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SUPERFIELD EXPANSION IN HIGHER DIMENSIONS

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INTRODUCTION

Recently there has been a renewed interest in higher dimensional theories of the Kaluza-Klein type [1,2]. In these theories we start with the Einstein Lagrangian in dimension $D = 4 + p$. It is then possible to associate a compact space with extra coordinates. The ordinary 4-dimensional physics is then obtained by a dimensional reduction from the $(4+p)$-dimensional theory: it arises as a low-energy approximation of the latter.

This idea has been revived several times, in particular, in connection with a possible unification of the gravitation with gauge fields, or in the context of the fiber bundle approach to Yang-Mills theories, when trying to associate the extra coordinates with the group space. The simplest dimensional reduction has been fruitful for the supersymmetric theories because it is possible to derive in this way extended supersymmetric theories from simple ones but in more space-time dimensions [3]. A connection has been shown between $N = 4$ Yang-Mills in 4 dimensions and $N = 1$ Yang-Mills in 10 dimensions [4], and $N = 8$ supergravity in 4 dimensions and $N = 1$ supergravity in 11 dimensions. Moreover, the dimensional reduction explains some of the hidden symmetries in extended supergravity. All these results were obtained in the framework of the component formalism.

On the other hand, the superspace and superfields provide an elegant and compact description of supersymmetric theories. They simplify the addition and multiplication of representations and are very useful in the construction of interaction Lagrangians. They also simplify the calculation of radiative corrections in quantized supersymmetric theories. The Feynman rules for supersymmetric theories may be stated in terms of superfield vertices and propagators. It already becomes easy to understand why the divergencies so miraculously cancel out in the supersymmetric theories.

The superfield formulation is also very profitable and essential in the investigations of such a problem as the finiteness of the theory. Recent results [5] show that the $N = 4$ Yang-Mills theory would be finite to all orders in perturbation theory provided the unconstrained $N = 4$ superfield formulation exists. It was also argued that even under weaker assumptions, namely, that the unconstrained $N = 2$ superspace formulation of the model exists, the finiteness could still be proved [6].

That's why it would be advantageous to find a superfield formulation of supersymmetric theories in higher dimensions. The first step in this direction, is to investigate the field content of superfields in D dimensions.

In this paper there is derived the field (i.e., SO(D-1,1) irreducible representations) content of N = 1 superfields in D = 6,8,9,10,11 dimensions. The paper is arranged as follows: in the first part we make a brief review of the possibility for existence of the Majorana spinors in different dimensions. In the second part some necessary information is given about irreducible representations of SO(N) groups. In part third we derive a field content of N = 1 superfields in D = 6,8,9,10,11 dimensions.

1. MAJORANA SPINORS IN D DIMENSIONS

The superspace in D dimensions is spanned by the coordinates (x_μ, θ_a), where x_μ denotes the D-dimensional space-time variable and θ_a are Majorana or Majorana-Weyl spinors. Consequently, it is essential to know in what dimensions Majorana or Majorana-Weyl spinors can be defined.

The Dirac spinor in D dimensions has $2^{[D/2]}$ complex components. For D even, it is a reducible representation with respect to SO(D) and it decomposes into two $2^{D/2-1}$ component Weyl spinors. A Majorana spinor is a spinor the Dirac conjugate of which is proportional to its Majorana conjugate and has $2^{[D/2]}$ real components. A Majorana-Weyl spinor obey both the Weyl and Majorana conditions and has $2^{D/2-1}$ real components.

Many of its properties are known from general group-theoretical arguments about the irreducible representations of SO(D) group. We briefly reproduce them here using explicitly the properties of the Dirac matrices in D dimensions [7].

