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WHY DIFFERENT FORMS OF THE LIGHT-CONE EXPANSION?

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1. INTRODUCTION

The light-cone expansion is a clear scheme to treat light-cone dominated scattering processes. The most popular form of the light-cone expansion represents the considered operator product as an infinite sum of local operators. This expansion has some unwanted aspects that restrict its applicability.

- After Fourier transform the scattering amplitude is represented as an infinite sum which converges for a restricted class of processes only. For example, for forward scattering only after Mellin transform a useful representation of the scattering amplitude can be given.
- It is not an operator identity, it is defined on a dense subset for the Fock space only.

Moreover by an application to nonforward scattering processes the usual local form of the light-cone expansion leads to complicated sets of renormalization group equations which are not easy to deal with. A diagonalization procedure leads to the so-called conformal light-cone expansions [1].

It is however very remarkable that there exist versions of light-cone expansions which are operator identities (in perturbation theory) and represent the scattering in a closed form by integral relations [2]. As an especially interesting version of this type of expansions we propose a "conformal" nonlocal light-cone expansion which leads directly to diagonal anomalous dimensions and represents the scattering amplitude as an integral relation known from summation techniques.

Starting with an important identity proved by S.A.Anikin and O.I.Zavialov we discuss shortly different forms of the light-cone expansion and its relations and applicability properties.

2. DIFFERENT FORMS OF THE LIGHT-CONE EXPANSION

A serious proof of the light-cone expansion has been given firstly in [2,3]. One of the main results there is the proof of the identity

$$R(j(x)j(0)S) = S_r \frac{1}{1+\mathfrak{M}^a(S_r-1)}\mathfrak{M}^a R(j(x)j(0)S) + Q^a(x) \qquad (2.1)$$

with the following notation: The symbol T for the time ordering is omitted everywhere, j(x) are scalar currents, S_r denotes the renormalized, S the unrenormalized S-matrix. R symbolizes the usual R-operation and $\mathfrak{M}^a$ is a special subtraction operator which acts on functionals containing external operators. Let

$$R(j(x)j(0)S) = \sum_{\ell} \int dq_1 dq_2 \ldots dq_\ell F_\ell (x, q_i) : \phi(q_1) \ldots \phi(q_\ell) : \qquad (2.2)$$

$\phi(q_i)$ - scalar field operator,

then

$$\mathfrak{M}^a R(j(x)j(0)S) = \qquad (2.3)$$

$$= \sum_{\ell=0}^{[a]} M_\sigma^{a-\ell} \int dq_1 \ldots dq_\ell F^{x\text{-prop}} \left(\frac{x_\sigma}{\sigma}, \sigma q_i\right) : \phi(q_1) \ldots \phi(q_\ell) :$$

whereby M_σ denotes the Taylor operator

$$M_\sigma^a f(\sigma) = \sum_{k=0}^{a} \left(\frac{\partial}{\partial\sigma}\right)^k \frac{1}{k!} f(\sigma)\Big|_{\sigma=0}$$

and

$$x_\sigma = \tilde{x} + \eta f_\sigma \qquad f_\sigma = \frac{\tilde{x}\eta}{\eta^2}\left[\sqrt{1 - \frac{\eta^2 x^2 \sigma^2}{(\tilde{x}\eta)^2}} - 1\right] \qquad (2.4)$$

$$\tilde{x} = ((\eta x)^2 - x^2\eta^2)^{1/2} \frac{\eta}{\eta^2} + x - (\eta x)\frac{\eta}{\eta^2}, \quad \eta^2 \neq 0, \quad \tilde{x}^2 = 0.$$

The importance of the relation (2.1) consists in the elimination of the reminder $Q^a(x)$ which behaves for $x^2 \to 0$ as $(x)^{a-2d-1}$, where d is the canonical dimension of the current j(x).

If we restrict ourselves to the minimal light-cone expansion (lowest twist), then we have to choose $a = 2$ and $\mathfrak{M}Rj(x)j(0)S$ takes the form

$$\mathfrak{M}R(j(x)j(0)S) = \int dq_1 dq_2 \frac{1}{2} F^{x\text{-prop}}(x^2, xq_i, q_i q_j = \mu_{ij}) : \phi(q_1)\phi(q_2) :$$

μ_{ij}- subtraction points of $\mathfrak{M}$.

