сообщения
объединенного
института
ядерных
исследований
дубна

E2-83-351

B.Geyer*, J.Hořejši, D.Robaschik

NONLOCAL CONFORMAL LIGHT-CONE EXPANSION IN QCD

* Sektion Physik der Karl-Marx-Universität, Leipzig, DDR.

1983

1. INTRODUCTION

There are various forms of the operator product expansion near the light cone. Generally known, however, is only the standard local light-cone expansion (LCE), which represents a scattering amplitude via an infinite sum of local operators. Such an expansion may be rearranged and rewritten in terms of local conformal operators, which possess diagonal anomalous dimensions at the one-loop level[1] This property is important for applications to exclusive light-cone dominated processes (see,e.g., refs.[1,2]).

Besides that, there exists the so-called nonlocal LCE, first introduced in[3], which unfortunately seems to be rather unfamiliar for most of physicists. This alternative expansion solves some intrinsic theoretical problems of the local LCE, but the general nonforward anomalous dimensions of its operators are nondiagonal, so that such a form is not very well suited for direct applications to exclusive scattering processes (for a straightforward application to the deep inelastic inclusive scattering, see, however,[4]).

These problems have been already considered in[5]. There has been proposed a new version of the nonlocal LCE which has the nice property that the relevant operators are sufficiently diagonal to all orders of the perturbation theory. All necessary properties have been proved there on the basis of the α-representation for a scalar field theory. Motivated by the scalar case, in the present paper we introduce the corresponding nonlocal "conformal"* LCE for QCD. We calculate the anomalous dimension of the flavour nonsinglet fermion operator and this turns out to coincide with the familiar Brodsky-Lepage kernel[6]. The connection with the local conformal LCE is also briefly discussed. Finally, in order to illustrate the virtues of our formalism, we apply the proposed nonlocal conformal LCE to an exclusive scattering process, discussed earlier in the literature from the point of view of the local conformal LCE (see

---

* Throughout this paper we employ rather loosely the adjective "conformal" although the conformal properties of the relevant nonlocal operators are never discussed. In fact it is the diagonality of the anomalous dimensions which matters. Our terminology is suggested by the existing results for the local LCE (see[1] for details).

ref.[2]). It becomes clear that within our formalism the derivation of an essential result (an evolution equation) is particularly compact and straightforward.

This paper should be understood as a direct continuation of ref.[5]. Therefore we do not explain the meaning of the symbols already defined there.

2. NONLOCAL CONFORMAL LIGHT-CONE EXPANSION

In this section we shall describe the essential steps leading to the nonlocal conformal LCE in QCD. For the sake of clarity and simplicity we wish to avoid all the (inessential) complications arising from the tensor structure of the product of two currents and therefore we restrict ourselves to the investigation of the scalar product of two electromagnetic currents. For this case the following nonlocal LCE has been derived earlier[7] in the acial gauge:

$$R(j^\mu(x)\; j_\mu(0)\, S) \underset{x^2\to 0}{\approx} \sum_i \int d\kappa_1 d\kappa_2\, \Sigma_i(x^2, \kappa_1, \kappa_2)\, \bar{R}(\Omega(\kappa_1\tilde{x}, \kappa_2\tilde{x})\, S) \tag{2.1}$$

$$\Omega_i(\kappa_1\tilde{x},\ \kappa_2\tilde{x}) = :\bar{\psi}(\kappa_2\tilde{x})\, \gamma_\mu \tilde{x}^\mu \lambda_i\, P\exp(-ig\int_{\kappa_1}^{\kappa_2} A^\mu(\tau\tilde{x})\tilde{x}_\mu d\tau)\, \psi(\kappa_1\tilde{x}): \tag{2.2}$$

Here ψ and $A_\mu = A_\mu^a t^a$ are the spinor and gluon fields of QCD. λ_i is a flavour matrix (the projector of the i-th flavour), the symbol P means path-ordering of the matrices appearing in the exponential along the straight line connecting $\kappa_1\tilde{x}$ with $\kappa_2\tilde{x}$. For the rest, see ref.[5]; note only that $\tilde{x}$ is a light-like vector corresponding to x, satisfying $x - \tilde{x} = 0(\sqrt{x^2})$. Furthermore, we have restricted ourselves to the flavour nonsinglet part of the LCE, which involves only the fermion operator. The coefficient function Σ is determined in the following way: take the x-proper functional of the renormalized product of two currents with two outgoint fermion lines (we drop for the moment flavour indices)

$$R(j^\mu(x)\; j_\mu(0)\, S) = \tag{2.3}$$
$$= \int dz_1 dz_2 F_{\alpha\beta}(x, z_1, z_2) :\bar{\psi}_\alpha(z_2)\, P\exp(-ig\int_{z_1}^{z_2} A_\mu dz^\mu)\, \psi_\beta(z_1): +\ldots$$

pick up the following term of the kinematical decomposition

$$F_{\alpha\beta}(x, z_1, z_2) = \gamma^\mu_{\alpha\beta} x_\mu F(x, z_1, z_2) + \ldots\ , \tag{2.4}$$

take the Fourier transform

$$\tilde{F}(x^2, xq_i, q_i q_j) = \int dz_1 dz_2 e^{-iz_1q_1 - iz_2q_2} F(x, z_1, z_2),$$

substitute into this $qx \to q\tilde{x}$, $q_i q_j \to \mu_{ij}$ (subtraction points of the light-cone subtraction procedure), and perform an additional Fourier transformation

$$\Sigma(x^2, \kappa_1, \kappa_2) = \tag{2.5}$$
$$= \frac{1}{(2\pi)^2} \int d\tilde{x}q_1\, d\tilde{x}q_2\, e^{-iq_1\tilde{x}\kappa_1 - iq_2\tilde{x}\kappa_2} F(x^2, \tilde{x}q_i, \mu_{ij}).$$

