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QUANTUM EFFECTS IN THE SCHWINGER MODEL

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1. Introduction

More than twenty years quantum electrodynamics in two dimensions $QED_{(1+1)}$ [1] is an example where new mathematical and physical ideas in quantum field theory are checked [2-7]. In the last years the interest in the Schwinger model has even grown because this is an infrared-unstable theory with topologically degenerated vacuum, which gives an example of confinement [6]. That is why the physical interpretation of the results in $QED_{(1+1)}$ from a new point of view and the study of the methods of their derivation are very important for understanding analogous problems in quantum chromodynamics.

The aim of this paper is: the investigation of some purely quantum effects of motions of the fermionic and bosonic vacua in the massless Schwinger model. We would like to emphasize that this model gives an example of dependence of the physical results in quantum theory on the choice of the vacuum and on the global properties of the gauge field configuration space.

In the last works on the Schwinger model its solution is obtained generally by the operator method [3]. It is not restricted to the physical sector only. This makes the understanding of the role of the vacuum more complicated and leads to a slightly formal interpretation of physical results.

In section 2 the Dirac vacuum collective motions are investigated.

Section 3 is devoted to the topological degeneration of the gauge field vacuum.

We keep in the theory the Plank constant $\hbar$ as a parameter to eliminate the incorrect interpretation of quantum effects via classical mechanisms [4].

2. The Jordan effect

We'll start with the Hamiltonian of the massless Schwinger model in Coulomb gauge ($A_1 = 0$) [4,7]:

$$H = \int dx \left[i\hbar \bar{\Psi} \gamma_1 \partial_1 \Psi - \frac{e^2}{2} j_0 (\partial_1^{-2}) j_0 \right], \tag{1}$$

where (∂_1^{-2}) is a formal notation for the Coulomb interaction.

The two-component spinors:

$$\Psi(x) = \begin{pmatrix} \Psi_2 \\ \Psi_1 \end{pmatrix}, \quad \Psi^+(x) = (\Psi_2^+, \Psi_1^+)$$

satisfy anticommutation relations

$$\begin{aligned} &\{\Psi_i, \Psi_j\} = \{\Psi_i^+, \Psi_j^+\} = 0 \\ &\{\Psi_i(x), \Psi_j^+(y)\} = \delta_{ij} \delta(x-y) \end{aligned} \tag{2}$$

Currents are defined as usual

$$\begin{aligned} &j_\mu(x) = \bar{\Psi}(x) \gamma_\mu \Psi(x) \\ &j_{5\mu}(x) = \bar{\Psi}(x) \gamma_5 \gamma_\mu \Psi(x) = \varepsilon_{\mu\nu} j^\nu(x), \quad \mu,\nu = 0,1 \end{aligned} \tag{3}$$

and γ-matrices are

$$\gamma_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \gamma_1 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad \gamma_5 = \gamma_0 \gamma_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{4}$$

Let us perform fermionic operators as plane wave expansions:

$$\begin{aligned} &\Psi(x) = \sqrt{\frac{1}{2\pi}} \int dp\, e^{ipx} \begin{pmatrix} a_2(p) \\ a_1(p) \end{pmatrix} \\ &\Psi^+(x) = \sqrt{\frac{1}{2\pi}} \int dp\, e^{-ipx} (a_2^+(p), a_1^+(p)) \end{aligned} \tag{5}$$

where p is the wave number and operators a_i, a_i^+ obey the following relations

$$\begin{aligned} &\{a_i(p), a_j(q)\} = \{a_i^+(p), a_j^+(q)\} = 0 \\ &\{a_i(p), a_j^+(q)\} = \delta_{ij} \delta(p-q), \quad i,j = 1,2 \end{aligned} \tag{6}$$

The Hamiltonian (1) becomes

$$H = \int_{-\infty}^{\infty} dp \left[\hbar p \left(a_1^+(p) a_1(p) - a_2^+(p) a_2(p) \right) + \left(\frac{e}{2\pi} \right)^2 \frac{1}{p^2} j_0(p) j_0(-p) \right] \tag{7}$$

$$j_0(p) = \int dx\, e^{ipx} j_0(x) = \sum_{i=1}^{\infty} \int dk\; a_i^+(k+p) a_i(k) \tag{8}$$