Inserting this expression into eq. (2.1) we obtain as basic relation

$$R(j(x)j(0)S) = S_r \frac{1}{1 + \mathfrak{M}(S_r - 1)} \int dq_1 dq_2 \frac{1}{2} F^{x\text{-prop}}(x^2, xq_i, \mu_{ij}) : \phi(q_1)\phi(q_2) :$$

$$+ Q(x), \qquad (2.5)$$

which we will exploit later on. Of course this relation is in this form nonapplicable because the operator written in brackets is too complicated. Different forms of the light-cone expansion may be found by applying different representations of the coefficient function $F^{x\text{-prop}}(x^2, \tilde{x}q_i, \mu_{ij})$. Hereby it is essential that this function is an entire analytic function of the variables $\tilde{x}q_i$ as it follows from its α-representation.

a) Local Light-Cone Expansion

The standard local light-cone expansion appears if the coefficient function is simply expanded in a Taylor series:

$$\frac{1}{2}F_2(x^2, \tilde{x}q_i, \mu_{ij}) = \sum_{n_1, n_2} F_{n_1 n_2}(x^2, \mu_{ij}) \frac{(\tilde{x}q_1)^{n_1}}{n_1!} \frac{(\tilde{x}q_2)^{n_2}}{n_2!}. \qquad (2.6)$$

After insertion of this series into the identity we get

$$R(j(x)j(0)S) = \sum_{n_1 n_2} F_{n_1 n_2}(x^2, \mu_{ij}) \bar{R}(O_{n_1 n_2} S) + Q \qquad (2.7)$$

$$O_{n_1 n_2} = \int dq_1 dq_2 (\tilde{x}q_1)^{n_1} (\tilde{x}q_2)^{n_2} (n_1!)^{-1} (n_2!)^{-1} : \phi(q_1)\phi(q_2) : . \qquad (2.8)$$

Thereby we have used /2/

$$S_r \frac{1}{1 + \mathfrak{M}(S_r - 1)} O(x) = \bar{R}(O(x), S). \qquad (2.9)$$

Eq. (2.7) represents the standard light-cone expansion. During its derivation we have to interchange the summations and integrations - to get nice local operators, but this is just the point that produces difficulties and restricts the physical applicability. The anomalous dimensions of the local operators can be formally defined by /4/

$$\bar{\mathfrak{M}}(\bar{R}O_{n_1 n_2}S) = -\sum_{n_1', n_2'} (\gamma_{n_1 n_2 n_1' n_2'} + \delta_{n_1 n_1'} \delta_{n_2 n_2'} \cdot 2\gamma_2) O_{n_1' n_2'}, \qquad (2.10)$$

with $\bar{\mathfrak{M}} = (\mu \frac{\partial}{\partial\mu} \mathfrak{M})$ where $\frac{\partial}{\partial\mu}$ acts on the μ-dependence introduced by $\mathfrak{M}$ itself.

The properties of the anomalous dimensions $\gamma_{(n)(n')}$ are well known.

b) Nonlocal Light-Cone Expansion

This form of the light-cone expansion was introduced in ref.[2]. It exploits the analyticity properties of the function $F_2(x^2, \tilde{x}q_i, \mu_{ij})$ through Fourier transforms

$$\frac{1}{2} F_2(x^2, \tilde{x}q_i, \mu_{ij}) = \int_0^1 d\kappa_1 d\kappa_2 F(x^2, \kappa_1, \kappa_2) e^{i\kappa_1 \tilde{x}q_1 + i\kappa_2 \tilde{x}q_2} . \qquad (2.11)$$

This leads to the following light-cone expansion

$$R(j(x)j(0)S) = \int_0^1 d\kappa_1 d\kappa_2 F(x^2, \kappa_1, \kappa_2) \bar{R}(O(\kappa_1, \kappa_2)S) + Q , \qquad (2.12)$$

$$O(\kappa_1, \kappa_2) = \int dq_1 dq_2 e^{i\kappa_1 \tilde{x}q_1 + i\kappa_2 \tilde{x}q_2} : \phi(q_1)\phi(q_2) : , \qquad (2.13)$$

which represents a true operator identity. The anomalous dimensions of the operators are defined by

$$\bar{\mathfrak{M}} R(O(\kappa_1, \kappa_2)S) = -\int d\kappa_1' d\kappa_2' (\gamma(\kappa_1, \kappa_2, \kappa_1', \kappa_2') + 2\gamma_2 \delta(\kappa_1 - \kappa_1')\delta(\kappa_2 - \kappa_2'))O(\kappa_1', \kappa_2') . \qquad (2.14)$$

