объединенный
институт
ядерных
исследований
дубна

E2-83-378

V.G.Makhankov, O.K.Pashaev, S.A.Sergeenkov

HOLE-LIKE EXCITATIONS IN MANY COMPONENT SYSTEMS

Submitted to "Physica Scripta"

1983

Integrable models have become the subject of comprehensive study lately especially due to theoretical and experimental investigations of quasi-one-dimensional crystals (magnets) at low temperatures. In particular, such models as sine-Gordon (SG) (describing easy plane ferromagnets [1]) and various versions of the Schrödinger equation with cubic nonlinearity (NLS, for easy axis ferromagnets [2] and Hubbard - like antiferromagnets [3] and so on [4]) are a sufficiently appropriate basis to develope corresponding theory at the definite area of the system parameters.

The gauge equivalence of Landau-Lifshitz equations and NLS is obtained to show the importance of studying the latter to understand the behaviour of more complicated systems.

If the scalar $U(1)$ version as well as the vector one of attractive NLS may be regarded as studied rather well the repulsive type NLS is yet nearly terra incognita. In fact after the first work in this direction [5] there appeared but a few publications (containing sometimes contradictious statements) concerning the scalar $U(0,1)$ version [6].

In one of the previous works of this series the $U(p,q)$ version of the NLS has been studied [7] but under vanishing boundary conditions. Whereby condensate states were excluded from consideration. In this case, however, a great attention should be paid namely to the properties of the condensate and its excitations that corresponds on the classical level to the solution of the Cauchy problem under nonvanishing (nontrivial) boundary conditions at both infinities. Such a picture can occur, e.g. when electro-magnetic waves go through a stable nonlinear medium with dispersion relation $\omega = k^2 + \kappa|\vec{E}|^2$ as well as in the case of a Bose gas with internal quasi-spin ("colour") degrees of freedom and a repulsion between particles.

The $U(2)$ NLSE model, direct generalization of the $U(1)$ NLSE, is described by the system of two coupled nonlinear equations for the two-component complex vector-function $q(x,t)$:

$$iq_t + q_{xx} - 2((q^+q) - \rho)q = 0 \qquad (1)$$

under nonvanishing boundary conditions

$$\begin{cases} q(x,t) \xrightarrow[x\to\pm\infty]{} q_\pm \\ q_x(x,t) \xrightarrow[|x|\to\infty]{} 0\,, \end{cases} \qquad (2)$$

where $(q^+q) \equiv |q_1|^2 + |q_2|^2$, $q^+ = (q^*)^T$.

From (1) by virtue of (2) it follows that correct setting of the problem implies

$$(q_+^+q_+) = (q_-^+q_-) = \rho\,. \qquad (3)$$

The possibility of the complete study of system (1) comes from corresponding linear problem:

$$\begin{cases} \phi_x = U\phi & (4) \\ \phi_t = V\phi\,, & (5) \end{cases}$$

where

$$U(x,\lambda) = -i\lambda\Sigma + Q(x)\,,$$

$$V(x,\lambda) = \begin{pmatrix} 4i\lambda^2 + i((q^+q) - \rho), & -2i\lambda q^+ + q_x^+ \\ 2i\lambda q + q_x\,, & -iq \otimes q^+ + i\rho I_2 \end{pmatrix}, \qquad (6)$$

$$\Sigma = \begin{pmatrix} 1 & 0 \\ 0 & -I_2 \end{pmatrix}, \qquad Q(x) = \begin{pmatrix} 0 & iq^+(x) \\ -iq(x) & 0\cdot I_2 \end{pmatrix}.$$

We shall follow the Zakharov-Shabat scheme of the Inverse Scattering Method [5] (see also [11]).

1. THE DIRECT PROBLEM

Consider the spectral problem (4) on the axis $-\infty < x < \infty$. Let us introduce two sets of the column Jost solutions which are determined by their asymptotic behaviour:

$$\begin{aligned} \Phi_{\pm1}(x,\lambda) &\xrightarrow[x\to\pm\infty]{} X_{\pm1}(x,\lambda) = e^{-i\zeta x}\begin{pmatrix} \lambda+\zeta \\ q_{\pm1} \\ q_{\pm2} \end{pmatrix}, \\ \Phi_{\pm2}(x,\lambda) &\xrightarrow[x\to\pm\infty]{} X_{\pm2}(x,\lambda) = e^{i\zeta x}\begin{pmatrix} \lambda-\zeta \\ q_{\pm1} \\ q_{\pm2} \end{pmatrix}, \end{aligned} \qquad (7)$$

$$\Phi_{\pm 3}(x,\lambda) \xrightarrow[x\to\pm\infty]{} X_{\pm 3}(x,\lambda) = e^{i\lambda x}\begin{pmatrix} 0 \\ -q^*_{\pm 2} \\ q^*_{\pm 1}\end{pmatrix},$$

where $\zeta^2 = \lambda^2 - \rho$.

