new contribution $\tilde{D}^c_{\mu\nu}$ to the propagator:

$$\frac{\partial}{\partial x_\mu} \tilde{D}^c_{\mu\nu}(x,y) = \frac{\partial}{\partial y_\nu} \tilde{D}^c_{\mu\nu}(x,y) = 0 \qquad (2.21)$$

$$\int d^4y\, ^sD^c_{\mu\nu}(x,y)(g^{\nu\rho}\partial^2 - \partial^\nu\partial^\rho)\, ^sD^c_{\rho\lambda}(y,z) = {}^sD^{c(\alpha=0)}_{\mu\lambda}(x,z) .$$

These relations can be proved by explicit calculations in momentum space or directly from its definition. Note that the validity of these relations is not restricted to the considered case of parallel plates.

3. CASIMIR FORCE

In the foregoing section we have derived the perturbative formulation of QED so that in principle arbitrary processes would be calculable. Of course, one has to check, that some version of renormalization theory works. However, we will not work out such a program rather restrict ourselves strictly to the problem of the Casimir force. That means we are interested in the properties of the vacuum state only. As an intermediate step we calculate the energy density of the vacuum state. This quantity can be determined from the known Z -functional (2.16) and the explicit expression of the energy momentum tensor (2.2). In a formal manner we get

$$<0|\, T_{00}(A, \bar{\psi}, \psi)\,|\, 0> \; = \; T_{00}\left(\frac{\delta}{\delta j}, \frac{\delta}{\delta \bar{\eta}}, \frac{\delta}{\delta \eta}\right) \frac{Z_B(j, \bar{\eta}, \eta)}{Z_B(0,0,0)}\Big|_{j = \bar{\eta} = \eta = 0} \; . \tag{3.1}$$

This expression contains Green functions with identified arguments, so that it appears as a highly divergent quantity, and some regularization procedure has to be introduced. As the simplest possibility we apply the point splitting for coinciding arguments. Our final task, however, is the calculation of the Casimir force which is a force per unit area. Its corresponding energetical quantity is the energy density per unit area

$$E(a) = \int_{-\infty}^{\infty} dx_3 <0|\, T_{00}(A, \bar{\psi}, \psi)\,|\, 0>. \tag{3.2}$$

This expression depends on the distance of the plates $a = |a_0 - a_1|$. The Casimir force appears as its derivative with respect to a. Because we are interested in the Casimir force as a well defined physical quantity only, we omit all a independent (divergent or not divergent) terms in E(a).

a) Casimir Force in Zero Order

Let us consider the $(e^2)^0$ approximation, i.e., the free field approximation. Remember that we have posed no boundary condition on the spinor field, so that its contribution does not depend on the distance and can be omitted completely. There remains the contribution of the electromagnetic field to the energy density

$$<0|\, T_{00}|\, 0>_{(e^2)^0}$$

$$= \left[-\frac{1}{2}\left(-\frac{\partial}{\partial x_\rho}\frac{\partial}{\partial y^\rho} + 2\frac{\partial}{\partial x_0}\frac{\partial}{\partial y_0}\right) g^{\mu\nu} + \frac{1}{2}\left(1 - \frac{1}{\alpha}\right)\frac{\partial}{\partial x_\mu}\frac{\partial}{\partial x^\nu}\right] i^s D^c_{\mu\nu}(x,y)\Big|_{x\to y}$$

$$= -\frac{1}{2}\left(-\frac{\partial}{\partial x_\rho}\frac{\partial}{\partial y^\rho} + 2\frac{\partial}{\partial x_0}\frac{\partial}{\partial y_0}\right)\left[4i\, D^c(x-y) + 2i\, \tilde{D}^c(x,y)\right]\Big|_{x\to y} . \tag{3.3}$$

For brevity we have introduced the scalar quantities

$$\tilde{D}^c(x,y) = \frac{1}{2} g^{\mu\nu} \tilde{D}^c_{\mu\nu}(x,y) \quad \text{and} \quad D^c(x-y) \quad (2.7). \tag{3.4}$$