The Dirac matrices obey the Clifford algebra

$$\{\Gamma_\mu, \Gamma_\nu\} = 2\eta_{\mu\nu}, \quad \eta_{\mu\nu} = (\overbrace{+,\dots,+}^{t}, \overbrace{-,\dots,-}^{s}),$$

where Γ_μ can be represented by $2^{[D/2]} \times 2^{[D/2]}$ unitary matrices. Since the matrices Γ_μ and Γ^*_μ (complex conjugated) satisfy the same Clifford algebra and the representation of Γ matrices is irreducible, there exists an invertible matrix B such that

$$\Gamma_\mu = a B^{-1} \Gamma^*_\mu B,$$

where $a = \pm 1$.

The matrix B obeys the following conditions

$B^+ B = 1, \quad B^+ B = \beta 1, \quad \beta = \pm 1.$

From the Majorana condition we may find that the Majorana spinors satisfy the reality condition

$$\Psi^* = B\Psi.$$

But this implies $\beta = 1$. If $\beta = -1$ and we have an even number of spinors Ψ^i, one can impose the reality condition

$$\Psi^{*i} = \Omega^{ij} B \Psi_j, \tag{1.1}$$

where Ω^{ij} must be a real antisymmetric matrix. We can consider it to be a symplectic metric, in this case the relation (1.1) is preserved under transformations of Ψ^i by the group USp(2n) for $i = 1,2,\dots,2n$. Using the Scherk method [8] it is easy to find that β must be a function of a, t and D, periodic in D with period 8 and in t with period 4.

In the case of one time-like dimension, Majorana ($a = -1$, $\beta = 1$), pseudo-Majorana ($a = 1$, $\beta = 1$), extended (pseudo)-Majorana ($\beta = -1$), and Majorana-Weyl-spinors can be defined as follows:

Majorana ($a = -1$, $\beta = 1$) for D = 2,3,4 mod 8 (1.2)
pseudo-Majorana ($a = 1$, $\beta = 1$) for D = 2,8,9 mod 8 (1.3)
extended-Majorana ($a = -1$, $\beta = -1$) for D = 6,7,8 mod 8 (1.4)
extended-pseudo-Majorana ($a = 1$, $\beta = -1$) for D = 4,5,6 mod 8 (1.5)
Majorana-Weyl for D = 2 mod 8 (1.6)
extended-Majorana-Weyl for D = 6 mod 8 (1.7)

We can define N = 1 superspace and superfields in D = 2,3,4, 6,8,9 mod 8. The superfield expansions are well known for D=2,3,4. We will investigate them in D = 6,8,10,11. The dimensions greater than 11 are not interesting for us because after the dimensional reduction they lead to an N > 8 theory, i.e., to the theory with spins greater than two.

2. IRREDUCIBLE REPRESENTATIONS OF SO(N)

The group SO(N, C) has two complex-analytical series of irreducible representations. Any representation from the first one is determined by and determines the greatest weight $m = (m_1, m_2, \dots, m_\nu)$, where m are integer numbers obeying the following conditions

$$m_1 \geq m_2 \geq \dots \geq m_{\nu-1} \geq |m_\nu|, \quad N = 2\nu, \tag{2.1}$$

$$m_1 \geq m_2 \geq \dots \geq m_{\nu-1} \geq m_\nu \geq 0, \quad N = 2\nu + 1. \tag{2.2}$$

Any representation from the second one is determined by and determines the greatest weight $m = (m_1, m_2, \dots, m_\nu)$, where m_i are half-integers obeying (2.1) and (2.2). The irreducible representations of $SO(N, C)$ associated with an integer greatest weight m are tensorial. The remaining representations are called spinorial. The fundamental spinorial representation in the case of $SO(2\nu+1, C)$ is with a greatest weight

$$m = (1/2,\ 1/2,\ \dots,\ 1/2)\ .$$

In the case of $SO(2\nu, C)$ there are two fundamental spinorial representations m_+ and m_-, where

