сообщения
объединенного
института
ядерных
исследований
дубна

E2-83-299

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THE PHASE-SPACE OF TACHYONS

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1983

1. INTRODUCTION

The aim of this paper is to discuss the phase-space (PS) of quantized systems that contain bradyons and tachyons, slower and faster than light particles*. At the beginning we present essential features of the concept of tachyons that we assume in present paper [1]. We would like to introduce the nonpractitioners and to avoid possible misunderstandings that could arise since several methods of solving the well-known causal paradoxes were found and several concepts of tachyons were consequently proposed [2-4].

The tachyon is described by space-like four momentum lying on the single-sheeted hyperboloid

$$p^{\mu}p_{\mu} = -m^2,$$

where m is real tachyon mass. From the above statement it follows that the energy of tachyon

$$E = \sqrt{\bar{p}^2 - m^2}$$

is real because $|\bar{p}| \geq m$. The reinterpretation principle (RP) [2] is assumed according to which the negative energy tachyon traveling backward in time has to be regarded as a positive energy antitachyon travelling forward in time. In the version more suitable for this paper RP can be formulated as follows: the negative energy tachyon in the final (initial) state of reaction has to be treated as a positive energy antitachyon in the initial (final) state of reaction.

Although progress towards understanding the tachyon dynamics has been made [5], no satisfactory field theoretical model has been found. For review of this interesting problem see refs. [6,7]. In such a situation it could be valuable to study the PS properties of tachyons since some problems of field models arise in these considerations but in a strongly simplified version. On the other hand, in the absence of dynamical theory the knowledge of PS could be intersting because in the threshold regions PS effects are expected to be dominant.

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*Some results of the following paper were presented in our previous article: St.Mrøwczyński, Lett.Nuovo Cim., 1983, 36, p. 340.

II. THE LORENTZ INVARIANT PHASE-SPACE

We define the volume of Lorentz invariant PS in the following explicitly invariant form

$$L^N(S,\pm m_1^2,\dots\dots\pm m_N^2) = \int \prod_{i=1}^{N} d^4p_i\,\delta(p_i^2 \mp m^2)\,\delta^{(4)}(P^\mu - \sum_{i=1}^{N} p_i^\mu) \qquad (1)$$

with $P^\mu P_\mu = S$,

$p_i^2 \equiv p_i^\mu p_{i_\mu}$.

Upper signs are for bradyons; and lower, for tachyons. The deltas under multiplication mark "keep" particles on the mass shell while the four-dimensional delta secures the conservation of four-momentum P^μ. In (1) we have to integrate over positive and negative energies of particles. In the case of bradyons the distinction of particles with negative and positive energies is Lorentz invariant since bradyons and antibradyons lie on two separated sheets of hyperboloid. So, we can, without destroying invariancy, neglect the negative energy part. Using the equality

$$\delta(p^2 \pm m^2) = \frac{\delta(p_0 - \sqrt{\vec{p}^2 \pm m^2}) + \delta(p_0 + \sqrt{\vec{p}^2 \pm m^2})}{2\sqrt{\vec{p}^2 \pm m^2}}$$

after integration over p_0, we get the well-known definition of invariant PS

$$L^N(S, m_1^2,\dots\dots, m_N^2) = \int \prod_{i=1}^{N} \frac{d^3p_i}{2E_i}\,\delta^{(4)}(P^\mu - \sum_{i=1}^{N} P_i^\mu),$$

where $E_i \equiv p_{0_i} = \sqrt{\vec{p}_i^2 + m_i^2}$.