In this way the coefficient function in (2.1) is defined. Now we make in a sense one step backwards and substitute this expression into the LCE (2.1); simultaneously we introduce the Fourier transform of the fermion operator

$$\Omega(\kappa_1\tilde{x},\ \kappa_2\tilde{x}) = \int dk_1 dk_2\, e^{ik_1\tilde{x}\kappa_1 + ik_2\tilde{x}\kappa_2} \tilde{\Omega}(k_1, k_2) \tag{2.6}$$

and get the result

$$R(j^\mu(x)\; j_\mu(0)\, S) \approx \bar{R} \int dq_1 dq_2 F(x^2, \tilde{x}q_i, \mu_{ij})\, \tilde{\Omega}(q_1, q_2)\, S. \tag{2.7}$$

This corresponds to the eq. (2.5) in[5]. Now we follow the construction of the nonlocal conformal LCE worked out for scalar fields: The expression (2.7) is difficult to handle with; the momenta must be eliminated from the coefficient functions. For this reason we introduce the variables $q_\pm = q_2 \pm q_1$ and $t = q_-\tilde{x}/q_+\tilde{x}$ and use the following representation

$$F(x^2, xq_i, \mu_{ij}) = \int dt\; d\kappa\; F^c(x^2, t, \kappa)\, \delta(t - \frac{\tilde{x}q_-}{\tilde{x}q_+})\, e^{i\kappa\tilde{x}q_+} \tag{2.8}$$

with

$$F^c(x^2, t, \kappa) = \frac{1}{2\pi} \int d\tilde{x}q\; e^{-i\kappa\tilde{x}q_+} F(x^2, xq_+, t\, xq_+, \mu_{ij}). \tag{2.9}$$

The insertion of this expression into eq. (2.7) leads to the desired new version of the nonlocal LCE

$$R(j^\mu(x)\; j_\mu(0)\, S) \approx \sum_i \int dt\; d\kappa\; F_i^c(x^2, t, \kappa)\; \bar{R}\, \Omega_i^c(t, \kappa)\, S, \tag{2.10}$$

$$\Omega_i^c = \int dq_1 dq_2\, \delta(t - \frac{\tilde{x}q_-}{\tilde{x}q_+})\, \tilde{\Omega}_i(q_1, q_2)\, e^{i\kappa\tilde{x}q_+}. \tag{2.11}$$

If we furthermore represent the δ-function by a Fourier integral, the expression for the light-cone operator takes the form

$$\Omega_i^c = \frac{1}{2\pi}\frac{\partial}{i\,\partial\kappa}\int d\lambda : \bar{\psi}(\tilde{x}(\kappa+\lambda(t-1)))\,\gamma\tilde{x}\lambda_i \,P\exp\Big(-ig\int_{\kappa+\lambda(t+1)}^{\kappa+\lambda(t-1)} A_\mu \tilde{x}^\mu d\tau\Big)\,\psi(\tilde{x}(\kappa+\lambda(t+1))): \tag{2.12}$$

which is a direct generalization of the corresponding operator in the scalar case.

Let us conclude this section with several remarks on the general properties of the LCE (2.10). The first remark concerns the integration range in (2.10): In ref.[5] it has been shown for scalar theories that by taking into account the support properties of matrix elements of the conformal light-cone operator, the integration region w.r.t. variable t can be restricted to the interval $(-1,1)$ which also implies a finite κ-integration range. We expect that an analogous result holds also for (2.10), although a detailed argument will not be given here. The second point concerns the anomalous dimensions $\gamma(\kappa,t,\kappa',t')$ of the operator (2.12). In the scalar case[5] an analogous object has been shown to be diagonal w.r.t. κ, i.e.,

$$\gamma(\kappa,\ t,\ \kappa',\ t') = \delta(\kappa-\kappa')\ \gamma(t,t') \tag{2.13}$$

to all orders of perturbation theory. Again, we expect the general validity of (2.13) for the operator (2.12) but the proof is not given. In the next section we shall calculate explicitly the anomalous dimension of (2.12) in the one-loop approximation, thereby confirming (2.13) at least in the lowest nontrivial order.

3. ANOMALOUS DIMENSION OF THE NONLOCAL CONFORMAL OPERATOR

We shall employ the following definition of the anomalous dimension of a composite operator in terms of Z-factors:

$$\mu\frac{\partial}{\partial\mu} Z(\kappa,t,\kappa',t') = \int d\kappa'' dt''\, Z(\kappa,t,\kappa'',t'')\,\gamma(\kappa'',t'',\kappa',t'); \tag{3.1}$$

$$Z_2^{-1}\int d\kappa' dt'\, Z(\kappa,t,\kappa',t')\,(\bar{\psi}\Omega(\kappa',t')\psi)^{1PI}_{ren} = (\bar{\psi}\Omega(\kappa,t)\psi)^{1PI}_{unren},$$

where the symbol 1PI means 1-particle irreducible graphs and Z_2 corresponds to the external legs. In a practical calculation this amounts to the evaluation of the uv-divergent parts of the relevant diagrams. Instead of dealing directly with the operator (2.12) we shall calculate the one-loop anomalous dimension of the simpler operator

$$\bar{\Omega}_i^c(\kappa,t) = \int\frac{d\lambda}{2\pi} : \bar{\psi}((\lambda(t-1)+\kappa)\tilde{x})\,\gamma\tilde{x}\lambda_i\,P\exp\Big(-ig\int_{\lambda(t+1)+\kappa}^{\lambda(t-1)+\kappa} A_\mu(\tau\tilde{x})\tilde{x}^\mu d\tau\Big)\,\psi((\lambda(t+1)+\kappa)\tilde{x}): \tag{3.2}$$

(i.e., with the derivative w.r.t. κ removed from (2.12)). In view of the diagonality in the variable κ this should yield the same result. One more important remark is in order here: Although eq. (2.1) (which has been the starting point of our considerations) has been derived in[7] in the axial gauge, we shall perform our calculation in the covariant Feynman gauge since on the one-loop level it makes no difference and the calculation itself is thus simplified considerably.