$$m_+ = (1/2,\ 1/2,\ \dots,\ 1/2)\ ,$$

$$m_- = (1/2,\ 1/2,\ \dots,\ -1/2)\ .$$

The objects which transform with respect to this representations are called spinors of I and II type. All these things take place also for the group $SO(p, q)$ /8/. For tensorial representations m_i are all integer and stand for the number of boxes in the rows of the Young tableau. The representation is therefore a tensor and $[m_1, m_2, \dots, m_{\nu-1}, m_\nu]$ gives the symmetry of its indices. In the spinorial case m_i are all half-integer and, the representation is a spinor-tensor, tensorial indices of which have the symmetry $[m_1 - 1/2,\ m_2 - 1/2, \dots, m_{\nu-1} - 1/2,\ m_\nu - 1/2]$. The tensors and spinor-tensors fulfill a convenient trace condition obtained by contraction with $\eta_{\mu\nu}$ and Γ_μ matrices which guarantees their irreducibility. In both the tensorial and spinorial cases m_i are also eigenvalues of the complete set of Casimir operators. We note that one writes the indices of the tensors and tensor-spinors always in the form of a Young tableau rotated by 90°.

We have computed the dimension of the representations using the standard formulae of the group theory /10/. The dimension of a given irreducible representation $[m_1, m_2, \dots, m_\nu]$ is

$$\dim(m_1, \dots, m_\nu) = \frac{2^{(\nu-1)}}{(2\nu-2)!\,(2\nu-4)!\,\dots\,2!}\prod_{p>q}^{\nu} (\ell_p^2 - \ell_q^2)\ , \qquad (2.4)$$

where $\ell_p = |m_p| + \nu - p$ in the case of $SO(2\nu)$ and

$$\dim[m_1, \dots, m_\nu] = \frac{(2\ell_1+1)\dots(2\ell_\nu+1)\,\Delta(\ell)}{(2\nu-1)!\,(2\nu-3)!\,\dots\,3!\,1!}\prod_{p<q}^{\nu} (\ell_p + \ell_q + 1), \qquad (2.5)$$

where $\Delta(\ell) = (\ell_1 - \ell_2)\dots(\ell_{\nu-1} - \ell_\nu) = \prod_{p<q} (\ell_p - \ell_q)$ in the case of $SO(2\nu+1, C)$.

To find the superfield expansion in terms of $SO(N)$ irreducible representations, we must know the decomposition into irreducible representations of the Kronecker product of two $SO(N)$ representations. Recently, a relatively simple algorithm has been proposed for this operation /11/.

Actually, this method reduces to the calculation of a finite number of products of generalized Young tableaux (GYT) for any two given $SO(N)$ irreducible representations. The GYT is a tableau which can include "negative" boxes. The product of two GYTs can be regarded as a natural extension of the usual product of two $SU(N)$ Young tableaux. Another attractive feature of this technique is that the rules for products involving tensor or spinor representations are essentially the same.

3. FIELD CONTENT OF SUPERFIELDS

The superfield $\Phi(x, \theta)$ is a field in the superspace (x_μ, θ_a) which should be understood in terms of its power series expansion in θ_a:

$$\Phi(x, \theta) = A + \bar\theta^{a}\psi_a + \dots + \theta_a\theta_\beta\dots\theta_\gamma M^{[a\beta\dots\gamma]} + \dots\ ,$$

where θ is a Majorana or Majorana-Weyl $2^{[D/2]}$ dimensional anticommuting spinor. The symbol like $[a, \beta, \gamma]$ means antisymmetrization. This expansion is finite, because of the anticommutation of the θ's. The superfield describes a finite set of ordinary fields - the coefficients $M^{[a\beta\dots\gamma]}$ of its power expansion in θ. These fields are, in general, a reducible representation of the D-dimensional Lorentz group, i.e., of $SO(D-1, 1)$. We must decompose them into irreducible representations of $SO(D-1,1)$. This is equivalent to the decomposition of an antisymmetrized Kronecker product of $\theta_a\theta_\beta\dots\theta_\gamma$ into irreducible representations. To solve this problem, we shall use the technique developed in ref. /11/. It is more simple in the cases we need