In the tachyon case it is not possible to remove the negative energies without violation of Lorentz invariancy since the hyperboloid of tachyon is single-sheeted. By invoking RP, we transfer the negative energies from the final to the initial state. However, PS defined in (1) by such a procedure loses its ordinary interpretation as a measure of a number of possible final states of the reaction where four-momentum P^μ is conserved. Let us consider the reaction which for different observers could be presented by graphs shown in Fig.1. The two-particle PS defined in (1) suitable for this situation

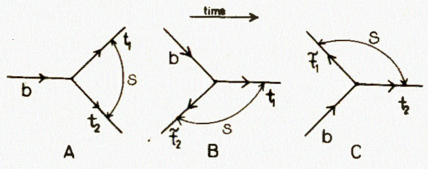

Fig.1. The decay of bradyon b into tachyons t_1 and t_2 seen by three different observers A, B and C. Due to RP, tachyon can be transformed from the final to the initial state by Lorentz transformations.

is a measure of the states of a system of two tachyons which can occur in the final and the initial states as well. The four-momenta of tachyons fulfill the condition

$$(\pm p_1^\mu \pm p_2^\mu)(\pm p_{1_\mu} \pm p_{2_\mu}) = S.$$

We see that Mandelstam variable S has different meanings for different observers; for A it is the square of energy of tachyons in their center of mass frame (CM), while for B and C it is four-momentum transfer between tachyons.

For symmetrical treating tachyons and bradyons we do not exclude the negative energies of bradyons from (1). Such an exclusion does not violate the Lorentz invariancy of (1). However, covariancy under superluminal transformations (boosts over the light velocity barrier that change space-like to time-like vectors and vice versa, see, e.g., ref. /1/) would be lost since such transformations do not conserve the sign of zero component of four-vector. In the next chapter we consider PS that has an ordinary meaning; however, it is neither Lorentz invariant nor covariant under superluminal transformations.

In the case of two particles L^2 is decomposed into four parts:

$$L^2 = L^2_{++} + L^2_{+-} + L^2_{-+} + L^2_{--},$$

where marks "+", "-" indicate the sign of energy of each particle. Final results are:

1. bradyon-bradyon

$$L^2(S, m_1^2, m_2^2) = \begin{cases} = A(S, m_1^2, m_2^2)\,; & (m_1 + m_2)^2 \le S \\ = 0.\ ; & (m_1 - m_2)^2 \le S < (m_1 + m_2)^2 \\ = A(S, m_1^2, m_2^2)\,; & 0. \le S < (m_1 - m_2)^2 \\ = +\infty\ ; & S < 0, \end{cases}$$

where

$$A(S, \pm m_1^2, \pm m_2^2) = \frac{\pi}{2}\sqrt{\frac{(S \mp m_1^2 \mp m_2^2)^2 - 4(\pm m_1^2)(\pm m_2^2)}{S^2}}$$

2. bradyon-tachyon or tachyon-tachyon

$$L^2(S, \pm m_1^2, \pm m_2^2) = \begin{cases} = A(S, \pm m_1^2, \pm m_2^2); & S \geq 0. \\ = +\infty; & S < 0. \end{cases}$$

The divergence that arise at $S < 0$ are linear and come from L^2_{+-} and L^2_{-+} terms. For finding L^N with $N > 2$, the recurrence formula (2) is introduced

$$L^{N+1}(S, \pm m_1^2, \ldots\ldots, \pm m_N^2, \pm m_{N+1}^2) =$$

$$= \int d^4 p_{N+1}\, \delta(p_{N+1}^2 \pm m_{N+1}^2)\, L^N(S', \pm m_1^2, \ldots\ldots, \pm m_N^2), \qquad (2)$$

where $P^\mu P_\mu = S, (P^\mu - P^\mu_{N+1})^2 = S'$.

The divergences found for L^2 make that L^N for $N > 2$ is divergent for any S. This unexpected result shows that there arise not only the interpretation difficulties quoted previously when we try to build the formalism of tachyons which is a simple extension of methods for particles slower than light. The above divergences are of a completely different nature than those in QED, for example, since they come from pure kinematics but do not depend on interaction phenomena. They arise for tachyons and bradyons as well, because such divergences are related to the way the negative energies are taken into account.