This light-cone expansion is in some sense an integral or summed up representation of the standard light-cone expansion. Both expansions, as well as their anomalous dimensions are essentially connected by Mellin transforms. A common feature of both expansions is the nondiagonality of their anomalous dimensions, which makes these expansions not very handable for nonforward processes. For forward processes all results obtained with the standard light-cone expansion or ladder summations can be immediately obtained also here. It is sufficient to remark that the anomalous dimensions for forward scattering are directly connected with the Altarelli-Parisi kernel[5]. The general anomalous dimensions $\gamma(\kappa_i, \kappa_i')$ satisfy two invariance relations

$$\gamma(\kappa_1, \kappa_2, \kappa_1', \kappa_2') = \gamma(\kappa_1 - \lambda, \kappa_2 - \lambda, \kappa_1' - \lambda, \kappa_2' - \lambda) , \qquad (2.15a)$$

$$\gamma(\kappa_1, \kappa_2, \kappa_1', \kappa_2') = \frac{1}{\lambda^2} \gamma(\lambda\kappa_1, \lambda\kappa_2, \lambda\kappa_1', \lambda\kappa_2') , \qquad (2.15b)$$

which will be proved in the Appendix.

c) Local Conformal Light-Cone Expansion

To apply light-cone expansions to nonforward scattering processes it is important to have diagonal anomalous dimensions.

Exploiting the ideas of conformal invariance of asymptotic physical processes one is led to the local conformal light-cone expansion[1]. For this purpose we expand $F_2(x^2, \tilde{x}q_i, \mu_{ij})$ in terms of Gegenbauer polynomials: For this reason we introduce as new variable $t = \frac{\tilde{x}q_-}{\tilde{x}q_+}$, $q_\pm = q_2 \pm q_1$. If the momenta q_i are time-like or light-like, then it holds $-1 \le t \le +1$. But this subspace is sufficient for an expansion

$$\frac{1}{2} F_2(x^2, \tilde{x}q_+, t\tilde{x}q_+, \mu_{ij}) = \sum_{n,m} F_{nm}(x^2, \mu^2)(\tilde{x}q_+)^{n+m} C_n^\alpha\left(\frac{\tilde{x}q_-}{\tilde{x}q_+}\right) , \qquad (2.16)$$

whereby

$$F_{nm} = \frac{1}{(\tilde{x}q_+)^n} \frac{1}{m!}\left(\frac{\partial}{\partial r}\right)^m \int_{-1}^{+1} dt (1-t^2)^{\alpha - \frac{1}{2}} \frac{C_n^\alpha(t)}{\eta_n^\alpha} \frac{1}{2} F_2(x^2, r, t\tilde{x}q_+)\Big|_{r=0} , \qquad (2.17)$$

note

$$\int_{-1}^{+1} dt (1-t^2)^{\alpha - \frac{1}{2}} C_n^\alpha(t) C_{n'}^\alpha(t) = \eta_n^\alpha \delta_{nn'} . \qquad (2.18)$$

It is important to note: whereas the expansion has been derived for a subspace only, however due to the holomorphy properties of $F_2(x^2, \tilde{x}q_+, \tilde{x}q_-)$ this expansion converges in the full space also. The intrinsic problem of all local light-cone expansions appears also here; we have to interchange an infinite sum with integrals. Doing this we get

$$R(j(x)j(0)S) = \sum_{n,m} F_{nm}(x^2, \mu^2) \bar{R}(O_{nm}^c S) , \qquad (2.19)$$

$$O_{nm}^c = \int d^4q_1 d^4q_2 (\tilde{x}q_+)^{n+m} C_n^\alpha\left(\frac{\tilde{x}q_-}{\tilde{x}q_+}\right) : \phi(q_1)\phi(q_2) : . \qquad (2.20)$$

The anomalous dimensions are formaly defined by eq. (2.10). It has been shown that they for scalar fields with $\alpha = \frac{3}{2}$ are diagonal, at least at the one loop level.