1) $D = 8$, $N = 1$ superfield

In $D = 8$ the superspace has eight bosonic and 16 fermionic coordinates. The Lorentz group is $SO(7.1)$. The irreducible representations are given by a set of 4 numbers $[\lambda_1, \lambda_2, \lambda_3, \lambda_4]$. The pseudo-Majorana spinor θ is a reducible representation of $SO(7.1)$. It splits into two 8-dimensional Weyl spinors $\theta^+([1/2,\ 1/2,\ 1/2,\ 1/2])$ and $\theta^-([1/2,\ 1/2,\ 1/2,\ -1/2])$ that are irreducible representations of $SO(7.1)$. We shall expand $\Phi(x, \theta^+, \theta^-)$ in powers of θ^+ and θ^-.

In Table 1 we present the SO (7.1) representations contained in $\Phi(x,\theta^{+},\theta^{-})$ up to and including the 8θ-sector (higher sectors can be obtained by reflection around the latter). Besides we wrote only terms with $\theta^{+m}\theta^{-n}(m\geq n)$. The terms with $\theta^{+n}\theta^{-m}$ can be derived from these with $\theta^{+m}\theta^{-n}$ by the mirror-conjugation (the mirror-conjugated of the irreducible representation $\lambda=[\lambda_1,\lambda_2,\dots,\lambda_{p-1},\lambda_p]$ is the representation $\bar{\lambda}=[\lambda_1,\lambda_2,\dots,\lambda_{p-1},-\lambda_p]$).

2) D = 9, scalar superfield

In D = 9 the superspace has 16 fermionic coordinates. The Lorentz group is SO (8.1). In this case the irreducible representations are given by a set of 4 numbers $[\lambda_1,\lambda_2,\lambda_3,\lambda_4]$. The pseudo-Majorana spinor θ_a is an irreducible representation of SO (8.1) and is given by [1/2, 1/2, 1/2, 1/2]. The dimensions of representations are calculated by the use of (2.5). The SO (8.1) representations contained in $\Phi(x,\theta)$ are presented in Table 2, up to the 8θ-sector.

3) D=10 scalar superfield

In the 10-dimensional space we can define Majorana-Weyl spinors. The Majorana spinor is 32-dimensional and in this case it splits into two 16-dimensional Majorana-Weyl spinors. The Lorentz group is SO (9.1). Its representations are characterized by the greatest weight $[\lambda_1,\lambda_2,\lambda_3,\lambda_4,\lambda_5]$. We calculate the dimensions of representations with the help of (2.4). We have two possibilities to define a superspace, namely, a "chiral superspace (x_μ,θ^{+}), where θ^{+} is the [1/2, 1/2, 1/2, 1/2, 1/2] representation of SO (9.1) and full superspace $(x_\mu,\theta^{+},\theta^{-})$, where θ^{-} is the [1/2, 1/2, 1/2, 1/2, -1/2] representation of SO (9.1).

In Table 3 we write the expansion of the N=1, chiral scalar superfield $\Phi(x,\theta^{+})$ in terms of SO (9.1) irreducible representations up to the 8θ-sector. The field content of the full N=1 superfields is very large and complicated and we do not give it here.

4) D=11 scalar superfield

In this case the Lorentz group is SO (10.1). The spinor coordinate is 32-dimensional. It is the [1/2, 1/2, 1/2, 1/2, 1/2] irreducible representation of SO (10.1). The dimensions are calculated according to (2.5). The result is given in Table 4 up to the 13θ-sector. The 14th, 15th and 16th θ-sectors are not explicitly given because they have a very rich structure and high dimensions (i.e., carry high spins) and are not intersting for the physical applications.