In a theory with vacuum

$$a_i |0\rangle = 0$$

all currents $j_\mu(p), j_{5\mu}(p)$ commute with each other. In such a theory one can find gauge transformation operators, entirely eliminating the interaction from the Hamiltonian (7) [8]. The theory (6)-(8) becomes equivalent to the free one, where currents j_μ, $j_{5\mu}$ are conserved

$$\partial_\mu j_\mu = \partial_\mu j_{5\mu} = 0 \tag{9}$$

But the representation (6) of commutation relations (2) has one essential physical flaw: the Hamiltonian (7) is not positive definite so there is no ground fermionic state with minimal energy (operators $a_1(-p), a_2(p)$, $p > 0$ create states with negative energies).

It's well known how to proceed in this case: we define the ground state of the fermions according to the Dirac prescription, i.e. consider all states with negative energies to be occupied.

Let us redefine the operators (5) in the following way:

$$
\begin{aligned}
a_1(p) &= b(p)\theta(p) + c^+(p)\theta(-p) \\
a_2(p) &= b(p)\theta(-p) + c^+(p)\theta(p) \\
a_1^+(p) &= b^+(p)\theta(p) + c(p)\theta(-p) \\
a_2^+(p) &= b^+(p)\theta(-p) + c(p)\theta(p)
\end{aligned}
\tag{10}
$$

where operators $b(p), b^+(p), c(p), c^+(p)$ satisfy the anticommutation relations

$$
\begin{aligned}
&\{b(p), b(q)\} = \{b^+(p), b^+(q)\} = 0 \qquad \{b(p), b^+(q)\} = \delta(p-q) \\
&\{c(p), c(q)\} = \{c^+(p), c^+(q)\} = 0 \qquad \{c(p), c^+(q)\} = \delta(p-q)
\end{aligned}
\tag{11}
$$

Here $\theta(p)$ is the step-function

$$
\theta(p) = \begin{cases} 1, & p \geq 0 \\ 0, & p < 0 \end{cases}
$$

It acts as a projection operator on the states with positive $(\theta(p))$ and negative $(\theta(-p))$ energies for the solutions of the free Dirac equation for $\Psi(x)$. Thus, we establish the ground state for the free Hamiltonian

$$
b\Psi_0 = c\Psi_0 = 0
$$

This is simultaneously a ground state for the total Hamiltonian because of the positive definiteness of its interaction part.

It is the Dirac filling of the negative energy states which causes the well-known anaomaly in the axial current commutator:

$$
[j_{50}(x), j_{51}(y)] = \frac{1}{\pi i}\partial_x \delta(x-y) \tag{12}
$$

This was first shown in the thirties in several works by Jordan, Born, Socolov, and others/9/, later recovered by Mattis and Lieb/10/

The commutator (12) coincides with the commutator for a scalar field (pion)

$$
j_{5\mu}(x) = \sqrt{\frac{1}{\pi\hbar}}\,\partial_\mu \hat{\Phi}(x) \tag{13}
$$

We may directly obtain anomalous divergence of the axial current from the Heisenberg equation of motion for the component $j_{50}(x)$ and using (12)

$$
\frac{d}{dt} j_{50} = \frac{i}{\hbar}[H, j_{50}]
$$

With (12) and (13) (see Appendix A), we obtain

$$
\partial_\mu j_{5\mu} = \frac{e^2}{\pi\hbar}\sqrt{\frac{1}{\pi\hbar}}\,\hat{\Phi} \tag{14}
$$

It's easy to find the spectrum of the model from (13) and (14)

$$
\partial_\mu^2 \hat{\Phi} = \frac{e^2}{\pi\hbar}\hat{\Phi} \tag{15}
$$

It consists of a scalar particle (pion) with the mass

$$
m^2 = \frac{e^2\hbar}{\pi}
$$

It becomes now evident that the appearance of the mass for the scalar field $\hat{\Phi}$ is an entirely quantum effect.

It's interesting to note that the importance of Dirac vacuum in the anomalous axial-current divergence in four-dimensional space-time has been recently discovered by Gribov /11/.