III. THE NONINVARIANT PHASE-SPACE

We consider PS that has an ordinary meaning and has been obtained from (1) by removing the negative energy parts:

$$L^N_>(P^\mu, \pm m_1^2, \ldots\ldots \pm m_N^2) = \int \prod_{i=1}^{N} \frac{d^3 p_i}{2E_i}\, \delta^{(4)}(P^\mu - \sum_{i=1}^{N} P_i^\mu).$$

As noted, when N particles are bradyons and luxons (massless particles), L^N is Lorentz invariant. The results for L^N, where N equals merely two, are so complicated in form that we discuss here some particular cases only, but general results (any two-particle system in any reference frame) are presented in Tables 1,2. In the center of mass frame($P^\mu = (\sqrt{S}, 0.)$, S in CM is always positive for any kind of particles) we have the following

$$L^2_>((\sqrt{S}, 0.), \pm m_1^2, \pm m_2^2) = \begin{cases} = A(S, \pm m_1^2, \pm m_2^2); & E_0^2 \leq S. \\ = 0.\ ; & S < E_0^2. \end{cases}$$

Table 1

1) tachyon-tachyon ; $m_1 \leq m_2$

$L^2_>(\mathcal{P}^\mu, -m_1^2, -m_2^2) =$

$= \frac{\pi}{2}\frac{\mathcal{E}}{\mathcal{P}}$;		$a^2 + 4\mathcal{E}^2 m_2^2 \leq 4m_2^2\mathcal{P}^2$	$S < 0.$
$= \frac{\pi}{2}\frac{\mathcal{E}S + \sqrt{a^2 + 4Sm_2^2}\,\mathcal{P}}{\mathcal{P}S}$;	$b < 0.$	$a^2 + 4\mathcal{E}^2(S-a) \leq 4m_2^2\mathcal{P}^2 < a^2 + 4\mathcal{E}^2 m_2^2$	$S < 0.$
$= \frac{\pi}{2}\frac{\mathcal{E}a + \sqrt{a^2 + 4Sm_2^2}\,\mathcal{P}}{2\mathcal{P}S}$;	$0. \leq b$	$a^2 + 4\mathcal{E}^2(S-a) \leq 4m_2^2\mathcal{P}^2 < a^2 + 4\mathcal{E}^2 m_2^2$	$S < 0.$
$= \frac{\pi}{2}\frac{\mathcal{E}a + \sqrt{a^2 + 4Sm_2^2}\,\mathcal{P}}{2\mathcal{P}S}$;		$a^2 \leq 4m_2^2\mathcal{P}^2 < a^2 + 4\mathcal{E}^2(S-a)$	$S < 0.$
$= 0.$;		$4m_2^2\mathcal{P}^2 < a^2$	$S < 0.$
$= A(S, -m_1^2, -m_2^2)$;	$m_2^2 - m_1^2 < (\mathcal{E} - \mathcal{P})^2$		$0. \leq S$
$= \frac{\pi}{2}\frac{\mathcal{E}a + \sqrt{b^2 + 4Sm_1^2}\,\mathcal{P}}{2\mathcal{P}S}$;	$(\mathcal{E}-\mathcal{P})^2 \leq m_2^2 - m_1^2 < (\mathcal{E}+\mathcal{P})^2$	$4m_1^2\mathcal{P}^2 < b^2 + 4\mathcal{E}^2(S-b)$	$0. \leq S$
$= 0.$;	$(\mathcal{E}+\mathcal{P})^2 \leq m_2^2 - m_1^2$		$0. \leq S$
$= \frac{\pi}{2}\frac{\mathcal{E}a + \sqrt{b^2 + 4Sm_1^2}\,\mathcal{P}}{2\mathcal{P}S}$;		$b^2 + 4\mathcal{E}^2(S-b) \leq 4m_1^2\mathcal{P}^2 < b^2$	$0. \leq S$
$= \frac{\pi}{2}\frac{\mathcal{E}}{\mathcal{P}}$;		$b^2 \leq 4m_1^2\mathcal{P}^2$	$0. \leq S$

$a \equiv S + m_1^2 - m_2^2$; $b \equiv S - m_1^2 + m_2^2$; $S = \mathcal{P}^\mu\mathcal{P}_\mu$; $\mathcal{P}^\mu = (\mathcal{E}, \bar{\mathcal{P}})$; $\mathcal{P} \equiv |\bar{\mathcal{P}}|$