Since the Jost solutions form the fundamental system of solutions, Φ_+ is a linear combination of Φ_-:

$$\Phi_-(x,\lambda) = \Phi_+(x,\lambda)\, S(\lambda), \tag{8}$$

where $S(\lambda)$ is the scattering matrix. Because of

$$\det\Phi_+(x,\lambda) = \det\Phi_-(x,\lambda) = \det X_\pm(x,\lambda) = 2\zeta\rho\, e^{i\lambda x} \tag{9}$$

from (8) we have the unimodularity of $S(\lambda)$:

$$\det S(\lambda) = 1\,. \tag{10}$$

Let us examine the symmtery properties of the Jost solutions. Consider for that the conjugate to (4) equation:

$$\Phi^+(-i\Sigma\partial_x + Q^+) = \lambda^*\Phi^+ .$$

Having used the fact that in our case

$$Q^+ = Q,\quad \Sigma^+ = \Sigma$$

and hence

$$\frac{d}{dx}(\Sigma\Phi^+(x,\lambda)\,\Sigma\Phi(x,\lambda)) = 0\,,$$

for real λ one obtains:

$$\Sigma\Phi^+(x,\lambda)\Sigma\Phi(x,\lambda) = A(\lambda) = \mathrm{const}\,. \tag{11}$$

The explicit form of the matrix $A(\lambda)$ is defined by the concrete choice of the Jost solutions. For the unnormalized ones (7) it reads:

$$A(\lambda) = \mathrm{diag}(2\zeta(\zeta+\lambda),\ 2\zeta(\zeta-\lambda),\ -\rho).$$

The appropriate choice of the Jost solutions

$$\tilde\Phi_{\pm 1}(x,\lambda) = \frac{1}{\sqrt{2\zeta(\zeta+\lambda)}}\Phi_{\pm 1}(x,\lambda),$$

$$\tilde\Phi_{\pm 2}(x,\lambda) = \frac{1}{\sqrt{2\zeta(\zeta-\lambda)}}\Phi_{\pm 2}(x,\lambda),$$

$$\tilde\Phi_{\pm 3}(x,\lambda) = \frac{1}{\sqrt{-\rho}}\Phi_{\pm 3}(x,\lambda)$$

gives instead of (11):

$$\bar\Phi(x,\lambda)\,\Phi(x,\lambda) = I,\quad \bar\Phi(x,\lambda) \equiv \Sigma\Phi^+(x,\lambda)\Sigma. \tag{12}$$

From (8) and (12) we have the pseudounitarity condition:

$$\bar S(\lambda)S(\lambda) = I, \tag{13}$$

where again $\bar S(\lambda) = \Sigma S^+(\lambda)\Sigma$, and a very important relation

$$S(\lambda) = \bar\Phi_+(x,\lambda)\Phi_-(x,\lambda) \tag{14}$$

which connects analytical properties of the S-matrix and the Jost solutions.

We have so far considered the properties of the Jost solutions and S-matrix for real λ and ζ. Let us define their analytic behaviour in the λ-plane. Note that the function $\zeta(\lambda) = \sqrt{\lambda^2-\rho}$ is defined on the two-fold Riemanian surface which first sheet is glued with the second one along cuts $(-\infty, -\sqrt{\rho})$ and $(\sqrt{\rho}, +\infty)$.