We have to calculate matrix elements of the operator (3.2) up to the order $O(g^2)$. The relevant graphs are depicted in fig.1.

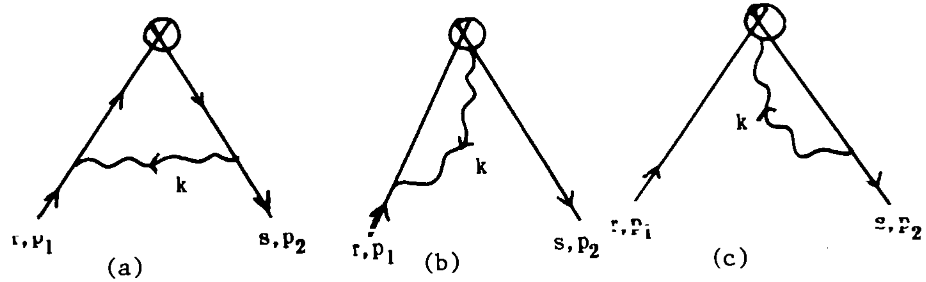
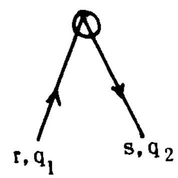
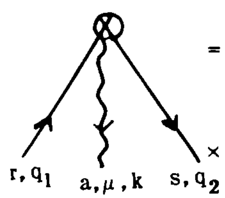


Fig.1. The one-loop 1PI diagrams contributing to $\gamma(\kappa,t,\kappa',t')$.

In what follows we denote the scalar product $a\cdot\tilde{x}$ by $\tilde{a}$ for any four-vector a. The Feynman rules for the operator vertices are (cf. also[4])



$$= \delta_{rs}\gamma\lambda_i\int\frac{d\lambda}{2\pi}\, e^{-\frac{i}{2}\tilde{q}_1(\lambda(t+1)+\kappa)+\frac{i}{2}\tilde{q}_2(\lambda(t-1)+\kappa)} \equiv \bar{\Omega}^c_{i;rs}(\kappa,t;q_1,q_2) \tag{3.3}$$



$$= -ig(t^a)_{rs}\tilde{x}_\mu\tilde{\gamma}\lambda_i\int\frac{d\lambda}{2\pi}\, e^{-\frac{i}{2}\tilde{q}_1(\lambda(t+1)+\kappa)+\frac{i}{2}\tilde{q}_2(\lambda(t-1)+\kappa)} \times \frac{e^{\frac{i}{2}\tilde{k}(\lambda(t-1)+\kappa)} - e^{\frac{i}{2}\tilde{k}(\lambda(t+1)+\kappa)}}{i\tilde{k}}.$$

The contribution of the diagram in fig.1(a) is

$$\Gamma_{1a} = i\,\delta_{rs}\,\lambda_i\,C_2\,g^2 \int \frac{d^4k}{(2\pi)^4}\int\frac{d\lambda}{2\pi}\,\gamma^\mu\,(\hat{k}+\hat{p}_2)\,\tilde{\gamma}\,(\hat{k}+\hat{p}_1)\,\gamma_\mu \times \tag{3.4}$$

$$\times\frac{1}{(k+p_1)^2\,k^2\,(k+p_2)^2}\times e^{-\frac{i}{2}(\tilde{k}+\tilde{p}_1)(\lambda(t+1)+\kappa)+\frac{i}{2}(\tilde{k}+\tilde{p}_2)(\lambda(t-1)+\kappa)},$$

where C_2 is the Casimir invariant, $C_2\delta_{rs}=(t^a t^a)_{rs}$. Proceeding in the usual way, we introduce Feynman parameters, shift the loop momenta, and perform symmetric integrations; we use the dimensional regularization in order to isolate the uv-divergent part. Note that the role of the light-like vector $\tilde{x}$ is twofold: On the one hand it introduces the uv-divergences and on the other hand it causes that only the first term of the Taylor expansion around $\tilde{k}=0$ of the shifted exponentials in (3.4) contributes when the symmetric integrations over $d^n k$ are carried out. The uv-divergent part of (3.4) is then given by (setting as usual $(4-n)^{-1}=\ln(\Lambda/\mu)$)

$$\Gamma^{div}_{1a} = C_2\,\frac{g^2}{4\pi^2}\ln\frac{\Lambda}{\mu}\,\delta_{rs}\,\tilde{\gamma}\,\lambda_i\int_0^1\frac{d\lambda}{2\pi}\int_0^1 y\,dy\,\times \tag{3.5}$$

$$\times\int dx\,e^{-\frac{i}{2}\tilde{p}_1(\lambda(t+1-2xy)+\kappa)+\frac{i}{2}\tilde{p}_2(\lambda(t+1-2y)+\kappa)}.$$

Now we perform a change of variables $(y,\lambda)\to(t',\lambda')$ in order to get

$$\lambda(t+1-2xy)=\lambda'(t'+1),\qquad \lambda(t+1-2y)=\lambda'(t'-1); \tag{3.6}$$

this implies

$$\lambda=\lambda'\,\frac{2+(1-x)\,(t'-1)}{(1-x)\,(t+1)},\qquad y=\frac{t+1}{2+(1-x)\,(t'-1)}. \tag{3.7}$$