Table 2

2) tachyon - bradyon

$L^2(\mathcal{P}^\mu, m_1^2, m_2^2)$			
$= \frac{\pi}{2}\frac{\mathcal{E}}{\mathcal{P}}$;	$a^2+4m_1^2\mathcal{E}^2 \le 4m_1^2\mathcal{P}^2$		$S<0$
$= \frac{\pi}{2}\frac{\mathcal{E}a+\sqrt{a^2+4Sm_1^2}\,\mathcal{P}}{2\mathcal{P}S}$;	$a^2 \le 4m_1^2\mathcal{P}^2 < a^2+4m_1^2\mathcal{E}^2$		$S<0$
$= 0$;	$4m_1^2\mathcal{P}^2 < a^2$		$S<0$
$= A(S, m_1^2, m_2^2)$;	$m_1^2+m_2^2 < (\mathcal{E}-\mathcal{P})^2$		$0 \le S$
$= \frac{\pi}{2}\frac{\mathcal{E}a+\sqrt{b^2-4Sm_2^2}\,\mathcal{P}}{2\mathcal{P}S}$;	$4m_2^2\mathcal{P}^2 < 4\mathcal{E}^2(b-S)-b^2$	$(\mathcal{E}-\mathcal{P})^2 \le m_1^2+m_2^2 < S$	$0 \le S$
$= A(S, -m_1^2, m_2^2)$;	$4\mathcal{E}^2(b-S)-b^2 \le 4m_2^2\mathcal{P}^2$	$(\mathcal{E}-\mathcal{P})^2 \le m_1^2+m_2^2 < S$	$0 \le S$
$= \frac{\pi}{2}\frac{\mathcal{E}a+\sqrt{b^2-4Sm_2^2}\,\mathcal{P}}{2\mathcal{P}S}$;	$4m_2^2\mathcal{P}^2 < 4\mathcal{E}^2(b-S)-b^2$	$S \le m_1^2+m_2^2 < (\mathcal{E}+\mathcal{P})^2$	$0 \le S$
$= 0$;	$4\mathcal{E}^2(b-S)-b^2 \le 4m_2^2\mathcal{P}^2$	$S \le m_1^2+m_2^2 < (\mathcal{E}+\mathcal{P})^2$	$0 \le S$
$= 0$;	$(\mathcal{E}+\mathcal{P})^2 \le m_1^2+m_2^2$		$0 \le S$

$a \equiv S - m_1^2 - m_2^2$; $b \equiv S + m_1^2 + m_2^2$; $S = \mathcal{P}^\mu \mathcal{P}_\mu$; $\mathcal{P}^\mu = (\mathcal{E}, \vec{\mathcal{P}})$; $\mathcal{P} \equiv |\vec{\mathcal{P}}|$

where E_0 is threshold energy and

$$E_0^2 = \begin{cases} (m_1 + m_2)^2 & \text{- for two bradyons} \\ m_1^2 + m_2^2 & \text{- for bradyon and tachyon} \\ |m_1^2 - m_2^2| & \text{- for two tachyons.} \end{cases}$$

The function A has been determined previously.

The existence of threshold in the systems containing tachyons is connected with the fact that the momentum of tachyon cannot be smaller than its mass; this corresponds to the property of bradyon the energy of which is always larger than its mass. The volume of PS for bradyons is a continuous and increasing function of energy, but when the system contains at least one tachyon, this function "jumps" at thresholds from zero to finite value and then decreases with energy. In Fig.2 is shown the volume of PS when the masses of particles are the same and equal to m. Let us notice that in this case there is no threshold in the two-tachyon system and L^2 goes to infinity when S reduces to zero. Such a property can lead to peculiar vacuum instability since the vacuum could decay into real (not virtual) tachyons, say, tachyon-antitachyon pair. These tachyons carry nearly zero energy but momenta higher than their masses. The possibility and some consequences of such vacuum instability have been discussed previously/8/, but this PS aspect of the quoted phenomena has not been examined.