The analytical properties of the Jost functions can be derived from the following integral equations:

$$\Phi_\pm(x,\lambda) = X_\pm(x,\lambda) - \int_x^{\pm\infty} dy\, X_\pm(x,\lambda)\, X_\pm^{-1}(y,\lambda)\,\Sigma\,(Q_\pm - Q(y))\,\Phi_\pm(y,\lambda) \tag{15}$$

which are equivalent to equations (4) under boundary conditions (2). Supposing that the potential $Q(x)$ tends to its asymptotics $Q_\pm$ fast enough, one can then ensure that Jost Φ_{+2} and Φ_{-1} can be analytically continued on the upper sheet of the Riemanian surface ($\mathrm{Im}\,\zeta > 0$); solutions Φ_{+1} and Φ_{-2} are analytical functions of λ on the lower sheet of the Riemanian surface ($\mathrm{Im}\,\zeta < 0$), and solutions Φ_{+3} and Φ_{-3} are defined on the real axis of λ-plane and have no analytical continuation. In their own regions of analyticity they have the following asymptotics (for large $|\lambda|$):

$$\Phi_{\pm j}(x,\lambda) \to X_{\pm j}(x,\lambda)\,(1 + O(1/|\lambda|)),\quad j = 1,2. \tag{16}$$

Note that (14) may be rewritten in components

$$S_{ik} = \bar\Phi_{+i}\,\Phi_{-k}\,. \tag{14'}$$

Thus (14') allows us to describe the S-matrix analytical properties via those of the Jost functions. In particular, from (14') it follows that the function $S_{11}(\lambda, \zeta)$ is analytical on the upper sheet of the Riemanian surface $(\mathrm{Im}\zeta > 0)$. The discrete spectrum of the problem (4) lies in the interval $(-\sqrt{\rho}, \sqrt{\rho})$ between cuts and is defined by the zeroes of the function $S_{11}(\lambda, \zeta)$. The continuous part of the spectrum (4) drastically differs from that of the U(1) NLSE model. In addition to the two-fold degenerated part of continuous spectrum lying on both the cuts of the Riemanian surface there is one more branch lying on the whole real axis of the λ-plane. Nevertheless, as we shall see the appearence of this branch does not effect the existence of pure soliton solutions in the system.

Therefore, at the points of the discrete spectrum we have:

$$\Phi_{-1}(x, \lambda_n) = c_n \Phi_{+2}(x, \lambda_n), \quad c_n \equiv S_{21}(\lambda_n, \zeta_n), \tag{17}$$

$$S_{11}(\lambda_n, \zeta_n) = 0, \quad S_{31}(\lambda_n, \zeta_n) = 0 . \tag{18}$$

It is interesting to notice that in the present case (unlike the U(1) model) one may say about the degeneration of the discrete spectrum, too.

Now let us obtain the time evolution of the spectral data. We use for that the technique of the Hamiltonian equations of motion for the S-matrix elements

$$S_t(\lambda, t) = \{H, S(\lambda, t)\}, \tag{19}$$

where $H = I_3 + 2\rho I_1$ is the Hamiltonian of the system (1), I_n are the expansion coefficients of the function $\ln S_{11}(\lambda, \zeta)$ in a series of λ^{-1} (i.e., the local involutive concervation laws of the system). Using the explicit form of the Poisson brackets between different elements of the S-matrix /9/ it is easy to obtain

$$iS_t(\lambda, t) = [\Gamma(\lambda), S(\lambda, t)], \tag{20}$$

where $\Gamma(\lambda) = \mathrm{diag}((\lambda+\zeta), (\lambda-\zeta), 0)$, or for components

$$\partial_t S_{kk}(\lambda, t) = 0, \quad \partial_t S_{km}(\lambda, t) = -\partial_t S_{mk}(\lambda, t), \quad k, m = 1,2,3,$$

$$\partial_t S_{12}(\lambda, t) = -4i\lambda\zeta S_{12}(\lambda, t), \tag{21}$$

$$\partial_t S_{13}(\lambda, t) = -i(\lambda+\zeta)^2 S_{13}(\lambda, t),$$

$$\partial_t S_{23}(\lambda, t) = -i(\lambda-\zeta)^2 S_{23}(\lambda, t).$$

2. INVERSE PROBLEM

From (14) making use of (15) one can derive the existence of the triangular representation for the Jost function $\Phi_+(x, \lambda)$:

$$\Phi_+(x, \lambda) = X_+(x, \lambda) - \int_x^\infty dy\, K(x, y)\, X_+(y, \lambda). \tag{22}$$

Inserting (22) into the linear problem (4) we get the differential equation:

$$\Sigma K_x(x, y) + K_y(x, y)\Sigma = iQ(x)K(x, y) - iK(x, y)Q_+ \tag{23}$$

with the boundary conditions

$$[K(x, x), \Sigma] = i(Q_+ - Q(x)), \qquad K(x, y) \xrightarrow[y\to\infty]{} 0. \tag{24}$$