So, a physical reason for the axial anomaly and nonconservation of the chiral charge

$$
Q_5 = \int dx\, j_{50}(x)
$$

is the Dirac vacuum **"polarization"** induced by gauge (Coulomb) field. From this point of view classical and quantum theories represent different physical theories with different symmetries although they start with the same Hamiltonian.

Note, that in such a treating of this problem there is no topological vacuum degeneration at all (we think that the introduction of such a degeneration in paper /3/ is based on the incorrect understanding of the physical reasons for chiral invariance breaking).

3. The Josephson effect

We'll show here that the transition from the classical action

$$S = \int dx\, dt\, \mathcal{L}(x) = \int dt\, L(t)$$

$$\mathcal{L}(x) = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\Psi}(x)(i\hbar\not{\partial} - e\not{A})\Psi(x) \tag{16}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad , \quad \mu,\nu = 0,1$$

to the Coulomb Hamiltonian (1) is not unique and depends on topological properties of the configuration space of the gauge field.

We'll begin with the simple case of a free gauge field in a gauge $A_0=0$:

$$\mathcal{L}_{em} = \frac{1}{2}(\partial_0 A_1)^2 \tag{17}$$

The Lagrangian (17) is still invariant under the stationary gauge transformation

$$A_1'(x,t) = A_1(x,t) + \frac{1}{e}\partial_1 \lambda(x) \tag{18}$$

In the pioneering works of Fock and Weyl /12/ these transformations were introduced as $U(1)$ phase transformations in quantum theory

$$A_1'(x,t) = e^{i\frac{\lambda(x)}{\hbar}} \left(A_1(x,t) + \frac{1}{e} i\hbar\partial_1\right) e^{-i\frac{\lambda(x)}{\hbar}} \tag{19}$$

The expression in brackets is just the Fock redefinition of the quantum-mechanical momentum $i\hbar\partial_1$ in the presence of an external field. Transformations (19) are defined on the class of smooth functions vanishing at infinity (or periodical), thus they couldn't influence the physical results of the theory:

$$\lim_{|x|\to\infty} \exp\{i\lambda^{(n)}(x)/\hbar\} = 1 \, . \tag{20}$$

$$\left[\lambda^{(n)}(\infty) - \lambda^{(n)}(-\infty) = 2\pi n\hbar\right]$$

Smooth transformations (19), (20) determine a map of the line R(1) into the circle U(1), characterized by the degree of mapping n . It shows how many times the line R(1) has turned around the circle U(1). By infinitesimal transformations only it's impossible to translate a field $A_1(x)$ from a class of the degree n to a class of degree (n+1). In other words the transformation group (19), and consequently, the configuration space $\{A_1(x)\}$ are not simply connected and are topologically equivalent to a ring $(\pi_1(U(1)) = Z)$. In such a theory there might exist constant vacuum fields without any external sources due to a purely quantum effect that appears as a field analogy of the Josephson effect /14,15/.

Let us find the stationary-states spectrum using the Schrödinger equation solution for the theory (17), (19):

$$H_0 \Psi_\varepsilon(A) = \varepsilon \Psi_\varepsilon(A) \quad , \quad H_0 = \frac{1}{2}\int \hat{E}^2 dx \tag{21}$$

$$\hat{E} = \partial_0 \hat{A}_1 = \frac{\hbar}{i} \frac{\delta}{\delta A_1(x)} \tag{22}$$

The solution of (21), (22) is just given by the eigenfunctional of the momentum operator (22), i.e. the plane wave in $\{A_1(x)\}$ space:

$$\Psi \sim \exp\left\{\frac{i}{\hbar}\int E(x)A_1(x)\,dx\right\}, \tag{23}$$

$$\hat{E}\Psi = E(x)\Psi \tag{24}$$

Because of the invariance of Ψ under topologically trivial infinitesimal transformations (with $n=0$)

$$\delta^{(0)}\Psi = 0 \Longrightarrow \partial_1 E(x)\Psi = 0 \Longrightarrow E(x) = const \tag{25}$$

field excitations in $\{A_1(x)\}$ -space are collective: there exists only one (collective) momentum for all $A_1(x)$. That's why the functional (23) depends on the collective variable $N[A]$ only:

$$N[A] = \frac{e}{2\pi\hbar}\int dx\,A_1(x) \tag{26}$$

This is just a continuous generalization of the Pontryagin index/13/

$$\nu = \frac{e}{4\pi\hbar}\int dx\,dt\,\varepsilon_{\mu\nu}F^{\mu\nu} = \int dt\,\dot{N} = N\Big|_{t=\infty} - N\Big|_{t=-\infty} \tag{27}$$