As a next step we have to calculate the energy per unit area, thereby we introduce $\delta = (0,0,0,\delta)$ as a point splitting parameter so that

$$E(a) = \lim_{\delta\to 0} E_\delta(a) = \tag{3.5}$$

$$= \lim_{\delta\to 0} \int_{-\infty}^{\infty} dx_3 \left(-\frac{1}{2}\right)\left(2\frac{\partial}{\partial x_0}\frac{\partial}{\partial y_0} - \frac{\partial}{\partial x_\rho}\frac{\partial}{\partial y^\rho}\right)\left[4i\, D^c(x-y) + 2i\, \tilde{D}^c(x,y)\right]\Big|_{x\to y} .$$

The first term here contains the nornal free field Green function which does not depend on a, so it can be dropped. The second term can be rewritten as

$$E(a) = \lim_{\delta\to 0} \int_{-\infty}^{\infty} dx_3\, i\, \partial_{\xi\xi}\, \tilde{D}^c(0; x_3, x_3 + \delta) =$$

$$= \lim_{\delta\to 0} \int d^4\xi\; \delta(\xi)\, \frac{1}{2}\, \partial_{\xi\xi} \int_{-\infty}^{\infty} dx_3\, 2i\, \tilde{D}^c(\xi_0, \xi_3 + x_3, x_3; \delta), \tag{3.6}$$

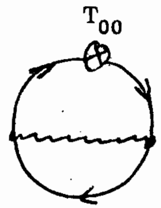
$$\delta_{\xi\xi} = 2\frac{\partial^2}{\partial \xi_0^2} + \frac{\partial}{\partial \xi_\rho}\frac{\partial}{\partial \xi^\rho} .$$

As it will be shown in ref.[5], the Casimir force $F(a) = -\frac{d}{da}E(a)$ is a finite quantity which needs no regularization or subtractions. This is contrary to energy expressions.

b) Radiative Corrections to the Casimir Force

As a next step we study the e^2 -approximation to the Casimir force. Looking at the general expression for the energy density (3.1), (2.2) we see that the inclusion of radiative corrections means that we have to take into account higher approximations for the Green functions. These are the electron and photon propagators with self-energy insertions. Note that the self-energy part of the electron propagator contains the a -dependent modified photon propagator so that also the electron field contributes now to the Casimir force. There are two diagrams

A: electromagnetic part of
the energy density



B: electron part of
the energy density

The corresponding explicit expressions are

$$\langle 0|\, T_{00}\, |0\rangle^{(A)}$$

$$= -i\left[-\frac{1}{2}\partial_{xy} g^{\mu\nu} + \frac{1}{2}\left(1-\frac{1}{\alpha}\right)\frac{\partial}{\partial x_\mu}\frac{\partial}{\partial x_\nu}\right]\int dz\,dz'\, {}^{S}D^{c}_{\mu\mu'}(x,z)\,\Pi^{\mu'\nu'}(z-z')\,{}^{S}D^{c}_{\nu'\nu}(z',y)\Big|_{x\to y} \tag{3.7}$$

and

$$\langle 0|\, T_{00} |0\rangle^{(B)} = -i\int dz\,dz'\, {}^{S}D^{c}_{\rho'\lambda'}(z,z')\, T_{00\rho\lambda}(x-z,\, x-z')\, g^{\rho\rho'} g^{\lambda\lambda'} \tag{3.8}$$

with

$$T_{\mu\nu\rho\lambda}(x-z,\, x-z') = -i\, e^2\, \mathrm{Tr}\left\{\gamma_\mu \frac{\partial}{\partial x^\nu}\, \hat{S}(x-z)\,\gamma_\rho\, \hat{S}(z-z')\,\gamma_\lambda\, \hat{S}(z'-x)\right\} \tag{3.9}$$

and the polarization operator /7/

$$\Pi_{\mu\nu}(z-z') = -i e^2\, \mathrm{Tr}\,\left\{\gamma_\mu \hat{S}(z-z')\,\gamma_\nu\, \hat{S}(z'-z)\right\}. \tag{3.10}$$

Of course, all these expressions are divergent because of the well-known uv-divergences. So one has to apply additionally a regularization procedure which we will not denote explicitly. In what follows it can be seen that the gauge parameter depending contributions either are independent of a or vanish (eq.2.20) so that they are in fact not present.