The volume of PS for bradyon-luxon and tachyon-luxon systems is presented in Fig.3. When we consider the reaction

$$B \to T + L,$$

where B denotes bradyon, T tachyon and L luxon, we find that the most favourable configuration from the point of view of kinematics arises when the rest masses of B an T are the same and the velocity of tachyon is infinite (tachyon carries zero energy). If there exists "tachyon electron", e_t, as an analog of "bradyon electron", e_b, i.e., ordinary electron, the above observation means that the following transitions or even oscillations, noticed in/9/,

$$e_b \leftrightarrow e_t + \gamma$$

could occur, γ denotes photon or, more generally, luxon.

We have to stress that the above physical interpretation is valid in CM only.

As an example of the volume of PS in any reference frame, we present a system of two tachyons with equal masses.

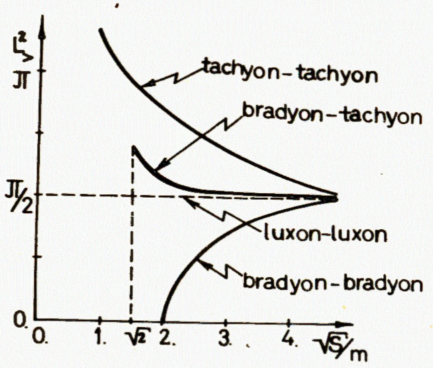

Fig.2. The volume of PS $L^2_>$ in CM for particles with equal masses as a function of energy.

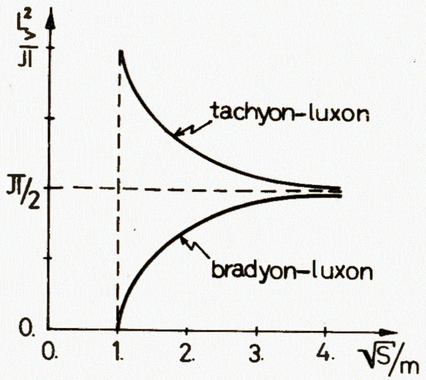

Fig.3. The volume of PS $L^2_>$ in CM for tachyon-luxon and bradyon-luxon systems as a function of energy.

$$L^2_>(\mathcal{P}^\mu,-m^2,-m^2)=\begin{cases}\frac{\pi}{2}\frac{\mathcal{E}}{\mathcal{P}}; & S^2+4m^2\mathcal{E}^2\le 4m^2\mathcal{P}^2\\ \frac{\pi}{2}\frac{\mathcal{E}S+\sqrt{S^2+4m^2S}\,\mathcal{P}}{\mathcal{P}S}; & S^2\le 4m^2\mathcal{P}^2 < S^2+4m^2\mathcal{E}^2\\ 0.;\quad 4m^2\mathcal{P}^2<S^2 & S<0.\\ \hline & S\ge 0.\\ \frac{\pi}{2}\frac{\mathcal{E}}{\mathcal{P}}; & 4m^2\mathcal{P}^2>S^2\\ A(S,-m^2,-m^2)=\frac{\pi}{2}\sqrt{1+\frac{4m^2}{S}}; & 4m^2\mathcal{P}^2<S^2\end{cases}$$

$S=\mathcal{P}^\mu\mathcal{P}_\mu$,

$\mathcal{P}^\mu=(\mathcal{E},\bar{\mathcal{P}})$,

$\mathcal{P}\equiv|\bar{\mathcal{P}}|$.

The differences of the volume of PS for the same reaction in different frames come from the fact that the energy of one or more particles in the configuration which conserves P^μ can be

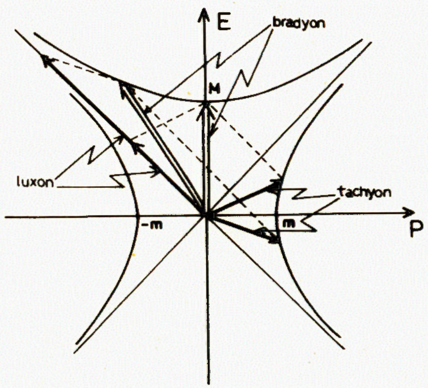

Fig.4. The decay of bradyon with mass M into luson and tachyon with mass m in the rest frame of bradyon (CM of products) and in the other moving frame. We see that the energy of tachyon in the second frame is negative.

positive in one frame and negative in another one. So in one frame such a configuration is taken into consideration while in the other one it is not.