The expression (3.5) may then be rewritten as (see (3.3))

$$\Gamma^{div}_{1a} = C_2\,\frac{g^2}{4\pi^2}\ln\frac{\Lambda}{\mu}\int d\kappa'\,dt'\,\delta(\kappa-\kappa')\,\times$$

$$\times\int_0^1 dx\,\theta\!\left(t'-\frac{t-x}{1-x}\right)\frac{|1+t|}{((1-x)t'+1+x)^2}\,\bar{\Omega}^{c}_{i;rs}\,(\kappa',t';p_1,p_2).$$

According to the definition (3.1) the contribution of fig.1(a) to the anomalous dimension $\gamma(\kappa,t,\kappa',t')$ is then

$$\gamma_{1a}(\kappa,t,\kappa',t') = \tag{3.8}$$

$$= -C_2\,\frac{g^2}{4\pi^2}\,\delta(\kappa-\kappa')\int_0^1 dx\,\theta\!\left(t'-\frac{t-x}{1-x}\right)\frac{|t+1|}{((1-x)t'+1+x)^2}.$$

In the subsequent sections we shall need $\gamma(\kappa,t,\kappa',t')$ only for

$$t,t'\in(-1,1). \tag{3.9}$$

The integration in (3.8) with the restriction (3.9) is easily done and we get

$$\gamma_{1a}(\kappa,t,\kappa',t') = -C_2\,\frac{g^2}{8\pi^2}\left[\theta(t'-t)\,\frac{1+t}{1+t'}+\theta(t-t')\,\frac{1-t}{1-t'}\right]. \tag{3.10}$$

The contribution of the graph in fig.1(b) is

$$\Gamma_{1b} = i g^2\,C_2\,\delta_{rs}\,\lambda_i\int\frac{d^4k}{(2\pi)^4}\,\tilde{\gamma}\,(\hat{k}+\hat{p}_1)\,\tilde{\gamma}\,\frac{1}{k^2(k+p_1)^2}\times$$

$$\times\int\frac{d\lambda}{2\pi}\,e^{-\frac{i}{2}(\tilde{k}+\tilde{p}_1)(\lambda(t+1)+\kappa)+\frac{i}{2}\tilde{p}_2(\lambda(t-1)+\kappa)}\,\frac{e^{\frac{i}{2}\tilde{k}(\lambda(t-1)+\kappa)}-e^{\frac{i}{2}\tilde{k}(\lambda(t+1)+\kappa)}}{\tilde{k}}.$$

For the uv-divergent part we obtain (after the standard manipulations along with the change $x\to 1-x$):

$$\Gamma^{div}_{1b} = C_2\,\frac{g^2}{4\pi^2}\ln\frac{\Lambda}{\mu}\,\delta_{rs}\,\tilde{\gamma}\,\lambda_i\int\frac{d\lambda}{2\pi}\int_0^1 dx\,\frac{x}{1-x}\Big[e^{-\frac{i}{2}\tilde{p}_1(\lambda(t+1)+\kappa-2\lambda(1-x))+\frac{i}{2}\tilde{p}_2(\lambda(t-1)+\kappa)} \tag{3.11}$$

$$-\,e^{-\frac{i}{2}\tilde{p}_1(\lambda(t+1)+\kappa)+\frac{i}{2}\tilde{p}_2(\lambda(t-1)+\kappa)}\Big].$$

Now we perform a change of variables $(x,\lambda)\to(t',\lambda')$ so as to have

$$\lambda(t+1)-2\lambda(1-x)=\lambda'(t'+1),\qquad \lambda(t-1)=\lambda'(t'-1).$$

This implies

$$x=\frac{t-1}{t'-1},\qquad \lambda=\lambda'\,\frac{t'-1}{t-1}. \tag{3.12}$$

After some manipulations the integral (3.11) may be rewritten as follows

$$\Gamma_{1b}^{div} = C_2 \frac{g^2}{4\pi^2} \ln\frac{\Lambda}{\mu} \int d\kappa' dt' \delta(\kappa-\kappa')[\theta(t-t')\frac{t-1}{|t'-1|}\frac{1}{(t'-t)_+} + \delta(t-t')]\,\bar{\Omega}^c_{i;rs}(\kappa',t';p_1,p_2).$$

Analogously, for fig.1(c) we get

$$\Gamma_{1c}^{div} = C_2 \frac{g^2}{4\pi^2} \ln\frac{\Lambda}{\mu} \int d\kappa' dt' \delta(\kappa-\kappa')[\theta(t'-t)\frac{t+1}{|t'+1|}\frac{1}{(t'-t)_+} + \delta(t-t')]\,\bar{\Omega}^c_{i;rs}(\kappa',t';p_1,p_2).$$

For the contribution of diagrams 1(b) and 1(c) to the anomalous dimension we then obtain (for t, t' satisfying (3.9))

$$\gamma_{1b+1c}(\kappa,t,\kappa',t') = -C_2 \frac{g^2}{4\pi^2}\delta(\kappa-\kappa')[\theta(t-t')\frac{1-t}{1-t'}\frac{1}{(t-t')_+} + \theta(t'-t)\frac{1+t}{1+t'}\frac{1}{(t'-t)_+} + 2\delta(t-t')].$$