5) D = 6 scalar superfield

As we have seen in section two, in the 6-dimensional space-time one can define only extended-Majorana or extended-Majorana-Weyl spinors. The Majorana spinor is a reducible representation of the SO (5.1) group and decomposes into two irreducible four-dimensional Majorana-Weyl spinors θ^{+i} and θ^{-i}. They are respectively [1/2, 1/2, 1/2] and [1/2, 1/2, -1/2] representations of SO (5.1). The spinors θ^{+i} and θ^{-i} transform according to the fundamental representation of the group USp(2N) for i = 1,2,... 2N. In analogy with the D =10 case we have two possibilities to define a superspace and superfields here. In Table 5 we give the expansion of the "chiral" superfield $\Phi(x,\theta^{+})$. The internal symmetry group chosen is $USp(2) \simeq SU(2)$.

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Table 1. D = 8 Scalar Superfield

θ-sector	Representation	Dimension
θ^{+}	[0.5, 0.5, 0.5, 0.5]	8
θ^{+2}	[1, 1, 1, 1]	28
$\theta^{+}\theta^{-}$	[1, 0, 0, 0] [1, 1, 1, 0]	8 56
θ^{+3}	[1.5, 0.5, 0.5, -0.5]	56
$\theta^{+2}\theta^{-}$	[0.5, 0.5, 0.5, -0.5] [1.5, 0.5, 0.5, 0.5] [1.5, 1.5, 0.5, -0.5]	8 56 160
θ^{+4}	[2, 0, 0, 0] [1, 1, 1 -1]	35 35

Table 1 (continued)

θ-sector	Representation	Dimension
$\theta^{+3}\theta^{-}$	[2,1,1−1]	224
	[2,1,0,0]	160
	[1,1,1,0]	56
	[1,0,0,0]	8
$\theta^{+2}\theta^{-2}$	[1,1,0,0]	28
	[1,1,0,0]	28
	[2,1,1,0]	350
	[2,1,1,0]	350
	[1,1,0,0]	28
θ^{+5}	[1.5,0.5,0.5,−0.5]	56
$\theta^{+4}\theta^{-}$	[2.5,0,5,0.5,−0,5]	224
	[1.5,0,5.0.5,0.5]	56
	[1.5,1.5,1,5,−1.5]	112
	[1.5,1.5,0.5,−0.5]	160
	[0.5,0.5,0.5,−0.5]	8
$\theta^{+3}\theta^{-2}$	[2.5,1.5,0.5,−0.5]	840
	[1.5,1.5,1.5,−0.5]	224
	[1.5,0.5,0.5,−0.5]	56
	[2.5,0.5,0.5, 0.5]	224
	[1.5,1.5,1.5,−0.5]	224
θ^{+6}	[1,1,0,0]	28
$\theta^{+5}\theta^{-}$	[2,1,1,−1]	224
	[2,1,0,0]	160
	[1,1,1,0]	56
	[1,0,0,0]	8
$\theta^{+4}\theta^{-2}$	[3,1,1−1]	840
	[2,1,1,0]	350
	[2,0,0,0]	35

Table 1 (continued)

θ-Sector	Representation	Dimension
$\theta^{+4}\theta^{-2}$	[2,0,0,0]	35
	[1,1,0,0]	28
	[2,1,1,0]	350
	[2,2,2,−2]	294
	[1,1,0,0]	28
$\theta^{+3}\theta^{-3}$	[3,2,0,0]	1400
	[2,2,1,0]	840
	[2,2,1,0]	840
	[1,1,1,0]	56
θ^{+7}	[0.5,0.5,0.5,0.5]	8
$\theta^{+6}\theta^{-}$	[1.5,1.5,0.5,−0.5]	160
	[1.5,0.5,0.5,0.5]	56
	[0.5,0.5,0.5,−0.5]	8
$\theta^{+5}\theta^{-2}$	[2.5,1.5,1.5,−1.5]	672
	[2.5,1.5,0.5,−0.5]	840
	[1.5,0.5,0.5,−0.5]	56
$\theta^{+4}\theta^{-3}$	[3.5,1.5,0.5,−0.5]	2800
	[2.5,0.5,0.5,−0.5]	224
	[1.5,1.5,1.5,0.5]	840
	[1.5,0.5,0.5,0.5]	56
θ^{+8}	[0,0,0,0]	1
$\theta^{+7}\theta^{-}$	[1,0,0,0]	8
	[1,1,1,0]	56
$\theta^{+6}\theta^{-2}$	[2,1,1,0]	350
	[1,1,0,0]	28
	[1,1,0,0]	28
	[2,1,1,0]	350
	[1,1,0,0]	28