Let us first discuss the expression A. If we use explicitly the representation of the photon propagator (2.17), then it splits into three contributions

$$\langle 0|\, T_{00}\, |0\rangle^{(A)} = \sum_{i=0}^{3} T^{(i)}$$

$$T^{(1)} = \frac{1}{2}\partial_{xy}\, g^{\mu\nu}\int dz\,dz'\, D^{c}_{\mu\mu'}(x-z)\,\Pi^{\mu'\nu'}(z-z')\, D^{c}_{\nu'\nu}(z'-y)\Big|_{x\to y} \tag{3.11a}$$

$$T^{(2)} = \frac{1}{2}\partial_{xy}\, g^{\mu\nu}\int dz\,dz'\, D^{c}_{\mu\mu'}(x-z)\,\Pi^{\mu'\nu'}(z-z')\, \tilde{D}^{c}_{\nu'\nu}(z',y)\Big|_{x\to y} \tag{3.11b}$$

$$T^{(3)} = \frac{1}{2}\partial_{xy}\, g^{\mu\nu}\int dz\,dz'\, \tilde{D}^{c}_{\mu\mu'}(x,z)\,\Pi^{\mu'\nu'}(z-z')\, \tilde{D}^{c}_{\nu'\nu}(z',y)\Big|_{x\to y}. \tag{3.11c}$$

The first expression contains the contributions of standard QED without boundary conditions and can be dropped (a-independent). Into the other terms we insert the usual representation of the polarization operator

$$\Pi_{\mu\nu}(z) = \left(g_{\mu\nu}\frac{\partial^2}{\partial z^2} - \frac{\partial}{\partial z^\mu}\frac{\partial}{\partial z^\nu}\right)\Pi(z^2) \tag{3.12}$$

with the result

$$T^{(2)} = 2\partial_{xy}\int dz\, \Pi(x-z)\, \tilde{D}^{c}(z,y)\Big|_{x\to y} \tag{3.13a}$$

$$T^{(3)} = \partial_{xy}\int dz\,dz'\, \tilde{D}^{c}(x,z)\,\Pi(z-z')\,\tilde{D}^{c}(z',y)\Big|_{x\to y}. \tag{3.13b}$$

The next step is the calculation of the energy per unit area. After the x_3-integration we have

$$E^{(A2)}_{(a)} = \lim_{\delta\to 0}(-2)\int d^4\xi\, \Pi(\xi^2)\,\partial_{\xi\xi}\int_{-\infty}^{\infty} dx_3\, \tilde{D}^{c}(\xi_\alpha;\, \xi_3+x_3,\, x_3+\delta)$$

$$E^{(A3)}_{(a)} = \lim_{\delta\to 0}\left(-\frac{1}{2}\right)\int d^4\xi\, \Pi(\xi^2)\,\partial_{\eta\eta}\int d^4 z' \int_{-\infty}^{\infty} dx_3$$

$$\times\, \tilde{D}^{c}(z_\alpha;\, z'_3,\, x_3)\frac{\partial}{\partial \xi_\lambda}\frac{\partial}{\partial \xi^\lambda}\tilde{D}^{c}(\xi_\alpha + z'_\alpha + \eta_\alpha;\, \xi_3+z'_3,\, x_3+\eta_3+\delta)\Big|_{\eta=a}.$$

This has to be inserted into the formula for the Casimir force

$$F(a) = -\frac{d}{da}E(a). \tag{3.14}$$

Let us now discuss the second contribution B to the energy density, that contains the self-energy part of the electron. In eq. (3.8) we replace the photon propagator ${}^{S}D^{c}_{\mu\mu'}$ by $\tilde{D}^{c}_{\mu\mu'}$ because this is the only a-dependent quantity in this formula

$$\langle 0|\, T_{\mu\nu}\, |0\rangle^{(B)} = \int dz\,dz'\, \tilde{D}^{c}_{\rho\lambda}(z,z')\, T_{\mu\nu\rho\lambda}(x-z,\, x-z'). \tag{3.15}$$