Incorporating in (3.1) the factor Z_2 for the external legs

$$Z_2 = 1 - C_2 \frac{g^2}{8\pi^2}\ln\frac{\Lambda}{\mu}.$$

we finally get the following expression for the anomalous dimension

$$\gamma(\kappa,t,\kappa',t') = \delta(\kappa-\kappa')\,\gamma(t,t'). \qquad (3.13a)$$

$$\gamma(t,t') = -C_2\frac{g^2}{8\pi^2}[\theta(t-t')\frac{1-t}{1-t'} + \theta(t'-t)\frac{1+t}{1+t'} - 2(\theta(t-t')\frac{1}{1-t'} + \theta(t'-t)\frac{1}{1+t'}) + 2(\theta(t-t')\frac{1}{(t-t')_+} + \theta(t'-t)\frac{1}{(t'-t)_+}) + 3\delta(t-t')]. \qquad (3.13b)$$

It is not difficult to verify that (3.13b) may be written as

$$\gamma(t,t') = -C_2\frac{g^2}{8\pi^2}(\gamma_0(t,t') + \gamma_{reg}(t,t')), \qquad (3.14)$$

where

$$\gamma_0(t,t') = \theta(t-t')\frac{1-t}{1-t'}(1+\frac{2}{t-t'}) + \theta(t'-t)\frac{1+t}{1+t'}(1-\frac{2}{t-t'}) \qquad (3.15)$$

and the regularizing part is

$$\gamma_{reg}(t,t') = -\delta(t-t')\int_{-1}^{1}\gamma_0(\tau,t')d\tau, \qquad (3.16)$$

or, explicitly

$$\gamma_{reg}(t,t') = -\delta(t-t')[\int_{t'}^{1}d\tau\frac{1-\tau}{1-t'}(1+\frac{2}{\tau-t}) + \int_{-1}^{t'}d\tau\frac{1+\tau}{1+t'}(1-\frac{2}{\tau-t'})]. \qquad (3.17)$$

However, the formulae (3.14) through (3,17) represent just the Brodsky-Lepage kernel $V_{BL}(t,t')$ /6,8,9/ in the usual notation $\gamma(t,t') = -\frac{g^2}{8\pi^2}V_{BL}(t,t')$. Note that the regularization (3.16) (or (3.17) resp.) coincides with that introduced in ref./8/.

4. CONNECTION WITH THE LOCAL CONFORMAL LCE

In order to establish a connection between the nonlocal expansion introduced in sect.2 (see (2.10), (2.11)) and the standard local conformal LCE /1/ we shall exploit the following particular representation of the δ-function:

$$\delta(t-t') = \sum_{n=0}^{\infty}\frac{1}{\eta_n^\alpha}(1-t^2)^{\alpha-\frac{1}{2}}C_n^\alpha(t)\,C_n^\alpha(t'), \qquad (4.1)$$

where C_n^α are Gegenbauer polynomials and η_n^α are the corresponding normalization factors

$$\int_{-1}^{1}dt\,(1-t^2)^{\alpha-\frac{1}{2}}C_n^\alpha(t)\,C_m^\alpha(t) = \eta_n^\alpha\delta_{nm}. \qquad (4.2)$$

Substitution of (4.1) with $t = \tilde{x}q_-/\tilde{x}q_+$ into (2.10) together with the Taylor expansion of $\exp(i\kappa xq_+)$ yield the result (for brevity we drop again flavour indices)

$$R(j_\mu(x)\,j^\mu(0)S) = \sum_{nm}F^c_{nm}(x^2,\mu^2)\,\bar{R}(O^c_{nm}S) \qquad (4.3)$$

with

$$O^c_{nm} = \int dq_1\, dq_2\, C^\alpha_n\left(\frac{\tilde{x}q_-}{\tilde{x}q_+}\right) (\tilde{x}q_+)^m\, \tilde{\Omega}(q_1, q_2) \tag{4.4}$$

$$F^c_{nm}(x^2, \mu^2) = \int_{-1}^{1} dt\, (1-t^2)^{\alpha-\frac{1}{2}} \int d\kappa \frac{(i\kappa)^m}{m!} \frac{1}{\eta^\alpha_n} C^\alpha_n(t)\, F^c(x^2, \kappa, t).$$

It is possible to see that (4.3) along with (4.4) coincides with the local conformal LCE/1/ for $\alpha = 3/2$.

According to (2.11) and (4.4) the nonlocal conformal operator may be written as

$$\Omega^c(\kappa, t) = \sum_{n,m} \frac{(i\kappa)^m}{m!} C^\alpha_n(t)\, (1-t^2)^{\alpha-\frac{1}{2}}\, O^c_{nm}. \tag{4.5}$$

The inverse relation reads

$$O^c_{nm} = \left(\frac{\partial}{i\,\partial\kappa}\right)^m\Big|_{\kappa=0} \int_{-1}^{1} dt\, C^\alpha_n(t)\, \frac{1}{\eta^\alpha_n}\, \Omega^c(\kappa, t). \tag{4.6}$$

Using the relations (4.5) and (4.6) we are now able to recover the well-known relation between the Brodsky-Lepage kernel and the local conformal anomalous dimensions/6,8/. We shall start from the formal definition/10/ of the anomalous dimensions, for the local and nonlocal operators resp.