Table 1 (continued)

θ-Sector	Representation	Dimension
$\theta^{+5}\theta^{-3}$	[3,2,2,-2]	1680
	[3,1,1,0]	1296
	[2,1,0,0]	160
$\theta^{+4}\theta^{-4}$	[4,1,1,0]	3675
	[2,2,2,0]	840
	[2,1,1,0]	350
	[2,0,0,0]	35

Table 2. D = 9 Scalar Superfield

θ sector	Representation	Dimension
θ	[0.5,0.5,0.5,0.5]	16
θ^2	[1,1,1,0]	84
	[1,1,0,0]	36
θ^3	[1.5,1.5,0.5,0.5]	432
	[1.5,0.5,0.5,0.5]	128
θ^4	[2,2,1,0]	1650
	[1,1,1,1]	126
	[2,0,0,0]	44
θ^5	[2.5,1.5,0.5,0.5]	2560
	[1.5,1.5,0.5,0.5]	432
	[1.5,1.5,1.5,1.5]	672
	[1.5,0.5,0.5,0.5]	128
	[2.5,0.5,0.5,0.5]	576

Table 2 (continued)

θ sector	Representation	Dimension
θ^6	[2,2,1,1]	2772
	[2,1,1,1]	924
	[3,1,0,0]	910
	[2,1,1,0]	594
	[2,2,1,1]	2772
	[1,1,0,0]	36
θ^7	[3.5,1.5,0.5,0.5	9504
	[3.5,0.5,0.5,0.5	1920
	[0.5,0.5,0.5,0.5	16
θ^8	[4,1,1,1]	12375
	[4,0,0,0]	450
	[1,1,0,0]	36
	[1,0,0,0]	9

Table 3. D = 10 "Chiral" Scalar Superfield

$\bar{\theta}$ sector	Representation	Dimension
θ^{+}	[0.5,0.5,0.5,0,5 0.5]	16
θ^{+2}	[1,1,1,0,0]	120
θ^{+3}	[1.5,1.5,0.5,0.5, -0.5]	560
θ^{+4}	[2,2,0,0,0]	770
	[2,1,1,1,-1]	1050
θ^{+5}	[2.5,1.5,0.5,0.5, -0.5]	3696
	[1.5,1.5,1.5,1.5, -1.5]	672

Table 3 (continued)

θ sector	Representation	Dimension
θ^{+6}	[3,1,1,0,0]	4312
	[2,2,1,1,−1]	3696
θ^{+7}	[3.5,0.5,0.5,0.5,0.5]	2640
	[2.5,1.5,1.5,0.5,−0.5]	8800
θ^{+8}	[4,0,0,0,0]	660
	[2,2,2,0,0]	4125
	[3,1,1,1,0]	8085

Table 4. D = 11 Scalar Superfield

θ sector	Representation	Dimension
θ	[0.5,0.5,0.5,0.5,0.5]	32
θ^2	[1,1,1,1,0]	330
	[1,1,1,0,0]	165
	[0,0,0,0,0]	1
θ^3	[1.5,1.5,1.5,0.5,0.5]	3520
	[1.5,1.5,0.5,0.5,0.5]	1408
	[0.5,0.5,0.5,0.5,0.5]	32
θ^4	[2,2,1,1,1]	17160
	[1,1,1,1,0]	330
	[1,1,1,0,0]	165
	[2,2,1,1,1]	17160
	[2,2,0,0,0]	1144
	[0,0,0,0,0]	1