This leads us to formula

$$E^{(B)}_{(a)} = \lim_{\delta\to 0} \int dz\, dz'\, \tilde{D}^c_{\rho\lambda}(z,z') \int_{-\infty}^{\infty} dx_3\, T_{00\rho\lambda}(x-z,\, x+\delta-z) \tag{3.16}$$

$$= \lim_{\delta\to 0} \int d^4\xi\, \tau^{\rho\lambda}(\xi) \int_{-\infty}^{\infty} dx_3\, \tilde{D}^c_{\rho\lambda}(\xi_a;\, \xi_3 + x_3,\, x_3 + \delta)$$

for the energy per unit area. As an auxiliary quantity we have introduced

$$\tau_{\rho\lambda}(\xi) = \int d^4\eta\, T_{00\rho\lambda}(-\xi-\eta,\, -\eta). \tag{3.17}$$

In the Appendix we derive a special representation of $\tau_{\rho\lambda}$ which is very suited (and also restricted) for our investigations

$$\tau_{\rho\lambda}(\xi) = \frac{1}{2} g_{\rho\lambda} \{ \partial_{\xi\xi} \Pi(\xi^2) + \frac{\partial}{\partial\xi_\rho} \frac{\partial}{\partial\xi^\rho} \frac{\partial^2}{\partial\xi_0^2} \xi^2 \Pi(\xi^2) \} \tag{3.18}$$

which relates all quantities to be calculated in perturbation theory to the standard polarization operator.

Let us collect finally all contributions to the Casimir force into the final expression

$$F(a) = \lim_{\delta\to 0} (-1) \frac{d}{da} \{ [\int d^4\xi (i\delta(\xi) - \Pi(\xi^2)) \partial_{\xi\xi} +$$

$$+ [\frac{\partial}{\partial\xi_\rho} \frac{\partial}{\partial\xi^\rho} \xi^2 \Pi(\xi^2)] \frac{\partial^2}{\partial\xi_0^2}] \times \int_{-\infty}^{\infty} dx_3\, \tilde{D}^c(\xi_a;\, \xi_3 + x_3,\, x_3 + \delta) - \tag{3.19}$$

$$- \int d^4\xi\, \Pi(\xi^2)\, \partial_{\eta\eta} \int d^4z' \int_{-\infty}^{\infty} dx_3 \times$$

$$\times \tilde{D}^c(z'_a;\, z'_3 + x_3) \frac{\partial}{\partial\xi^\rho} \frac{\partial}{\partial\xi_\rho} \tilde{D}^c(\xi_a + z'_a + \eta_a;\, \xi_3 + z'_3,\, x_3 + \eta_3 + \delta)|_{\eta=0} \}.$$

In the subsequent paper/5/ we will explicitly show that this expression is finite and calculate its values in the physical reasonable limit $a\to\infty$.

APPENDIX

The aim of this appendix is the foundation of eq. (3.18)

$$\tau_{\rho\lambda}(\xi) = \frac{1}{2} g_{\rho\lambda} \{ \partial_{\xi\xi} \Pi(\xi^2) + \frac{\partial}{\partial\xi_\rho} \frac{\partial}{\partial\xi^\rho} \frac{\partial^2}{\partial\xi_0^2} \xi\, \Pi(\xi^2) \} \tag{A.1}$$

for the quantity $\tau_{\rho\lambda}$. We start with the investigation of the more general expression

$$\tau_{\mu\nu\rho\lambda}(\xi) = \int d^4\eta\, T_{\mu\nu\rho\lambda}(-\xi-\eta,\, -\eta), \qquad \tau_{\rho\lambda} \equiv \tau_{00\rho\lambda}. \tag{A.2}$$

$T_{\mu\nu\rho\lambda}$ is given by the Feynman diagram

$$T_{\mu\nu\rho\lambda}(x-z,\, x-z') = -ie^2 \mathrm{Tr}\{ \gamma_\mu \frac{\partial}{\partial x^\nu} \hat{S}(x-z) \gamma_\rho \hat{S}(z-z') \gamma_\lambda \hat{S}(z'-x) \} \tag{A.3}$$

and satisfies the relation

$$\frac{\partial}{\partial x_\mu} T_{\mu\nu\rho\lambda}(x-z,\, x-z') = -\frac{1}{2}\Pi_{\rho\lambda}(x-z) \frac{\partial}{\partial x^\nu} \delta(x-z') - \frac{1}{2}\Pi_{\rho\lambda}(x-z') \frac{\partial}{\partial x^\nu} \delta(x-z') \tag{A.4}$$