We see that the condition, for example $4m^2\mathcal{P}^2\ge S$, defines the reference frame, more precisely the class of reference frame. This situation is shown in Fig.4. However, $L^N_>$ is noninvariant there are some reference frames where $L^N_>$ is the same. For example,

$$L^2_>(\mathcal{P}^\mu,-m^2,0.)=\frac{\pi}{2}\left(1+\frac{m^2}{S}\right)$$

if $(\mathcal{E}-\mathcal{P})^2\ge m^2$ and $S\ge 0$. See Table 1.

$L^N_>$ for $N>2$ can be calculated from the following recurrence formula(3)

$$L^{N+1}_>(\mathcal{P}^\mu,\pm m_1^2,\dots\pm m_{N+1}^2)=\int\frac{d^3p_{N+1}}{2E_{N+1}}L^N_>(\mathcal{P}^\mu-p^\mu_{N+1},\pm m_1^2,\dots\pm m_N^2).\quad(3)$$

We see that for finding $L^{N+1}_>$ even in one frame, e.g., CM, we have to know $L^N_>$ in any frame. Let us consider the two simplest cases of three-particle PS, namely two luxons and bradyon and two luxons and tachyon in CM.

$$L^3_>((\sqrt{S},0.),0.,0.,\pm m^2)=\int\frac{d^3p}{2E}L^2_>(\mathcal{P}^\mu-p^\mu,0.,0.).$$

But $L^2_>(\mathcal{P}^\mu,0.,0.)=\frac{\pi}{2}$, so after integration we get:

$$L^3_>((\sqrt{S},0.),0.,0.,\pm m^2)=\frac{\pi^2}{8}S\left(1-\frac{m^4}{S^2}\pm 2\frac{m^2}{S}\ln\frac{m^2}{S}\right).$$

For luxons $L^N_>$ can be found for any N. The recurrence formula (3) is simplified to the form

$$L_{>}^{N+1}(S, 0., \ldots, 0.) = 2\pi \int_{0}^{\sqrt{S}/2} dE\,E\,L_{>}^{N}(S - 2\sqrt{S}\,E, 0., \ldots 0.)$$

and by induction it can be proved that

$$L_{>}^{N}(S, 0., \ldots 0.) = \left(\frac{\pi}{2}\right)^{N-1} \frac{S^{N-1}}{(N-2)!\,(N-1)!} .$$

We conclude that it is easy to construct PS that is free of difficulties discussed in the previous section, however at the expense of Lorentz invariancy. We accept the view of the authors of /7/ that "the first major problem to be overcome in developing a quantum theory of tachyons is in reconciling the apparent conflict between Lorentz invariancy and need to have only positive energies capable of being observed to the theory".

The author would like to express his thanks to Professor I.Birula-Białynicki for helpful discussions.

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Received by Publishing Department
on May 5,1983.

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Мрувчински С. E2-83-299
Фазовое пространство тахионов

До сих пор не удалось получить удовлетворительного описания тахионов в теории поля, поэтому предложено рассмотреть модель фазового объема, которую можно считать нулевым приближением динамической теории. Исследовано фазовое пространство квантовых систем, в которых содержатся тахионы. Если рассматривать лоренц-инвариантный объем фазового пространства, появляются трудности в интерпретации понятия фазового объема и неожиданные расходимости. Указанные проблемы можно преодолеть, теряя лоренц-инвариантность. В работе делается вывод о невозможности построить лоренц-инвариантную теорию тахионов простым обобщением методов известных из теорий досветовых частиц.

Работа выполнена в Лаборатории высоких энергий ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1983

Mrówczyński St. E2-83-299
The Phase-Space of Tachyons

The phase-space of quantized systems that contain tachyons has been investigated. Interpretation difficulties and unexpected divergences are found when we consider the volume of Lorentz invariant phase-space. These problems can be overcome however at the expense of Lorentz invariancy.

The investigation has been performed at the Laboratory of High Energies, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1983