Table 4 (continued)

θ sector	Representation	Dimension
θ^5	[2.5,1.5,1.5,1.5,1.5]	36960
	[2.5, 2.5,1.5,1.5,0.5]	160160
	[1.5,1.5,1.5,1.5,1.5]	4224
	[0.5,0.5,0.5,0.5,0.5]	32
θ^6	[3,2,2,1,1]	289775
	[3,3,2,2,0]	616616
	[0,0,0,0,0]	1
θ^7	[3.5,2.5,2.5,1.5,1.5]	2013440
	[2.5,2.5,2.5,1.5,1.5]	274560
	[2.5,2.5,2.5,1.5,0.5]	292864
	[3.5,1.5,1.5,1.5,1.5]	183040
	[3.5,2.5,1.5,0.5,0.5]	573440
	[2.5,1.5,1.5,0.5,0.5]	28512
θ^8	[4,3,2,2,1]	7900750
	[3,3,3,1,1]	1115400
	[3,3,3,1,1]	1115400
	[4,3,1,0,0]	386750
θ^9	[4.5,2.5,2.5,2.5,0.5]	8580000
	[4.5,2.5,2.5,1.5,0.5]	9123840
	[3.5,2.5,1.5,1.5,0.5]	1034880
	[3.5,3.5,3.5,1.5,1.5]	6864000
	[3.5,3.5,3.5,0.5,0.5]	2446080
θ^{10}	[5,3,3,3,0]	22415250
	[3,2,2,2,0]	188760
	[4,4,3,2,1]	39309270
	[4,4,4,0,0]	2598960

Table 4 (continued)

θ sect.	Representation	Dimension
θ^{11}	[2.5,2.5,2.5,2.5,0.5]	251680
	[4.5,4.5,2.5,2.5,0.5]	66193920
	[4.5,3.5,3.5,1.5,0.5]	43130880
	[3.5,3.5,3.5,1.5,1.5]	6864000
	[3.5,3.5,3.5,1.5,0.5]	6726720
	[3.5,3.5,2.5,1.5,0.5]	5857280
θ^{12}	[3,3,3,3,1]	1656369
	[3,3,3,2,0]	1002001
	[3,3,2,2,1]	1274130
	[5,5,2,2,0]	58953960
	[4,4,3,3,0]	15169440
	[5,4,4,1,0]	61725300
	[4,4,4,2,0]	19059040
	[4,4,4,1,1]	18232500
	[[illegible],3,3,1,1]	30604288
	[4,3,3,2,0]	6891885
	[4,3,3,1,0]	4332042
	[4,3,3,2,0]	6891885
θ^{13}	[3.5,3.5,3.5,3.5,1.5]	8328320
	[3.5,3.5,3.5,2.5,0.5]	8968960
	[3.5,2.5,2.5,2.5,0.5]	1921920
	[2.5,2.5,2.5,2.5,0.5]	251680
	[3.5,3.5,3.5,2.5,0.5]	8968960
	[4.5,4.5,2.5,2.5,0.5]	66193920
	[4.5,4.5,2.5,2.5,0.5]	66193920
	[4.5,3.5,3.5,2.5,0.5]	59488000
	[4.5,3.5,2.5,2.5,0.5]	35143680
	[4.5,4.5,4.5,1.5,0.5]	91914240

Table 4 (continued)

θ sect.	Representation	Dimension
θ^{14}	471435600	
θ^{15}	565722720	
θ^{16}	601080390	