d) Nonlocal Conformal Light-Cone Expansion

Let us now come to the nonlocal version of the conformal light-cone expansion. In our opinion the most important step for the derivation of the conformal light-cone expansion is the introduction of the variable $t = \frac{\tilde{x}q_-}{\tilde{x}q_+}$. Choosing the most trivial representation of $\frac{1}{2}F_2(x^2, xq_+, xq_-)$ namely

$$\frac{1}{2}F_2(x^2, \tilde{x}q_+, \tilde{x}q_-) = \int_{-\infty}^{+\infty} dt \int_{-\infty}^{+\infty} d\kappa\, F^c(x^2, t, \kappa)\,\delta\left(t - \frac{\tilde{x}q_-}{\tilde{x}q_+}\right) e^{i\kappa \tilde{x} q_+}, \qquad (2.21)$$

we get

$$R(j(x)\, j(0) S) = \int_{-\infty}^{+\infty} dt \int_{-\infty}^{+\infty} d\kappa\, F^c(x^2, t, \kappa)\, R(O(t, \kappa) S) \qquad (2.22)$$

with

$$O(t, \kappa) = \int dq_1\, dq_2\, \delta\left(t - \frac{\tilde{x}q_-}{\tilde{x}q_+}\right) e^{i\kappa \tilde{x} q_+} : \phi(q_1)\phi(q_2) : \qquad (2.23)$$

as nonlocal conformal operator. In the following we will discuss this expansion.

First some remarks concerning the integration region. The α-representation for the coefficient functions $F_2(x^2, \tilde{x}q_i, \mu_{ij})$ can be written as[2]

$$F_2(x^2, \tilde{x}q_i, \mu^2) = \sum_{\text{graphs}} \int d\alpha_1 \ldots d\alpha_L\, G(\alpha_k, x^2)\, e^{-i(B_1(\alpha)\tilde{x}q_1 + B_2(\alpha)\tilde{x}q_2)} \qquad (2.24)$$

Here we have used a short notation which exhibits in an explicit form the important dependence on the variables $\tilde{x}q_i$ only. The coefficients satisfy the following conditions (see Appendix)

$$0 \le B_i(\alpha) \le 1, \qquad B_1 + B_2 = 1, \qquad (2.25)$$

so that

$$F_2(x^2, xq_i, \mu^2) = \int d\alpha_1 \ldots d\alpha_L\, G(\alpha, x^2)\, e^{-\frac{i}{2}\tilde{x}q_+ - \frac{i}{2}\tilde{x}q_+ t(B_2 - B_1)} .$$

A Fourier transform with respect to the variables $\tilde{x}q_+$, $\int \frac{d\tilde{x}q_+}{2\pi} F_2(x^2, \tilde{x}q_i, \mu^2)\, e^{-i\kappa\tilde{x}q_+}$ produces the δ-function $\delta(\kappa - \frac{1}{2}(1+(B_2-B_1)t)$ that leads to the restriction $-1 \le \frac{2\kappa - 1}{t} \le +1$ in contrast to the restriction $0 \le \kappa_i \le 1$ for the κ-variables of the usual nonlocal expansion (2.12).

Let us now study the operators (2.23). Representing the δ-function by the Fourier integral and introducing the Fourier transform of the field operator $\tilde{\phi}(x) = \int dq\, \phi(q) \exp iqx$ we obtain

$$O(t, \kappa) = \frac{1}{2\pi} \int d\lambda\, dq_1 dq_2 \exp i(\kappa \tilde{x}q_+ - \lambda(\tilde{x}q_- - \tilde{x}q_+ t)) \cdot \tilde{x}q \; : \phi(q_1)\phi(q_2) : \qquad (2.26)$$

$$= \frac{1}{2\pi} \frac{\partial}{\partial i\kappa} \int d\lambda : \tilde{\phi}(\tilde{x}(\kappa + \lambda(t+1))\tilde{\phi}(\tilde{x}(\kappa + \lambda(t-1)) : \; .$$

These operators have a structure similar to the usual nonlocal operators (22.13): $\tilde{\phi}(\kappa_1\tilde{x})\tilde{\phi}(\kappa_2\tilde{x})$: and are simply related to them by the just derived equation.