One can then express the potential $q(x)$ through the elements of the kernel $K(x, y)$:

$$q_k(x) = q_{+k} + 2iK^*_{1, k+1}(x, x), \quad k = 1,2. \tag{25}$$

In addition:

$$K^*_{1n}(x, x) = K_{n1}(x, x), \quad n = 2,3.$$

To get the Marchenko equation let us rewrite the first column of (8) in the form:

$$\frac{1}{S_{11}}\Phi_{-1} - X_{+1} = \Phi_{+1} - X_{+1} + r_{21}\Phi_{+2} + r_{31}\Phi_{+3},$$

multiplying it by $\frac{1}{2\pi\zeta}e^{i\zeta y}$ integrate along the infinite circle at the complex λ-plane on the upper sheet of the Riemanian surface $(\mathrm{Im}\zeta > 0)$. One can apply the residue technique at points λ_n to the left-hand side of this relation (under condition $y > x$). The result is as follows:

$$\sum_n \frac{\Phi_{-1}(x, \lambda_n)e^{-\nu_n y}}{\nu_n S'_{11}(\lambda_n, i\nu_n)} = \sum_n \frac{c_n\Phi_{+2}(x, \lambda_n)e^{-\nu_n y}}{\nu_n S'_{11}(\lambda_n, i\nu_n)} \equiv$$

$$\equiv \sum_n \mu_n \Phi_{+2}(x, \lambda_n)e^{-\nu_n y},$$

where

$$\zeta_n = \sqrt{\lambda_n^2 - \rho} = i\sqrt{\rho - \lambda_n^2} \equiv i\nu_n, \quad S_{11}(\lambda_n, i\nu_n) = 0.$$

The right-hand side (which is the continuous part of the spectrum) one can represent in the form (supposing the existence of corresponding limits for $\Phi_{\pm 3}$ and r_{31} near the real axis):

$$\int \frac{d\lambda}{2\pi\xi} e^{i\xi y} \left(\Phi_{+1} - X_{+1} + r_{21}\Phi_{+2} + r_{31}\Phi_{+3}\right) = K(x,y) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} +$$

$$+ \begin{pmatrix} F_1^{(1)}(x+y) + iF_2^{(1)\prime}(x+y) \\ q_{+1} F_2^{(1)}(x+y \\ q_{+2} F_2^{(1)}(x+y) \end{pmatrix} + F_3^{(1)}(x,y) \begin{pmatrix} 0 \\ -q^*_{+2} \\ q^*_{+1} \end{pmatrix} -$$

$$- \int_x^\infty ds\, K(x,s) \left[\begin{pmatrix} F_1^{(1)}(s+y) + iF_2^{(1)\prime}(s+y) \\ q_{+1} F_2^{(1)}(s+y) \\ q_{+2} F_2^{(1)}(s+y) \end{pmatrix} + F_3^{(1)}(s,y) \begin{pmatrix} 0 \\ -q^*_{+2} \\ q^*_{+1} \end{pmatrix} \right],$$

where

$$F_1^{(1)}(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\xi\, b_1(\xi)\, e^{i\xi z},$$

$$F_2^{(1)}(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\xi\, b_2(\xi)\, e^{i\xi z},$$

$$F_3^{(1)}(x,y) = \frac{1}{2\pi} \int \frac{d\lambda}{\xi} r_{31}(\lambda,\xi)\, e^{i(\lambda x + \xi y)},$$

$$b_1(\xi) = \frac{1}{2}[r_{21}(\lambda,\xi) + r_{21}(-\lambda,\xi)],$$

$$b_2(\xi) = \frac{1}{2\lambda}[r_{21}(\lambda,\xi) - r_{21}(-\lambda,\xi)], \quad \xi = \mathrm{Re}\zeta.$$

