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**MANY-BODY PROBLEM
IN QUANTUM MECHANICS
AND HOPF MAPS**

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supplementary conditions (SC) $\dot{\xi}\xi - \bar{\xi}\dot{\xi} = 0$ in classical theory and

$$(\xi\bar{\pi} - \bar{\xi}\pi) | > = 0, \quad \text{or} \quad (m(\bar{\xi}\xi)(\dot{\xi}\xi - \bar{\xi}\dot{\xi}) + i\hbar) | > = 0,$$

or
$$\left(\xi \frac{\partial}{\partial \xi} - \bar{\xi} \frac{\partial}{\partial \bar{\xi}}\right) \langle \xi, \bar{\xi} | > = 0 \quad (4)$$

1. Embedding into spaces of higher dimensionalities and imposing relevant supplementary conditions (constraints) can lead to some useful forms of dynamics in the classical and quantum theories^{/1-6/}. The Hopf maps (fiber bundles) bear interesting possibilities. Thus, the fiber bundle $S^3 \rightarrow S^2$ can be written as a transformation from a complex 2-component spinor to Cartesian coordinates

$$x_m = \bar{\xi} \sigma_m \xi \quad (m=1,2,3), \quad r = \bar{\xi}\xi, \quad (1)$$

where σ_m are the Pauli matrices. Eq. (1) realizes the map at each value of r ($\rho = \sqrt{r}$ and r are radii of spheres S^2 and S^3 , respectively). S^2 is the base space, and $S^1(e^{i\lambda}\xi)$ is a fiber. The change of variables (1) may also be of interest, on the one hand, if we tend to come to the spinors throughout (such a tendency is observed in the contemporary field theory) and, on the other hand, because the quantum mechanics takes a form similar to popular CP^1 -models.

2. Two-body problem. The change of variables (1) leads to the Lagrangian

$$L = \frac{m}{2} \dot{\vec{x}} \dot{\vec{x}} + \frac{e^2}{r} = 2m(\bar{\xi}\dot{\xi})(\dot{\xi}\xi) + \frac{m}{2}(\dot{\xi}\xi - \bar{\xi}\dot{\xi})^2 + \frac{e^2}{\bar{\xi}\xi} \quad (2)$$

(any other potential $V(r) = V(\bar{\xi}\xi)$, $V(\vec{x}) = V(\bar{\xi}\sigma\xi)$ can be accepted), which is invariant under gauge transformations $\xi(t) \rightarrow e^{i\lambda(t)}\xi(t)$. It is convenient to omit the second term of the last expression of eq. (2), to assume

$$\tilde{L} = 2m(\bar{\xi}\dot{\xi})(\dot{\xi}\xi) + \frac{e^2}{\bar{\xi}\xi} \quad (\text{or with any other } V) \quad (3)$$

(recall an analogous approach in electrodynamics) and to impose the

in quantum theory. The Lagrangian \tilde{L} is invariant under the phase transformations only with $\lambda = \text{const}(t)$ ($(\bar{\xi}\xi)(\dot{\xi}\xi - \bar{\xi}\dot{\xi})$ is a relevant integral of motion), but is O_4 -symmetric, while the Lagrangian L and SC (4) are only O_3 -symmetric. When energy (Hamiltonian) is fixed, equations of motion turn out to be those for a 4-dimensional oscillator^{/1/} (see ref. ^{/6/} for details in terms adopted here and for a group-theoretical meaning of SC).

A Schrödinger picture consideration can be performed independently of the above Heisenberg picture approach. To this end we solve the relations

$$\frac{\partial}{\partial \xi_a} = \frac{\partial x_m}{\partial \xi_a} \frac{\partial}{\partial x_m}, \quad \frac{\partial}{\partial \bar{\xi}_a} = \frac{\partial x_m}{\partial \bar{\xi}_a} \frac{\partial}{\partial x_m} \quad (5)$$

with respect to $\partial/\partial x_m$. This set is overdetermined and as a compatibility condition we get SC (4) again. Then, applied to the functions satisfying SC (4) the relations

$$2r \frac{\partial}{\partial x_m} = \xi \sigma_m^T \frac{\partial}{\partial \xi} + \bar{\xi} \sigma_m \frac{\partial}{\partial \bar{\xi}} \quad (6)$$

and the following connection between Laplacians in R_3 and R_4

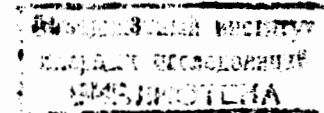
$$\Delta_{(3)} \equiv \frac{\partial}{\partial x_m} \frac{\partial}{\partial x_m} = \frac{1}{4r} \Delta_{(4)} \equiv \frac{1}{\bar{\xi}\xi} \frac{\partial}{\partial \xi_a} \frac{\partial}{\partial \bar{\xi}_a} \quad (7)$$

are valid. The Schrödinger equation, say, for Green functions

$$\left[-\frac{\hbar^2}{2m} \Delta_{(3)} - \frac{e^2}{r} - E\right] G(\vec{x}, \vec{x}_0, E) = -i\hbar \delta(\vec{x} - \vec{x}_0) \quad (8)$$

can be replaced by the Schrödinger equation in R_4

$$\left[-\frac{\hbar^2}{8m} \Delta_{(4)} - e^2 - E\right] \tilde{G}(\xi, \bar{\xi}, \xi_0, \bar{\xi}_0, e^2) = -i\hbar \frac{\pi}{4} \delta(\xi - \xi_0) \delta(\bar{\xi} - \bar{\xi}_0), \quad (9)$$



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Полубаринов И.В.
Проблема нескольких тел в квантовой механике
и отображения Хопфа

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В задаче многих тел в квантовой механике рассматривается переход от декартовых координат к спинорным переменным.

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Many-Body Problem in Quantum Mechanics
and Hopf Maps

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Transformation from the Cartesian coordinates to spinor variables is considered in many-body problem in quantum mechanics.

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