Consider now the anomalous dimensions of these operators. We will show here that these anomalous dimensions are diagonal in the variables κ, i.e., $\gamma(\kappa, t, \kappa', t') = \delta(\kappa - \kappa')\gamma(t, t')$. To show this, we need the α-representation for the coefficient functions H of the renormalized operators

$$R(O(\kappa, t)S) = \sum_{s=0}^{\infty} \frac{1}{s!} \int dp_1 \ldots dp_s\, H(\kappa, t; \tilde{x}, p_1, \ldots, p_s) : \phi(p_1) \ldots \phi(p_s) :$$

We use the α-representation of H again in a reduced form that exhibits the essential dependence on the variable $\tilde{x}q_i$ explicitly. After the application of the modified subtraction operator $\bar{\mathfrak{R}}$ we get

$$\bar{\mathfrak{R}} R(O(\kappa, t)S) = \sum_{\text{graphs}} \int d\alpha_1 \ldots d\alpha_L \int \frac{d\lambda}{2\pi} D(\alpha, \mu^2) \frac{\partial}{\partial i\kappa} \int dq_1 dq_2 : \phi(q_1)\phi(q_2) : \qquad (2.27)$$

$$\cdot \exp\{ i\tilde{x}(\lambda(t+1) + \kappa) B_{1\alpha} q_\alpha + i\tilde{x}(\lambda(t-1) + \kappa) B_{2\alpha} q_\alpha \} .$$

Reordering the exponential

$$\{\ \} = i\tilde{x}q_+ \cdot \frac{1}{2}((\kappa + (t+1)(B_{11} + B_{12}) + (\kappa + (t-1)\lambda)(B_{21} + B_{22}))$$
$$+ i\tilde{x}q_- \frac{1}{2}((\kappa + (t+1)\lambda)(B_{12} - B_{11}) + (\kappa + (t-1)\lambda)(B_{22} - B_{21}))$$

and taking into account (Appendix)

$$B_{12} + B_{22} = 1, \qquad B_{11} + B_{21} = 1, \qquad 0 \le B_{ij} \le 1 \qquad (2.28)$$

we obtain

$$\bar{\mathfrak{R}} R(O(\kappa, t)S) = \sum_{\text{graphs}} \int \frac{d\alpha}{2\pi} \int d\alpha_1 \ldots d\alpha_L D(\alpha, \mu^2) \frac{\partial}{\partial i\kappa} \int dq_1 dq_2 : \phi(q_1)\phi(q_2) : \cdot$$

$$\cdot \exp i\tilde{x}q_+ \{(\kappa + \lambda t) + \frac{1}{2}\lambda(B_{11} + B_{12} - B_{21} - B_{22}) +$$

$$+ \frac{1}{2} t\lambda (B_{12} - B_{11} - B_{22} + B_{21})\} : \; .$$

After the substitution $\lambda' = \lambda\tilde{x}q_+$ and an explicit integration this expression takes the form

$$\bar{\mathfrak{M}}\, R(O(\kappa,t)S) = \int dt'd\kappa' \sum_{graphs} \int d\alpha_1 \ldots d\alpha_L D(\alpha,\mu^2)\ \cdot$$

$$\cdot\ \delta(t+\frac{1}{2}(B_{11}+B_{12}-B_{21}-B_{22})+\frac{1}{2}t'(B_{12}-B_{11}-B_{22}-B_{21})\cdot\delta(\kappa-\kappa')\ \cdot$$

$$\cdot \int dq_1 dq_2 \exp i\tilde{x}q_+\kappa' \cdot \delta(t' - \frac{\tilde{x}q_-}{\tilde{x}q_+}) : \phi(q_1)\phi(q_2) : \quad .$$

By comparison with the definition of the anomalous dimension we receive the result

$$\gamma(\kappa,t,\kappa',t') = -\sum_{graphs} \int d\alpha_1 \ldots d\alpha_L D(\alpha,\mu^2)\,\delta(t+(B_{11}+B_{12}-B_{21}-B_{22})\frac{1}{2} +$$

$$+\frac{1}{2}t'(B_{12}+B_{21}-B_{11}-B_{22}))\cdot\delta(\kappa-\kappa') .$$

So we have shown that these operators have diagonal anomalous dimensions in all orders of perturbation theory. Because of the direct connection of anomalous dimensions with Z-factors or the subtraction mechanism we have shown that the set of nonlocal conformal operators is closed by renormalization (only operators of this type mix with themselves during the renormalization procedure).

Let us add here some remarks concerning support restrictions in the t-variable. For this reason we eliminate not necessary quantities as

$$B_{21} = 1 - B_{11}, \quad B_{22} = 1 - B_{12} ,$$

so that the argument of the δ function in eq. (2.30) gives the restriction $t = 1 - B_{11}(1-t') - B_{12}(1+t')$. If we assume $|t'| \le 1$ then because of $0 \le B_{11}(1-t') + B_{12}(1+t') \le 1 - t' + 1 + t' \le 2$ it follows $|t| \le 1$. This is an important restriction for later applications.