Finally we obtain the solution (23) in the form

$$\Psi = \frac{1}{\sqrt{2\pi}}\,e^{ipN}\quad,\quad \hat{E}\Psi = \frac{pe}{2\pi}\Psi \tag{28}$$

The theory is also invariant under topologically nontrivial transformations. So, the points $N, N+1, N+2, \ldots$ are physically identical:

$$\Psi(N+1) = e^{i\theta}\Psi(N)\quad,\quad |\theta| \leq \pi \tag{29}$$

This boundary condition determines the constant electric-field spectrum

$$p = 2\pi K + \theta\quad,\quad K = 0, 1, 2, \ldots \tag{30}$$

The field state functional (28) has a finite energy density

$$\mathcal{E} = \frac{1}{2}\left(\frac{ep}{2\pi}\right)^2 V\quad,\quad V = \int dx \tag{31}$$

The phase of the state functional is not homogeneous in the whole configuration space. This leads to a nonvanishing eigenvalue of the electric field in the ground state of this "ring" (when $K=0$)

$$E = \frac{e\theta}{2\pi} \tag{32}$$

This is a field analogy of the Josephson effect which consists in the existence of undamped currents when the wave function phase is not homogeneous. This nontrivial gauge-field dynamics in the two-dimensional space-time leads to nontrivial Green's function considered in Appendix B.

We'll now show that collective motions of this gauge field are directly connected with the infrared dynamics. Consider the action (16) without gauge fixing. The time component A_0 has no canonical conjugated momentum. So we'll treat it as a classical quantity. Its equation of motion

$$\frac{\delta S}{\delta A_0} = 0$$

which for (16) gives

$$\partial_1^2 A_0 = \partial_1\partial_0 A_1 - e j_0$$

is, in fact, a constraint. Its general solution is

$$A_0 = Cx + \partial_0(\partial_1^{-1})A_1 - e(\partial_1^{-2})j_0 + C_0 \tag{33}$$

Here C is a functional over dynamical variables. The solution (33) includes the zero mode of the differential operator $\partial_1^2 = 0$, taking into account collective dynamics of the system. We have already seen that it is connected with topological properties of the gauge field. We'll come to the topological dynamics (31) if consider the Pontryagin index (27) on the solutions (33) as a constraint equation between the functional C and independent topological variable $\dot{N}$:

$$\dot{N} = \frac{e}{2\pi\hbar}\int dx\, F_{01} = \frac{e}{2\pi\hbar}\left[CV - e\int dx\, \partial_1^{-1} j_0\right] \tag{34}$$

or

$$C = \left(\frac{2\pi\dot{N}\hbar}{e} + e\int dx\, \partial_1^{-1} j_0\right)\frac{1}{V} \tag{35}$$

It's not difficult now to obtain the **Hamiltonian** in terms of the topological momentum $\hbar p = \delta S / \delta \dot{N}$

$$H = \int dx\left[i\hbar\bar{\Psi}\gamma_1\partial_1\Psi - \frac{e^2}{2}j_0(\partial_1^{-2})j_0 - \hat{p}\frac{e^2}{2\pi}xj_0\right] + \frac{1}{2}\left(\frac{e\hat{p}}{2\pi}\right)^2 V \tag{36}$$

where $\hat{p}$ takes values (30).

