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ON THE IDEAL GAS OF TACHYONS

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In the following paper we consider properties of the ideal gas of classical (nonquantum) tachyons [1-3].

One can say that it is senseless to study tachyons if their existence has not been experimentally confirmed. In our opinion, there are two possibilities to refute this objection.

1. In order to propose the experiment that could give an answer to the question on the existence of tachyons, theoretical investigations are needed, because we already know that properties of tachyons (if they exist) are quite different from those of particles slower than light - bradyons. It should be stressed that a theoretical support of the majority of experiments which have been done is very weak. So, their result cannot be counted as conclusive [1].

2. Even if we a priori assume that tachyons do not exist, a study of their properties can be valuable since it makes deeper our understanding of the theory of bradyons.

We propose to a reader to choose one of the above possibilities and read what follows.

Firstly we formulate some basic notions of the theormodynamics of tachyons following Huang's book [4]. We start from the microcanonical ensemble and the postulate of "a priori equal probabilities" that are the same as in bradyon physics. Also without any differences we define entropy of a system and, through the entropy, the temperature of a subsystem. In this way we are led to the canonical ensemble and the partition function of N particles

$$Q_N(V,T) \overset{df}{=} \int \frac{dp^{3N} dq^{3N}}{N!} e^{-\frac{H(\bar{p}_1 \dots \bar{p}_N, \bar{q}_1 \dots \bar{q}_N)}{T}},$$

where V is the volume of the system; T, the temperature; and $H(\bar{p}_1 \dots \bar{p}_N, \bar{q}_1 \dots \bar{q}_N)$, the Hamiltonian of the system of N particles which depends on their momenta, $\bar{p}$, and positions, $\bar{q}$. Integration is taken over momenta greater than the rest masses of tachyons, m. We use the units where $k = c = \hbar = 1$.

The thermodynamical quantities: pressure, P, entropy, S, and energy, U, are defined through Helmholtz's free energy, F, which is related to Q_N by the equation

$$F \overset{df}{=} -T \ln Q_N(V,T), \tag{1}$$

$$p \stackrel{df}{=} -\left(\frac{\partial F}{\partial V}\right)_T ,$$
$$S \stackrel{df}{=} -\left(\frac{\partial F}{\partial T}\right)_V , \qquad (2)$$
$$U \stackrel{df}{=} F + TS.$$

In the case of free tachyons

$$H(\bar{p}_1, \dots \bar{p}_N, \quad \bar{q}_1, \dots \bar{q}_N) = \sum_{i=1}^{N} \sqrt{p_i^2 - m^2}$$

and

$$Q_N(V,T) = \frac{1}{N!}\left[\int_{|p|\geq m} d^3p\, e^{-\frac{\sqrt{\bar{p}^2 - m^2}}{T}}\right]^N$$

$$\int_{|p|\geq m} d^3p\, e^{-\frac{\sqrt{\bar{p}^2-m^2}}{T}} = 4\pi m^3 \int_0^\infty dt\, ch^2 t\, sh t\, e^{-z sh t} = 4\pi m^3 \frac{1}{z} S_{02}(z),$$

where $z = m/T$ and $S_{02}(z)$ is the so-called Lommel function [5]. Finally we get

$$Q_N(V,T) = \frac{[4\pi m^2 TVS_{02}(m/T)]^N}{N!} .$$

For the gas of relativistic bradyons the partition function is similar in form, although the Lommel function $S_{02}(z)$ has to be changed into the Macdonald function $K_2(z)$ [6,7].

Using (1) and (2), we find the equation of state which is the same as for bradyons, namely:

$$pV = NT.$$

Let us notice that at zero temperature the pressure is zero. In the case of tachyons it is not a trivial property if we remember that zero energy tachyons carry momenta equal to their masses. So, at first sight we could expect that, similar to the Fermi-Dirac gas, the value of pressure at zero temperature would be finite.