Of course, the local conformal light-cone expansion and the corresponding nonlocal light-cone expansion are directly connected. For example a standard representation of the δ-function

$$\delta(t - \frac{\tilde{x}q_-}{\tilde{x}q_+}) = \sum_n (\eta_n^\alpha)^{-1} (1-t^2)^{\alpha-\frac{1}{2}} C_n^\alpha(t)\, C_n^\alpha(\frac{\tilde{x}q_-}{\tilde{x}q_+})$$

in the interval $(-1,+1)$ (which is however valid for entire analytic function on the complete t-axis) and a power series expansion of the exponential $\exp i\kappa\tilde{x}q_+ = \sum (i\kappa\tilde{x}q_+)^n (n!)^{-1}$ substituted into the expression for the nonlocal conformal expansion

$$R(j(x)j(0)S) = \int dt\,d\kappa\, F^c(x^2,t,\kappa)\, \bar{R}\int dq_1 dq_2\, \delta(t - \frac{\tilde{x}q_-}{\tilde{x}q_+})\ .$$

$$\cdot\ e^{i\kappa\tilde{x}q_+} : \phi(q_1)\phi(q_2) : S + Q$$

give

$$R(j(x)j(0)S) = \sum_{n,m} \int dt(1-t^2)^{\alpha-\frac{1}{2}} \int d\kappa \frac{(i\kappa)^m}{m!} \frac{C_n^\alpha(t)}{\eta_n^\alpha} F^c(x^2,t,\kappa)\ \cdot$$

$$\cdot\ \bar{R}\int dq_1 dq_2\, C_n^\alpha(\frac{\tilde{x}q_-}{\tilde{x}q_+})(\tilde{x}q_+)^m : \phi(q_1)\phi(q_2) : S,$$

that is a version of the local conformal light-cone expansion.

3. APPLICATION OF THE NONLOCAL CONFORMAL LIGHT-CONE EXPANSION

We claim that the nonlocal conformal light-cone expansion is the clearest scheme to handle nonforward scattering processes. We will illustrate this with the simplest nonforward scattering process: the meson production by two virtual photons in a certain kinematical region. This process was already treated by different techniques as diagram summation or local conformal light-cone expansions /6/. We denote the momenta of the two incoming photons by q_1 and q_2, the outgoint meson momentum by k. It lies in the spirit of the foregoing considerations that we discuss here a scalar model of this process. Into the expression for the scattering amplitude

$$T(Q,k) \sim \int d^4x\, e^{iQx} \langle 0|\, Rj(\frac{x}{2})j(\frac{-x}{2})S|k\rangle, \quad Q = \frac{1}{2}(q_2 - q_1) = 2q_-$$

we insert the nonlocal light-cone expansion (2.22) with the result

$$T \sim \int d^4x\, e^{iQx} \int dt\,d\kappa\, F(x^2,t,\kappa)\ \cdot \tag{3.1}$$

$$\cdot\ \langle 0|\, \bar{R}\int dp_1 dp_2\, \delta(t - \frac{\tilde{x}p_-}{\tilde{x}p_+})\, e^{i\kappa\tilde{x}p_+} : \phi(p_1)\phi(p_2) : S|k\rangle .$$

As first step of the investigation of this expression we have to study the matrix elements of the nonlocal operator

$$<0|\bar{R}\int dp_1 dp_2 \delta(t-\frac{\tilde{x}p_-}{\tilde{x}p_+})e^{i\kappa\tilde{x}p_+}:\phi(p_1)\phi(p_2):|k> = \chi(t,k^2,\mu^2)e^{i\kappa\tilde{x}k}\ . \qquad (3.2)$$

Here we have exploited: translation invariance and homogeneity properties with respect to $\tilde{x}\to\lambda\tilde{x}$. An essential question is support restriction of χ with respect to the variable t. So we have to study the coefficient functions of the renormalized operator $Q(\kappa,t)$ again. The short version of their α-representation we write similar to eq. (2.27)

$$\bar{R}(O(\kappa,t)S) = \sum_{\ell}\sum_{graphs}\int\frac{d\lambda}{2\pi}\int d\alpha_1\dots d\alpha_L\tilde{D}(\alpha,q_i q_j)\frac{\partial}{\partial i\kappa}\int dq_1\dots dq_\ell\ \cdot$$

$$\exp i(\tilde{x}(\lambda(t+1)+\kappa)B_{1\alpha}q_\alpha+\tilde{x}(\lambda(t-1)+\kappa)B_{2\alpha}q_\alpha):\phi(q_1)\dots\phi(q_\ell):.$$