Finally the Marchenko equations become:

$$K(x,y)\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} F_1(x+y) + iF_2'(x+y) \\ q_{+1} F_2(x+y) \\ q_{+2} F_2(x+y) \end{pmatrix} + F_3^{(1)}(x,y) \begin{pmatrix} 0 \\ -q^*_{+2} \\ q^*_{+1} \end{pmatrix} - \tag{26}$$

$$- \int_x^\infty ds\, K(x,s) \left[\begin{pmatrix} F_1(s+y) + iF_2'(s+y) \\ q_{+1} F_2(s+y) \\ q_{+2} F_2(s+y) \end{pmatrix} + F_3^{(1)}(s,y) \begin{pmatrix} 0 \\ -q^*_{+2} \\ q^*_{+1} \end{pmatrix} \right] = 0,$$

where

$$F_1^{(2)}(z) = -\sum_n \mu_n \lambda_n e^{-\nu_n z},$$

$$F_2^{(2)}(z) = -\sum_n \mu_n e^{-\nu_n z},$$

$$F_{1,2}(z) = F_{1,2}^{(1)}(z) + F_{1,2}^{(2)}(z).$$

In the case of reflectionless potentials the Marchenko equations may be explicitly solved. So we look for a solution of (26) in the form:

$$K(x,y) = \sum_{n=1}^{N} K_n(x)\, X^+_{+2}(y, \lambda_n) \tag{27}$$

with $K_n(x)$ being the column vector.

System (26) with kernel (27) reduces to the system of 2N (where N is the number of eigenvalues λ_n) linear algebraic equations for $K_n(x)$ which immediately leads to the N-soliton solution. In particular, the single-soliton kernel $K(x,y)$ $(N = 1)$ assumes the form:

$$K(x,y) = \frac{\nu e^{\nu(x-y)}}{\rho\left(1 + \frac{\nu}{\mu} e^{2\nu x}\right)} \begin{pmatrix} \rho & q^*_{+1}(\lambda - i\nu) & q^*_{+2}(\lambda - i\nu) \\ q_{+1}(\lambda + i\nu) & |q_{+1}|^2 & q_{+1} q^*_{+2} \\ q_{+2}(\lambda + i\nu) & q_{+2} q^*_{+1} & |q_{+2}|^2 \end{pmatrix} \tag{28}$$

From (18) and (21) the time dependence of $\mu(\lambda, t)$ is as follows:

$$\mu(\lambda, t) = \mu(\lambda, 0)\, e^{4\lambda\nu t}, \tag{29}$$

therefore we have the following single-soliton solution to the problem (1)-(2):

$$q_k(x,t) = q_{+k}\frac{(\lambda+i\nu)^2\rho^{-1} + e^{2\nu(x-x_0-2\lambda t)}}{1+e^{2\nu(x-x_0-2\lambda t)}},\quad k=1,2, \tag{30}$$

where

$$e^{2\nu x_0} = \frac{\mu(\lambda,0)}{\nu},\quad \lambda^2+\nu^2=\rho=|q_{+1}|^2+|q_{+2}|^2 .$$

One can now verify the validity of the boundary conditions for the solution. Really at $x\to+\infty$, $q_k\to q_{+k}$ and at $x\to-\infty$, $q_k\to q_{+k}e^{i\alpha}=q_{-k}$ then the condition $|q_{+1}|^2+|q_{+2}|^2=|q_{-1}|^2+|q_{-2}|^2$ is fulfilled which guarantees the spectra of the "asymptotic" operators $U_\pm$ coinciding and the correctness of setting of the problem (1)-(2).

3. THE SINGLE-SOLUTION STABILITY

The next important question is the stability of the above solution under small continuous perturbations. Unlike the U(1) case here we have two different continuous modes connected with functions r_{21} and r_{31} respectively. Using the methods of paper /10/ it will be shown that appearence of the additional perturbation source does not in this case break the single-solution stability.

Let the spectral function F be presented in the form:

$$F(x,y;t) = F_d(x,y;t) + F_c(x,y;t),$$

where the continuous branch F_c consists of two modes related to r_{21} and r_{31}, respectively. The Marchenko equations become

$$K(x,y;t) + F_d(x,y;t) + F_c(x,y;t) - \int_x^\infty ds\,K(x,s;t)\,[F_d(s,y;t)+F_c(s,v;t)] = 0. \tag{31}$$