$$\bar{\mathfrak{M}}(\bar{R}O_{nm}S) = -\sum_{n'm'}(\gamma_{nmn'm'} + 2\gamma_2\, \delta_{nn'}\, \delta_{mm'})\, O_{n'm'} \tag{4.7a}$$

$$\bar{\mathfrak{M}}(\bar{R}O(\kappa, t)S) = -\int d\kappa'\, dt' (\gamma(\kappa, t, \kappa', t') + 2\gamma_2\, \delta(\kappa-\kappa')\, \delta(t-t'))\, O(\kappa', t') \tag{4.7b}$$

with $\bar{\mathfrak{M}} \equiv \mu\frac{\partial}{\partial\mu}\mathfrak{M}$, where $\frac{\partial}{\partial\mu}$ acts on the μ-dependence introduced by the subtraction operator $\mathfrak{M}$ and γ_2 denotes the anomalous dimension of the corresponding external field operators. Now we insert (4.5) into the l.h.s. of eq. (4.7b), use (4.7a) and express the local operators in terms of nonlocal ones via (4.6). Further, we employ the identity

$$\left(\frac{d}{dx}\right)^m\Big|_{x=0} f(x) = \int dx\, f(x)\, (-1)^m \left(\frac{d}{dx}\right)^m \delta(x) \tag{4.8}$$

and eq. (4.1); the terms involving γ_2 then cancel and we finally obtain the relation

$$\gamma(\kappa, t, \kappa', t') =$$

$$= \sum_{nmn'm'} i^{m-m'} (-1)^{m'} \left(\frac{d}{d\kappa'}\right)^{m'} \delta(\kappa')\, \frac{\kappa}{m!}\, \frac{(1-t^2)^{\alpha-\frac{1}{2}}}{\eta^\alpha_{n'}}\, C^\alpha_n(t)\, C^\alpha_{n'}(t')\, \gamma_{nmn'm'} \quad ; \tag{4.9a}$$

the inverse then reads

$$\gamma_{nmn'm'} = i^{m'-m} \left(\frac{\partial}{\partial\kappa}\right)^m \int_{-1}^{1} dt\, C^\alpha_n(t)\, \eta^\alpha_n \int dt'\, d\kappa' (1-t'^2)^{\alpha-\frac{1}{2}} \frac{\kappa'^{m'}}{m'!} C^\alpha_{n'}(t'). \tag{4.9b}$$

The diagonality of $\gamma(\kappa, t, \kappa', t')$ in κ (see (2.13)) implies the diagonality of $\gamma_{nm\,n'm'}$ in the indices m, m' and vice versa. In such a case we have

$$\gamma_{nm\,n'm'} = \delta_{mm'} \int_{-1}^{1} dt\, C^\alpha_n(t)\, \eta^\alpha_n \int_{-1}^{1} dt'\, (1-t'^2)^{\alpha-\frac{1}{2}} C^\alpha_{n'}(t')\, \gamma(t, t')$$

and thus it is clear that the diagonality in n, n' is an exceptional case, occurring if C^α_n are the eigenfunctions of $(1-t\;)^{\alpha-1/2}\, \gamma(t, t')$. As is well known, the local anomalous dimensions in (4.9) are diagonal on the one-loop level just for $\alpha = 3/2$/1/:

$$\gamma_{nm\,n'm'} = \gamma_n\, \delta_{nn'}\, \delta_{mm'}. \tag{4.10}$$

where $\gamma_n = \frac{g^2}{8\pi^2}\bar{\gamma}_n$ with

$$\bar{\gamma}_n = C_2\left(\frac{2}{(n+1)\,(n+2)} - 1 + 4\sum_{j=2}^{n+1}\frac{1}{j}\right).$$

From (4.9) and (4.10) we then immediately obtain for the one-loop nonlocal anomalous dimensions

$$\gamma(\kappa, t, \kappa', t') = \delta(\kappa-\kappa') \sum_{n=0}^{\infty} (1-t^2)\, \frac{1}{\eta^{3/2}_n}\, C^{3/2}_n(t)\, C^{3/2}_n(t')\, \gamma_n \tag{4.11}$$

or, in view of the results of sect.3 (see (3.13a) through (3.17))

$$V_{BL}(t, t') = \sum_{n=0}^{\infty} (1-t^2)\, \frac{1}{\eta^{3/2}_n}\, C^{3/2}_n(t)\, C^{3/2}_n(t')\, \bar{\gamma}_n. \tag{4.12}$$

Of course, (4.12) expresses the well-known fact that $C^{3/2}_n$ are the eigenfunctions and $\bar{\gamma}_n$ the eigenvalues of the Brodsky-Lepage kernel[6,8,9]:

$$\int_{-1}^{1} V_{BL}(t,t')\, C_n^{3/2}(t)\,dt = \bar{\gamma}_n\, C_n^{3/2}(t').$$

5. AN APPLICATION

The purpose of this section is to illustrate methodical virtues of the formalism developed in sections 2 and 3.

Let us consider a simple light-cone dominated exclusive process, discussed earlier in the literature (see, e.g.,[2]), namely the two-photon production of a spin-zero, flavour nonsinglet meson

$$\gamma^*(q_1) + \gamma^*(q_2) \to M \tag{5.1}$$

in the kinematical region

$$q_1^2 \le 0, \qquad q_2^2 \to -\infty, \tag{5.2}$$

i.e., (denoting $Q = (q_2 - q_1)/2$, $P = q_1 + q_2$)

$$Q^2 \to -\infty, \quad P\cdot Q \to -\infty; \quad w = \frac{Q^2}{P\cdot Q} \ \text{fixed} \tag{5.3}$$

Note that $w \ge 1$ for q_1, q_2 satisfying (5.2). We shall work in the c.m. system and consider transversely polarized photons with equal helicity. The amplitude of the process (5.1) may be written as

$$T(Q^2,w) = \epsilon_1^{\mu}\epsilon_2^{\nu} \int d^4x\, e^{iQx} \langle M|\, R j_{\mu}(\tfrac{x}{2})\, j_{\nu}(-\tfrac{x}{2}) S|0\rangle, \tag{5.4}$$

where ϵ_1, ϵ_2 are the polarization vectors of photons and j_{μ} is the electromagnetic current. For simplicity we shall only deal with the case when the final-state meson is scalar. Such a restriction will allow us to circumvent the (otherwise inessential) complications stemming from the tensor structure of the product of two currents in (5.4) (which we have also suppressed in sect.2).