To perform the λ-integration we collect all λ dependent terms

$$\int\frac{d\lambda}{2\pi}\exp i\lambda(t\cdot\sum_\alpha q_\alpha\tilde{x}+(B_{1\alpha}-B_{2\alpha})\tilde{x}q_\alpha).$$

Eliminating the variables $B_{2\alpha}$ by $B_{2\alpha}=1-B_{1\alpha}$ and introducing $\lambda'=\lambda\sum q_\alpha\tilde{x}$ as new integration variable we have

$$\int\frac{d\lambda'}{2\pi}\exp i\lambda'(t-1+2B_{2\alpha}\frac{\tilde{x}q_\alpha}{\sum q_\beta\tilde{x}}).$$

Let us assume that the outgoing meson is represented by a state of the Fock space (sum of products of creation operators integrated with suitable weight functions). In this case all momenta q_α are time-like or light-like $q_\alpha\in\bar{V}_+$ with $\sum q_\alpha=k$. The argument of the appearing δ-function leads to

$$t=1-2B_{2\alpha}xq_\alpha\frac{1}{\sum_\beta\tilde{x}q_\beta}\le 1$$

because of

$$0\le B_{2\alpha}\frac{\tilde{x}q_\alpha}{\sum\tilde{x}q_\beta}\le\frac{\sum_\alpha\tilde{x}q_\alpha}{\sum_\beta\tilde{x}q_\beta}=1\ .$$

So we have proved the important result $|t|\le 1$.

Returning to the T-amplitude we insert eq. (3.2) for the matrix element into eq. (3.1), so that

$$T\sim\iint d^4x\,e^{iQx+i\kappa\tilde{x}k}F^c(x^2,t,\kappa,\mu^2)\chi(t,k^2,\mu^2)dt\,d\kappa$$

$$\sim\int dt d\kappa\,\tilde{F}^c((Q+k)^2,t,\kappa,\mu^2)\chi(t,k^2,\mu^2) \qquad (3.3)$$

$$\sim\int dt\,G(Q^2,\zeta,t,\mu^2)\chi(t,k^2,\mu^2),\qquad \zeta=-\frac{Q^2}{2Qk}\ .$$

χ represents the matrix element of the nonlocal operator and satisfies a renormalization group equation with the anomalous dimension $\gamma(t,t')$

$$\mu\frac{d}{d\mu}\chi(t,k^2,\mu^2)=\int_{-1}^{+1}dt'\gamma(t,t')\chi(t',k^2,\mu^2). \qquad (3.4)$$

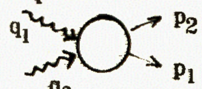
If we restrict ourselves to the light-like approximation, then it is possible to choose $\mu^2=Q^2$ and $\chi(t,k^2,\mu^2)\to\chi(t,k^2,Q^2)$ which satisfies now the evolution equation

$$Q^2\frac{d}{dQ^2}\chi(t,k^2,Q^2)=2^{-1}\int_{-1}^{+1}dt'\gamma(t,t')\chi(t',k^2,Q^2), \qquad (3.5)$$

whereas the T-amplitude itself reads

$$T\sim\int_{-1}^{+1}dt\,G(Q^2,\zeta,t,\mu^2=Q^2)\chi(t,k^2,Q^2). \qquad (3.6)$$

With our choice of the subtraction point it is possible to choose for the function G the Born approximation



$$e^{iQx}F(x^2,\tilde{x}q_i)\sim\frac{1}{x^2}e^{\frac{i}{2}(q_2-q_1)\tilde{x}}(e^{i(p_2-p_1)\frac{1}{2}\tilde{x}}+e^{-i(p_2-p_1)\tilde{x}\frac{1}{2}})$$

or

$$e^{iQx}F(x^2,t,\kappa)\sim\frac{1}{x^2}e^{iQx}(\delta(\frac{t}{2}-\kappa)+\delta(\frac{t}{2}+\kappa))$$

so that

$$T\sim\int dt d\kappa\frac{1}{(Q+k\kappa)^2}(\delta(\frac{t}{2}-\kappa)+\delta(\frac{t}{2}+\kappa))\chi(t,k^2,Q^2)$$

$$\sim\int dt\frac{\xi}{Q^2}(\frac{1}{\xi+t}+\frac{1}{(\xi-t)})\chi(t,k^2,Q^2).$$

But this is the standard expression known from literature /6/.