Table 5. D = 6 "Chiral" Scalar Superfield

θ sector	Representation	Dimension	field
θ^{+}	$[0.5,0.5,0.5]^{K}$	$4^{K} = 8$	Ψ_{α}^{K}
θ^{+2}	$[1,0,0]^{(ij)}$	$6^{(ij)} = 18$	$A_{\mu}^{(ij)}$
	$[1,1,1]$	10	$T_{\mu\nu\rho}$
θ^{+3}	$[0.5,0.5,-0.5]^{(ijK)}$	$4^{(ijK)} = 16$	$\bar{\chi}_{\alpha}^{(ijK)}$
	$[1.5,0.5,0.5]^{K}$	$20^{K} = 40$	$\Psi_{\mu\alpha}^{K}$
θ^{+4}	$[0,0,0]^{(ijk\ell)}$	$1^{(ijk\ell)} = 5$	$A^{(ijk\ell)}$
	$[1,1,0]^{(k\ell)}$	$15^{(k\ell)} = 45$	$T_{\mu\nu}^{(k\ell)}$
	$[2,0,0]$	20	$E_{(\mu\nu)}$
θ^{+5}	$[1.5,0.5,-0.5]^{K}$	$20^{K} = 40$	$\bar{\xi}_{\mu\alpha}^{K}$
	$[0.5,0.5,0.5]^{(ijK)}$	$4^{(ijK)} = 16$	$\xi_{\alpha}^{(ijK)}$
θ^{+6}	$[1,1,-1]$	10	$\bar{P}_{\mu\nu\rho}$
	$[1,0,0]^{(ij)}$	$6^{(ij)} = 18$	$B_{\mu}^{(ij)}$
θ^{+7}	$[0.5,0.5,-0.5]^{K}$	$4^{K} = 8$	$\bar{\varkappa}_{\alpha}^{K}$
θ^{+8}	$[0,0,0]$	1	C

REFERENCES

1. Kaluza Th., Sitzunsber-Prenss. Akad.Wiss. Berlin, Math. Phys. Kl, 1921, p. 966.
2. Klein D. Z.Phys., 1926, 37, p. 897.
3. Cremmer E., Julia B., Scherk J. Phys.Lett., 1978, 76B, p.409; Cremmer E., Julia B. Phys.Lett., 1978, 80B, p. 48; Nucl.Phys., 1979, B159, p. 141.
4. Gliozzi F., Olive D., Scherk J. Nucl.Phys., 1977, B122, p. 253. Brink L., Scherk J., Schwarz J. Nucl.Phys., 1977, B121, p. 77.
5. Grisaru M., Siegel W. Nucl.Phys., 1981, B187, p. 149. Grisaru M., Siegel W. Nucl.Phys., 1982, B201, p. 292.
6. Howe P., Stelle K., Townsend P. CERN, preprint TH-3271, p. 1982.
7. Kugo T., Townsend P. preprint LPTENS, 1982, 82/21.
8. Scherk J. In: Recent Developments in Gravitation. (Cargese 1978), eds M.Levy, S.Deser, Plenum Press, 1979, No. 4.
9. Barut A., Raczka X. 1980, Theory of Group Representations and Applications, vol.1, Moscow, Mir (in Russian).
10. Murnaghan F.D. The Unitary and Rotation Groups, Spartan Books, Washington, D.C. 1962.
11. Girardi G., Sciarrino A., Sorba P. CERN preprint TH-3103, 1981. Girardi G., Sciarrino A., Sorba P. preprint, LAPP-TH-45,1981.

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Разложение суперполей в высших размерностях

В работе получено разложение $N = 1$ суперполей в $(D, 2^{[D/2]})$-мерном суперпространстве в терминах неприводимых представлений группы $SO(D-1,1)$ для $D = 6,8,9,10,11$.

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Litov L.B. E2-83-467

Superfield Expansion in Higher Dimensions

In this paper the expansion of $N = 1$ superfields in $(D, 2^{[D/2]})$ -dimensional superspace for $D = 6,8,9,10,11$ is derived. The result is given in terms of $SO(D-1,1)$ irreducible representations.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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