which connects $T_{\mu\nu\rho\lambda}$ with the polarization operator $\Pi_{\mu\nu}$. We basically use this relation for the proof of eq. (3.18). If we consider eq. (A.4) as a differential equation for $T_{\mu\nu\rho\lambda}$, then its general solution appears as a sum of a special solution of this equation and the general solution of the homogeneous equation $\frac{\partial}{\partial x_\mu} T^h_{\mu\nu\rho\lambda} = 0$. To this representation of $T_{\mu\nu\rho\lambda} = T^h_{\mu\nu\rho\lambda} + T^s_{\mu\nu\rho\lambda}$ there corresponds via eq. (A.2) an analogous splitting of

$$\tau_{\mu\nu\rho\lambda}(\xi) = \tau^h_{\mu\nu\rho\lambda}(\xi) + \tau^s_{\mu\nu\rho\lambda}(\xi). \tag{A.5}$$

At first we prove $\tau^h_{00\rho\lambda} = 0$. This would follow from $T^h_{00\rho\lambda} = 0$. For simplicity we perform the argumentation in momentum space.

$$T_{\mu\nu\rho\lambda}(x-z,\, x-z') = \int \frac{dp\, dp'}{(2\pi)^8} e^{ip(x-z)+ip'(x-z')} \tilde{T}_{\mu\nu\rho\lambda}(p,p'). \tag{A.6}$$

The homogeneous differential equation now reads

$$(p+p')^\mu \tilde{T}^h_{\mu\nu\rho\lambda}(p,p') = 0, \quad (p^0 + p'^0) \tilde{T}^h_{0\nu\rho\lambda}(p,p') = -(p+p')^k \tilde{T}^h_{k\nu\rho\lambda}(p,p'). \tag{A.7}$$

This equation is true in all systems of references, of course, also in the system with $(p+p')^k = 0$. If there are no singularities at this point (as it should be because $\tilde{T}_{\mu\nu\rho\lambda}$ originates from a Feynman diagram), then we have

$$(p^0 + p'^0)\, \tilde{T}^h_{0\nu\rho\lambda}(p,p')_{(p+p')^k=0} = 0$$

and

$\tilde{T}^h_{0\nu\mu\rho}|_{(p+p')^k=0} = 0$ for $p^0 + p'^0 \neq 0$. If we want to extend this relation also to $p^0+p'^0=0$, then we have again to take into account that $T_{\mu\nu\rho\lambda}$ is a Feynman expression which excludes possible terms of the type $\delta(p^0-p'^0)$. So, we get $T^h_{0\nu\rho\lambda}(p,p')_{(p+p')^k=0}=0$.

Let us now consider the remaining inhomogeneous equation in momentum space

$$(p+p')^\mu\,\tilde{T}_{\mu\nu\rho\lambda}(p,p') = -\frac{1}{2}p'_\nu\,\tilde{\Pi}_{\rho\lambda}(p) - \frac{1}{2}p_\nu\,\tilde{\Pi}_{\rho\lambda}(p') \tag{A.8}$$

$$\Pi_{\mu\nu}(x) = \int\frac{d^4p}{(2\pi)^4}\,e^{ipx}\,\tilde{\Pi}_{\mu\nu}(p). \tag{A.9}$$

The special solution will be constructed by explicitly using a kinematical decomposition of $T_{\mu\nu\rho\lambda}$. Here we take into account that $\tau_{\rho\lambda}$ appears only in connection with $\tilde{D}^c_{\mu\nu}$ which is transversal, so that terms proportional to p_ρ, p_λ can be omitted. This becomes clear if we rewrite eq. (A.2) in momentum space

$$\tau_{\mu\nu\rho\lambda}(\xi) = \int\frac{dp}{(2\pi)^4}\,e^{-ip\xi}\,\tilde{T}_{\mu\nu\rho\lambda}(p,-p). \tag{A.10}$$