We wish to recover the standard evolution equation[2,6,8] for the "meson wave function" in the leading order in QCD. To this end, let us calculate first the coefficient functions of the nonlocal conformal LCE in the Born approximation. It is sufficient to calculate the product of two currents in (5.4) with one quark and one antiquark external line, in the free-field theory. One thus easily obtains the coefficient functions of the

expansion corresponding to (2.7), pertaining to the operators not involving γ_5 (only these are relevant to our scalar-meson case):

$$F_i(x^2, xq_{\pm}) = \frac{1}{2\pi^2}\, e_i^2\, \frac{1}{(x^2+i\epsilon)^2}\left(e^{\frac{i}{2}xq_-} - e^{-\frac{i}{2}xq_-}\right), \tag{5.5}$$

with e_i being the charge of the i-th flavour. In arriving at the last expression we have used the Feynman propagator of a massless fermion in the x-representation

$$S_F(x) = -\frac{1}{2\pi^2}\, \frac{\gamma\cdot x}{(x^2+i\epsilon)^2}.$$

Note that (5.5) multiplies the following combination of operators:

$$(g_{\mu\nu}g_{\alpha\beta} - g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha})\,\tilde{\Omega}_i^{\alpha\beta}. \tag{5.6}$$

where $\tilde{\Omega}_i^{\alpha\beta}$ schematically denotes the operator analogous to (2.7), with $\tilde{x}\,\gamma$ replaced by $\tilde{x}_{\alpha}\gamma_{\beta}$ (and corresponding to the i-th flavour). However, the last two terms in (5.6) do not contribute to the amplitude (5.4) (as will become obvious later) and thus we are effectively left only with the standard nonlocal operators discussed in sections 2,3. From (5.5) we may now pass to the coefficient functions of the nonlocal conformal LCE via (2.9); we get

$$\begin{aligned} F^c(x^2, \kappa, t) &= \frac{e_i^2}{2\pi^2}\int \frac{dr}{2\pi}\, e^{-i\kappa r}\, \frac{1}{(x^2+i\epsilon)^2}\left(e^{\frac{i}{2}rt} - e^{-\frac{i}{2}rt}\right) = \\ &= \frac{e_i^2}{2\pi^2}\, \frac{1}{(x^2+i\epsilon)^2}\left(\delta(\kappa - \tfrac{t}{2}) - \delta(\kappa + \tfrac{t}{2})\right). \end{aligned} \tag{5.7}$$

Next we shall consider matrix elements of the nonlocal conformal operator (2.12) between the meson state and vacuum. Using the covariance under translations and the homogeneity properties of $\Omega^c(\kappa,t)$ w.r.t. $\tilde{x}$, we may write

$$\langle M|\, \bar{R}\Omega_i^c(\kappa,t) S|0\rangle = \tilde{x}\cdot P\, \chi_i(t, P^2, \mu^2)\, e^{i\kappa\tilde{x}P}. \tag{5.8}$$

For the operators involving $\tilde{x}_{\nu}\gamma_{\mu}$ or $\tilde{x}_{\mu}\gamma_{\nu}$ resp., instead of $g_{\mu\nu}x\cdot\gamma$ (see (5.6)), the matrix elements corresponding to (5.8) would be proportional to P_{μ} or P_{ν} resp. and this would lead to factors $P\cdot\epsilon_1$ or $P\cdot\epsilon_2$ in the amplitude (5.4). However, $P\cdot\epsilon_1 = P\cdot\epsilon_2 = 0$ for transversely polarized photons in the c.m. system.

The amplitude (5.4) in the kinematical region (5.3) may be then written as (we omit the P^2-dependence of the χ)

$$
\begin{aligned}
T(Q^2, w) &\approx \epsilon_1 \cdot \epsilon_2 \int d^4x e^{iQx} \sum_i \int d\kappa\, dt\, F_i^c(x^2, \kappa, t)\, \tilde{x} P e^{i\kappa \tilde{x} P} \chi_i(t, \mu^2) \\
&\approx \sum_i e_i^2 \int d\kappa\, dt\, Q P \frac{1}{Q^2(1+\frac{2\kappa}{w})} \left(\delta(\kappa - \frac{t}{2}) - \delta(\kappa + \frac{t}{2}) \right) \chi_i(t, \mu^2) \qquad (5.9) \\
&= \sum_i e_i^2 \int_{-1}^{1} dt \left(\frac{1}{w+t} - \frac{1}{w-t} \right) \chi_i(t, \mu^2) .
\end{aligned}
$$

In arriving at the last expression we have used (5.7), performed the Fourier transformation setting $\tilde{x} = x$, and neglected everywhere terms of the order $O(P^2/Q^2)$. Moreover, we have used $\epsilon_1 \cdot \epsilon_2 = 1$. Note also that we take for granted the support properties of χ w.r.t. variable t (cf. the end of sect.2).