We estimate the correction to a pure solution at $t\to+\infty$ under the conditions:

$$|r_{21}(\lambda,\xi)| \ll 1,\quad |r_{31}(\lambda,\xi)| \ll 1. \tag{32}$$

Representing the kernel in (31) as

$$K(x,y;t) = K(x,t)\,\overset{+}{X}_{+2}(y,\lambda) + \delta K(x,y;t)$$

we have for δK:

$$\delta K(x,y;t) - \int_x^\infty ds\,\delta K(x,s;t)\,F_d(s,y;t) = \delta G(x,y;t), \tag{33}$$

where

$$\delta G(x,y;t) = -F_c(x,v;t) + K(x,t)\int_x^\infty ds\,\overset{+}{X}_{+2}(s,\lambda)\,F_c(s,y;t),$$

and we neglect the term $\delta K\cdot F_c$ as the higher order one with respect to δK. Using the relation between F_c and $F^{(1)}_{1,2,3}$ one then obtains

$$F_c(x,y;t) = \int_{-\infty}^{\infty} d\xi\, g_1(\xi;x,y)\,e^{itf_1(\xi)} + \int_{-\infty}^{\infty} d\xi\, g_2(\xi;x,y)\,e^{itf_2(\xi)}, \tag{34}$$

where $f_1(\xi) = 4\lambda(\xi)\xi$, $f_2(\xi) = (\lambda(\xi)+\xi)^2$,

$$g_1(\xi,x,y) = \frac{S_{21}(\lambda,\xi)}{\lambda S_{11}(\lambda,\xi)}e^{i\xi(x+y)},$$

$$g_2(\xi;x,y) = \frac{S_{31}(\lambda,\xi)}{\lambda S_{11}(\lambda,\xi)}e^{i(\lambda x+\xi y)}.$$

It is easy to verify that equations

$$\frac{df_{1,2}(\xi)}{d\xi} = 0$$

have no real solutions in the definition region of ξ. As is well known, this means that the phase functions $f_{1,2}(\xi)$ have no stationary points. Therefore the main contribution is of the following asymptotical behaviour at $t\to+\infty$:

$$F_c(x,y;t) \sim \frac{g_1(\infty;x,y)}{it}e^{itf_1(\infty)} + \frac{g_2(\infty;x,y)}{it}e^{itf_2(\infty)}. \tag{35}$$

Estimating $g_{1,2}(\xi; x, y)$ at large ξ needs asymptotical representation (for $|\lambda| \to \infty$) of the elements S_{11}, S_{21} and S_{31} which due to (14') and (17) are

$$S_{11}(\lambda, \xi) \simeq \frac{(\lambda+\xi)^2 - (\overset{+}{q}_+ q_-)}{2\xi(\lambda+\xi)},$$

$$S_{21}(\lambda, \xi) \simeq \frac{\rho + (\overset{+}{q}_+ q_-)}{2\xi\sqrt{-\rho}},$$

$$S_{31}(\lambda, \xi) \simeq \frac{q_{+1} q_{-2} - q_{+2} q_{-1}}{(\lambda+\xi)\sqrt{2\xi(\xi-\lambda)}}.$$

Finally we have relations

$$|r_{21}(\lambda, \xi)| \sim \frac{1}{\xi}, \qquad \xi \to \infty$$
$$|r_{31}(\lambda, \xi)| \sim \frac{1}{\xi^2},$$

which are consistent with above conditions (32). Thus the integral $F_c(x, y; t)$ at $t \to +\infty$ is of the order

$$|F_c(x, y; t)| \leq \frac{c_1}{t} + \frac{c_2}{t} \tag{36}$$

with c_1 and c_2 corresponding to $r_{21}(\lambda, \xi)$ and $r_{31}(\lambda, \xi)$.We can now obtain the estimate of δG in (33) which is due to (36)

$$|\delta G(x, y; t)| \leq \left(\frac{c_1}{t} + \frac{c_2}{t}\right)\left(1 + \frac{|K(x)|}{2\nu}\right) \leq \frac{c_1'}{t} + \frac{c_2'}{t}.$$

Look for a solution of (33) in the form:

$$\delta K(x, y; t) = \delta K(x, t)\, \overset{+}{X}_{+2}(y, \lambda) + \delta G(x, y; t).$$

Inserting it into (33) we get for $\delta K(x,t)$:

$$\delta K(x, t)\, \overset{+}{X}_{+2}(y, \lambda) - \delta K(x, t) \int_{-\infty}^{\infty} ds\, \overset{+}{X}_{+2}(s, \lambda) F_d(s, y; t) =$$

$$= \int_x^{\infty} ds\, \delta G(x, s; t)\, F_d(s, y; t).$$

Having used the explicit form of the single-soliton spectral function F_d one can found the estimate of δK at $t \to +\infty$

$$|\delta K(x, t)| \leq \frac{c'}{t} + \frac{c''}{t} \tag{37}$$

which shows that the weakly perturbed $U(0, 2)$ single-soliton solution is asymptotically "cleaned" as $1/t$ so resembling the $U(0, 1)$ one.