To solve the inhomogeneous equation (A.8), we start with the ansatz

$$\tilde{T}^s_{\mu\nu\rho\lambda}(p,p') = g_{\rho\lambda}\{g_{\mu\nu}h + p_\mu p_\nu b_1 + p'_\mu p'_\nu b_2 + p_\mu p'_\nu b_3 + p'_\mu p_\nu b_4\} \tag{A.11}$$

$$+ p_\rho p_\lambda[\ldots] + \ldots$$

The insertion of (A.11) in eq. (A.8) gives the following restrictions on the coefficients

$$h = \frac{1}{2}[u(b_1+b_4) + (s-t)(b_1-b_4)] + \frac{1}{2}t\,\tilde{\Pi}(t) \tag{A.12}$$

$$h = -\frac{1}{2}[u(b_2+b_3) + (s-t)(b_3-b_2)] + \frac{1}{2}s\,\tilde{\Pi}(s) \tag{A.13}$$

$$s = p^2, \quad t = p'^2, \quad u = (p+p')^2.$$

Therefore $\tilde{T}^s_{\mu\nu\rho\lambda}(p,-p)$ can be represented by

$$\tilde{T}^s_{\mu\nu\rho\lambda}(p,-p) = g_{\rho\lambda}[g_{\mu\nu}h_1 + p_\mu p_\nu(b_1+b_2-b_3-b_4)]\,|_{s=t,\,u=0}. \tag{A.14}$$

We are left to determine h and $b_1+b_2-b_3-b_4$ for $s=t$ and $u=0$. The compatibility of eq. (A.12) and (A.13) gives

$$u(b_1-b_2-b_3+b_4)+(s-t)(b_1+b_2-b_3-b_4) = t\,\tilde{\Pi}(t) - s\,\tilde{\Pi}(s) \tag{A.15}$$

which leads for $s=t$, $u=0$ to

$$b_1+b_2-b_3-b_4 = -(\tilde{\Pi}(s) + s\,\tilde{\Pi}'(s)) \tag{A.16}$$

then both equations (A.12), (A.13) coincide and give

$$h(s) = \frac{1}{2}s\,\tilde{\Pi}(s). \tag{A.17}$$

This leads finally to

$$\tilde{T}^s_{\mu\nu\rho\lambda}(p,-p) = g_{\rho\lambda}[(g_{\mu\nu}\frac{1}{2}p^2 - p_\mu p_\nu)\tilde{\Pi}(p^2) - p_\mu p_\nu p^2\,\tilde{\Pi}'(p^2)] \tag{A.18}$$

which gives together with $\tau^h_{00\rho\lambda}=0$ the desired result.

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Бордаг М., Робашик Д., Вицорек Э. E2-83-488
Эффект Казимира с точки зрения квантовой теории поля.
Процедура квантования и теория возмущений в ковариантной калибровке

Нетривиальная трактовка эффекта Казимира с точки зрения квантовой теории поля требует квантования спинорной электродинамики с граничными условиями в ковариантной калибровке. Граничные условия реализуются посредством двух параллельных бесконечно тонких сверхпроводящих пластин. Исследование ведется с помощью метода функционального интеграла. Показано, что в теории возмущения обычные правила Фейнмана остаются в силе, требуется только модификация фотонного пропагатора. Одним из достоинств подхода является вывод замкнутого выражения для модифицированного фотонного пропагатора, что,в свою очередь, позволяет явно вычислить петлевые диаграммы. Вычислены до второго порядка по теории возмущений радиационные поправки к выражению энергии вакуума. В итоге получено формальное выражение во втором порядке по теории возмущений для силы Казимира, которое будет обсуждаться в следующей работе.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1983

Bordag M., Robaschik D., Wieczorek E. E2-83-488
Quantum Field Theoretic Treatment of the Casimir Effect.
Quantization Procedure and Perturbation Theory in Covariant Gauge

A nontrivial quantum field theoretical treatment of the Casimir effect demands the quantization of spinor electrodynamics with boundary conditions in a covariant gauge. The boundary conditions are realized by two superconducting infinitely thin parallel plates. As technical tool we use the path integral method. It is shown that in perturbation theoretical calculations the standard Feynman rules remain valid up to a modification of the photon propagator. One advantage of our procedure is the derivation of a closed expression for this modified photon propagator which allows the explicit calculation of loop diagrams. Up to the order e^2 we determine the radiative corrections for vacuum energy expressions. We end up with a formal expression for the Casimir force up to the same order in e^2 which will be discussed in a following paper.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1983