We underline however that our treatment by the nonlocal light-cone expansion can be easily generalized to more complicated processes.

The authors are indebted to J.Hořejši for useful discussions.

APPENDIX

Invariance Properties of the Nonlocal Anomalous Dimensions (Relations between the Coefficients of the α-Representation)

Here we will prove the invariance properties (2.15) of the nonlocal anomalous dimensions $\gamma(\kappa, \kappa')$. In their definition (2.14) we introduce the coefficient functions H of the renormalized operator $O(\kappa_1, \kappa_2)$. As next we use their α-representation and apply the subtraction operator $\mathfrak{M}$. In this way we get

$$\mathfrak{M} H(x, p_1 \ldots p_s) = \mathrm{const}\, \delta_{s2} \sum_{\text{graphs}} \int d\alpha_1 \ldots d\alpha_L D(\alpha, \mu^2) e^{i\kappa_i B_{i\alpha} \tilde{x} p_\alpha} .$$

After Fourier transform with respect to $\tilde{x}p_i$ we can read off the explicit expression for the anomalous dimensions

$$\gamma(\kappa, \kappa') + 2\gamma_2 \delta(\kappa - \kappa') = c\,(-1) \sum_{\text{graphs}} \int d\alpha_1 \ldots d\alpha_L D(\alpha, \mu^2) \prod_{\rho=1}^{2} \delta(\kappa'_\rho - \sum_1^2 \kappa_i B_{i\rho}(\alpha))$$

Also here it is important that $\sum_i B_{i\rho}(\alpha) = 1$. This follows from the explicit representation of these coefficients /7;9-25,9-26/

$$B_{i\rho}(\alpha) = D^{-1}(\alpha) \sum_{P_\rho} [P_\rho : i] D_P .$$

Hereby P_ρ denotes all paths within the considered graph going from the vertex ρ to the vertex O corresponding to the operator $O(\kappa)$. The symbol $[P_\rho : i]$ has the following values

$$[P_\rho : i] = \begin{cases} 1 \text{ if the path } P_\rho \text{ contains the line } i \\ 0 \text{ if the path } P_\rho \text{ does not contain the line } i . \end{cases}$$

The interesting quantity reads now

$$\sum_i B_{i\rho}(\alpha) = D^{-1}(\alpha) \sum_{P_\rho} \sum_i [P_\rho : i] D_P .$$

Because of $\sum_i [P_\rho : i] = 1$ /7;3.20/ and $\sum_{P_\rho} D_P = D$ we arrive at $\sum_i B_{i\rho} = 1$. From this relation it follows

$$\gamma(\kappa_i + a, \kappa'_i + a) = \gamma(\kappa_i, \kappa'_i)$$

because of

$$\delta(\kappa'_\rho + a - \sum_i (\kappa_i + a) B_{i\rho}) = \delta(\kappa'_\rho - \sum_i \kappa_i B_{i\rho}) .$$

The relation $\gamma(\lambda\kappa_i, \lambda\kappa'_i) = \lambda^{-2}\gamma(\kappa_i, \kappa'_i)$ is an immediate consequence of the structure of the α-representation that contains κ-depending terms only in the form of δ-functions.

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Бордаг М., Гейер Б., Робашик Д. E2-83-301
Почему применяются различные виды операторного разложения?

Показано, что различные виды операторного разложения на световом конусе получаются из операторного тождества, доказанного С.А.Аникиным и О.И.Завьяловым. Предлагается нелокальное конформное операторное разложение, которое применяется к случаю рассеяния не вперед. Показано, что аномальные размерности соответствующих операторов диагональны для всех порядков теории возмущения. Из этого следует существование простых уравнений эволюции. Для простоты все рассмотрения проведены на основе скалярной теории поля.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1983

Bordag M., Geyer B., Robaschik D. E2-83-301
Why Different Forms of the Light-Cone Expansion?

Different types of light-cone expansions are traced back to an operator identity proved by S.A.Anikin and O.I.Zavialov. As a new type of light-cone expansion, a nonlocal conformal one, is proposed and applied to a non-forward scattering process. It is shown that the anomalous dimensions of the corresponding light-cone operators are diagonal in one of the two parameters to all orders of perturbation theory. This leads to one-parameter evolution equations. For simplicity all considerations here are based on scalar field theory.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1983