Besides, we show that appearence of the additional continuous mode does not violate the soliton stability. Note that in contrast to the attractive $U(2)$ NLSE the soliton stability is here influenced by the "medium" of a finite density ρ which accelerates the release process of perturbed soliton from a weak continuous spectrum.

Ultimately we should note that in addition to soliton solution (30) there are others. In paper [7], for instance, the solution

$$\begin{cases} q_1(x, t) = a_1 e^{i\theta(x, t)} \operatorname{sech}\kappa z, \\ q_2(x, t) = a_2\left(\operatorname{th}\kappa z + \dfrac{iv}{2\kappa}\right) \end{cases}$$

has been found, in which

$$\theta = \frac{v}{2}x - \omega_1 t, \quad \omega_1 = \frac{v^2}{4} - \kappa^2, \quad \kappa^2 = a_1^2 + a_2^2, \quad z = x - vt - x_0.$$

This solution does not embed in the scope of the Hermitian linear problem (which only we have studied) and requires to proceed to an appropriate non-self-adjoint operator. We also say nothing about such an interesting problem as coloured kinks scattering. They are now in progress.

REFERENCES

1. Steiner M. JMMM,31-34, 1983, p. 1277.
2. Fedyanin V.K., Yushankhai V.Uy. Phys.Lett., 1981, 85A, p.100; Fedyanin V.K. JMMM 31-34. 1983, p. 1237.

3. Lindner U., Fedyanin V. phys.stat.sol., 1978, 89B, p. 123.
Makhankov V.G. Phys.Lett., 1981, 81A, p. 156;
Makhankov V.G., Makhaldiani N.V., Pashaev O.K. Phys.Lett., 1981, 81A, p. 161.
4. Kundu A., Makhankov V.G., Pashaev O.K. JINR, E17-82-677, Dubna, 1982; Physica Scripta (accepted for publication).
5. Zakharov V.E., Shabat A.B. Sov.Phys. JETP, 1973, 64, p.1627.
6. Kawata T., Inoue H. J.Phys.Soc.Jpn., 1979, 44, p. 1722;
Gerdjikov V.S., Kulish P.P. Bulg. J.Phys., 1978, 5, p.337.
Kulish P.P., Manakov S.V., Faddeev L.D. TMF, 1976, 28, p. 38.
7. Makhankov V.G., Pashaev O.K. TMF, 1982, 53, p. 55.
8. Zakharov V.E., Shabat A.B. Sov.Phys., LETP, 1971, 61, p. 118.
9. Pashaev O.K., Sergeenkov S.A. JINR, E17-82-602, Dubna, 1982.
10. Makhankov V.G., Pashaev O.K., Sergeenkov S.A. JINR, P2-83-186, Dubna, 1983.
11. Manakov S.V. ZhETF, 1973, 65, p. 505.

Received by Publishing Department
on June 7,1983.

Маханьков В.Г., Пашаев О.К., Сергеенков С.А. E2-83-378
Квазидырочные возбуждения в многокомпонентных системах

Методом обратной задачи рассеяния изучается задача Коши для векторного НУШ отталкивающегося типа при нетривиальных граничных условиях на полевые переменные. Для случая безотражательных потенциалов построены точные N-солитонные решения. Доказана устойчивость односолитонного решения относительно малых возмущений непрерывным спектром. Показано, что слабовозмущенное решение асимптотически стремится к чисто солитонному, как 1/t.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1983

Makhankov V.G., Pashaev O.K., Sergeenkov S.A. E2-83-378
Hole-Like Excitations in Many Component Systems

The Cauchy problem for repulsive vector nonlinear Schrödinger equation under nonvanishing boundary conditions is studied via the inverse transform. For reflectionless potentials exact N-soliton solutions are constructed. The single solution stability is proved as well under small perturbations of continuous spectrum. The perturbed soliton is shown to tend to a pure one asymptotically as 1/t.

The investifation has been performed at the Laboratory of Computing Techniques and Automation, JINT.

Preprint of the Joint Institute for Nuclear Research. Dubna 1983