The energy of tachyon gas is expressed by the formula

$$U = NT\left(1 - z\frac{S'_{02}(z)}{S_{02}(z)}\right) = NT\left(3z\frac{S_{-13}(z)}{S_{02}(z)} - 1\right).$$

In two extreme cases $z \gg 1$ and $\frac{1}{z} \gg 1$, $S_{02}(z)$ can be approximated

$$S_{02}(z) = \begin{cases} \frac{2}{z^2} + \mathcal{O}(\ell n\, z); & \text{for } \frac{1}{z} \gg 1 \\ \frac{1}{z} + \mathcal{O}\left(\frac{1}{z^2}\right); & \text{for } z \gg 1. \end{cases}$$

So, we find

$$U = \begin{cases} NT\left(3 + \mathcal{O}\left(\frac{m^2}{T^2}\ell n\,(m/T)\right)\right); & \text{for } T \gg m \\ NT\left(2 + \mathcal{O}(T/m)\right); & \text{for } T \ll m. \end{cases}$$

The high temperature limit is the same as for bradyons [6,7]. It is connected with the fact that high energy tachyons and bradyons behave as luxons. The low temperature limit is obviously different since for bradyons

$$U = N\left(\frac{3}{2}T + m\right).$$

Specific heat is defined as follows

$$c_V \stackrel{df}{=} \frac{1}{N}\left(\frac{\partial U}{\partial T}\right)_V$$

and is described by

$$c_V = 1 + z^2\frac{S''_{02}(z)}{S_{02}(z)} - \left(z\frac{S'_{02}(z)}{S_{02}(z)}\right)^2 =$$
$$= 3z^2\left(\frac{5S_{-24}(z) - \frac{1}{z^2}S_{-13}(z)}{S_{02}(z)} - \frac{3S^2_{-13}(z)}{S^2_{02}(z)}\right) - 1,$$

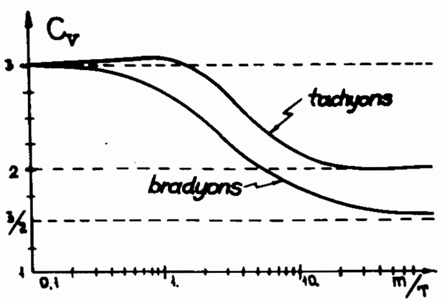
$$c_V = \begin{cases} 3 + \mathcal{O}\left(\frac{m^2}{T^2}\ell n\,(m/T)\right); & \text{for } T \gg m \\ 2 + \mathcal{O}(T/m); & \text{for } T \ll m. \end{cases}$$

In the Table we show the numerically found values of c_V and U/pV versus m/T. Let us notice that specific heat is not a monotonical function of m/T and at $m/T \approx 1$ there is a weak maximum. An analogous table for bradyons is presented in Ref.7.

We conclude that all properties of the classical gas of tachyons and the gas of bradyons are similar and no new phenomena have been found in the case of tachyons.

Table

m/T	C_V	U/pV
0.	3.000	3.000
0.1	3.004	2.995
0.2	3.014	2.983
0.5	3.052	2.917
1.	3.073	2.768
2.	2.930	2.505
3.	2.725	2.337
5.	2.427	2.171
10.	2.151	2.054
20.	2.043	2.014
∞	2.000	2.000



The specific heat of the ideal gas of tachyons and the gas of bradyons as a function of m/T.

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Об идеальном газе тахионов

Рассмотрены свойства идеального газа классических /неквантовых/ частиц быстрее света - тахионов. С этой целью вводятся основные понятия термодинамики тахионов. Найдена статистическая сумма для невзаимодействующих тахионов и вычислены термодинамические функции тахионного газа. Найдено уравнение состояния, которое совпадает с соответствующим уравнением газа частиц медленнее света - брадионов. Обсуждается поведение внутренней энергии и удельной теплоемкости в пределах очень больших и очень низких температур. Показано, что в высокотемпературном пределе свойства газа тахионов точно совпадают со свойствами брадионного газа. Приведены результаты численных расчетов внутренней энергии и теплоемкости для разных температур. Показано, что во всем интервале температур характеристики тахионного и брадионного газов сходные.

Работа выполнена в Лаборатории высоких энергий ОИЯИ.

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On the Ideal Gas of Tachyons

The properties of the ideal gas of classical (nonquantum) faster than light particles - tachyons have been considered. The basic notions of thermodynamics of tachyons have been introduced. We have found the partition function and other thermodynamical quantities for the ideal tachyon gas. The equation of state which we have found for tachyons is exactly the same as for the ideal gas of particles slower than light - bradyons. The internal energy and the specific heat have been discussed at low and at very high temperatures. It has been shown that in high temperature limit the properties of gas of tachyons and gas of bradyons are the same. The numerical calculations concerning the internal energy and specific heat at different temperatures were performed and the results have been presented. It has been shown that in full interval of temperature the characteristics of gas of tachyons are similar to those of gas of bradyons.

The investigation has been performed at the Laboratory of High Energies, JINR.

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