In order to obtain the desired evolution equation, we shall employ the renormalization group equation for the renormalized nonlocal conformal operator; owing to the diagonality property (3.13a) of the relevant anomalous dimension this immediately implies the following equation for χ:

$$\mu \frac{d}{d\mu} \chi_i(t, \mu^2) = - \int_{-1}^{1} \gamma(t, t') \, \chi_i(t', \mu^2) \; . \qquad (5.10)$$

Finally, we set $\mu^2 = Q^2$ and use the one-loop approximation for $\gamma(t, t')$ obtained in sect.3. Thus, we may write, within the leading order approximation

$$T(Q^2, w) = \sum e_i^2 \int_{-1}^{1} dt \frac{1}{w+t} \Phi_i(t, Q^2), \qquad (5.11)$$

where

$$\Phi_i(t, Q^2) = \chi_i(t, Q^2) - \chi_i(-t, Q^2)$$

and Φ satisfies the equation

$$Q^2 \frac{\partial}{\partial Q^2} \Phi(t, Q^2) = \frac{\alpha_s(Q^2)}{4\pi} \int_{-1}^{1} V_{BL}(t, t') \, \Phi(t', Q^2) dt' . \qquad (5.12)$$

Note that eq. (5.12) follows from (5.10) and the following symmetry property of $V_{BL}(t, t')$ (which is obvious from (3.13b))

$$V_{BL}(t, t') = V_{BL}(-t, -t') .$$

Eqs. (5.11) and (5.12) represent the desired result, which has

been derived earlier by different methods [2,6,8]. Although this may be the matter of personal taste, we feel that the above derivation is particularly compact and straightforward and illustrates nicely the utility of the formalism developed in this paper.

The authors are indebted to M.Bordag and Th.Braunschweig for useful discussions.

REFERENCES

1. Craigie N.S., Dobrev V.K., Todorov I.T. Trieste preprint IC/82/63 and reference therein.
2. Del Aguila F., Chase M.K. Nucl.Phys., 1981, B193, p.517.
3. Anikin S.A., Zavialov O.I. Ann.Phys., 1978, 116, p.135.
4. Braunschweig Th., Hořejši J., Robaschik D. JINR, E2-82-747, Dubna, 1982.
5. Bordag M., Geyer B., Robaschik D. JINR, E2-83-301, Dubna, 1983.
6. Brodsky S.J., Lepage G.P. Phys.Rev., 1980, D22, p.2157.
7. Bordag M. et al. Yad.Fiz., 1983, 37, p.193.
8. Chase M.K. Nucl.Phys., 1980, B167, p.125.
9. Mueller A.H. Phys.Rep., 1981, 73, p.239; Ohrndorf Th. Nucl.Phys., 1981, B186, p.153.
10. Bordag M., Robaschik D. TMF, 1981, 49, p.330.

Received by Publishing Department
on May 31, 1983.

WILL YOU FILL BLANK SPACES IN YOUR LIBRARY?

You can receive by post the books listed below. Prices - in US $, including the packing and registered postage

D-12965	The Proceedings of the International School on the Problems of Charged Particle Accelerators for Young Scientists. Minsk, 1979.	8.00
D11-80-13	The Proceedings of the International Conference on Systems and Techniques of Analytical Computing and Their Applications in Theoretical Physics. Dubna, 1979.	8.00
D4-80-271	The Proceedings of the International Symposium on Few Particle Problems in Nuclear Physics. Dubna, 1979.	8.50
D4-80-385	The Proceedings of the International School on Nuclear Structure. Alushta, 1980.	10.00
	Proceedings of the VII All-Union Conference on Charged Particle Accelerators. Dubna, 1980. 2 volumes.	25.00
D4-80-572	N.N.Kolesnikov et al. "The Energies and Half-Lives for the α- and β-Decays of Transfermium Elements"	10.00
D2-81-543	Proceedings of the VI International Conference on the Problems of Quantum Field Theory. Alushta, 1981	9.50
D10,11-81-622	Proceedings of the International Meeting on Problems of Mathematical Simulation in Nuclear Physics Researches. Dubna, 1980	9.00
D1,2-81-728	Proceedings of the VI International Seminar on High Energy Physics Problems. Dubna, 1981.	9.50
D17-81-758	Proceedings of the II International Symposium on Selected Problems in Statistical Mechanics. Dubna, 1981.	15.50
D1,2-82-27	Proceedings of the International Symposium on Polarization Phenomena in High Energy Physics. Dubna, 1981.	9.00
D2-82-568	Proceedings of the Meeting on Investigations in the Field of Relativistic Nuclear Physics. Dubna, 1982	7.50
D9-82-664	Proceedings of the Symposium on the Problems of Collective Methods of Acceleration. Dubna, 1982	9.20
D3,4-82-704	Proceedings of the IV International School on Neutron Physics. Dubna, 1982	12.00

Orders for the above-mentioned books can be sent at the address:
Publishing Department, JINR
Head Post Office, P.O.Box 79 101000 Moscow, USSR

Гейер Б., Горжейши И., Робашик Д. E2-83-351
Нелокальное конформное операторное разложение в КХД

Предлагается новый вариант нелокального операторного разложения на световом конусе для КХД. Аномальная размерность соответствующего фермионного оператора, несинглетного по аромату, вычислена в однопетлевом приближении и показано, что она совпадает с ядром Бродского-Лепажа. Обсуждается связь нашего подхода со стандартным локальным конформным операторным разложением. В рамках предложенного формализма рассматривается простой эксклюзивный процесс в ведущем порядке в КХД. Эволюционное уравнение, полученное ранее другими методами, восстановлено особенно простым и прямолинейным путем.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1983

Geyer B., Hořejši J., Robaschik D. E2-83-351
Nonlocal Conformal Light-Cone Expansion in QCD

A new variant of the nonlocal light-cone expansion in QCD is proposed. The anomalous dimension of the corresponding fermion flavour nonsinglet operator is calculated in the one-loop approximation and shown to coincide (up to a trivial diagonal factor) with the Brodsky-Lepage kernel. The connection of our approach with the standard local conformal light-cone expansion is briefly discussed. An exclusive light-cone dominated process is considered within the framework of our formalism. The evolution equation obtained earlier by different methods is recovered in a particularly straightforward and compact manner.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1983