The Hamiltonian (36) reproduces results of the gauge field quantization in a gauge $A_0 = 0$ (31) including the field analogy of the quantum Josephson effect (32). And what is more, the collective motion Lagrangian

$$L = \frac{1}{2V}\left(\frac{2\pi\hbar}{e}\right)^2 \dot{N}^2 \equiv \frac{1}{2}M\dot{N}^2$$

makes evident that the region where quantum theory is applicable (L_Q) is proportional to the one-space "volume" V ($L_Q \sim M^{-1}$, $M \sim V^{-1}$). Thus, infrared regularization must not be removed at the intermediate stage of "classical" theory.

The interaction of the current j_0 with "external" field in (36) formally coincides with Coleman's expression /4/. Coleman has introduced a constant classical field $E = e\theta/2\pi$ for reproducing the result of the chiral vacuum degeneracy /3/. In classical theory the variational principle requires the fields and their derivatives to vanish on the boundary of the space where they are defined. So, the introduction of constant fields always implies the existence of their sources. But in the action (16) there are no such sources. In the proposed here theory constant vacuum fields exist due to a purely quantum effect. The mechanism, giving them rise, is just a field analogy of the Josephson effect.

The Hamiltonian (36) is equivalent to the following effective Hamiltonian for the scalar field (pion)

$$H_{eff} = \frac{1}{2}\int dx\left\{(\partial_0\hat{\Phi})^2 + (\partial_1\hat{\Phi})^2 + \frac{e^2}{\pi\hbar}\left[\hat{\Phi} + \sqrt{\pi\hbar}\left(\frac{\hat{p}}{2\pi}\right)\right]^2\right\} \tag{37}$$

Its invariance under transformations

$$\sigma^n H_{eff}\sigma^{-n} = H_{eff} , \tag{38}$$

$$\sigma^n = \exp\{i\pi n(Q_5 + 2N)\} \tag{39}$$

is obvious.

So physical states in the Schwinger model are indeed degenerated under simultaneous transformations of the Dirac and gauge vacua. The operator $\mathfrak{d}$ acts as a "chiral"-transformation operator in the new theory with a Dirac vacuum. Without fermions H_{eff} coincides with the free gauge-field Hamiltonian, describing collective motions of its "real" vacuum.

Discussion of the results

The Schwinger model gives a good example of the dependence of the vacuum on the global symmetry of the quantum theory, and vice versa, of the influence of the choice of the ground state (vacuum) on the symmetry of the theory.

The model Hamiltonian may have the ground state with minimal energy (corresponding to the introduction of the Dirac vacuum) or may not have it. This is the reason for different symmetries in both the cases. The gauge field induces the physical manifestation of the Dirac vacuum through the axial anomaly and chiral topological invariance breaking for the positive definite Hamiltonian. Thus, it becomes evident that comparing symmetries of the initial classical lagrangian and effective quantum Hamiltonian we fall into mistake. These might be theories with the same local dynamics and different vacua. Understanding of the physical reason for the chiral symmetry breaking makes unnecessary its restitution by the θ- vacuum construction /3,6/.

We've shown that the topological vacuum degeneration in $QED_{(1+1)}$ is connected with topological properties of the gauge field itself (regarding the gauge invariance principle as a quantum theory symmetry following Fock and Weyl). Then the gauge--field configuration space is not simply connected and has the topology of a ring. This leads to the nontrivial infrared dynamics and to the existence of constant electric fields (a field analogy of the Josephson effect). The exact solution of the Schwinger model indicates the physical role of the gauge field topology in the infrared dynamics collectivization (apart from the instanton approach). As is shown in papers /14,15/, a generalization of the Josephson field effect to quantum chromodynamics gives rise to a physical picture of the "infrared vacuum" as a quantum liquid with singularities forming bags.

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Appendix A

The axial current divergence is

$$\partial^{\mu} j_{5\mu} = \partial_0 j_{50} - \partial_1 j_{51} \tag{A.1}$$

The first term in (A.1) may be evaluated with the use of the Heisenberg equation of motion for j_{50} :

$$\partial_0 j_{50} = \frac{i}{\hbar}\left[H, j_{50}\right] = \frac{i}{\hbar}\left[H, N_1 - N_2\right] ; \tag{A.2}$$

where H is the total Hamiltonian. On the other hand, it's easy to obtain the commutation relation between j_{51} and H_0 , H_0 being the free Hamiltonian

$$\left[H_0, j_{51}\right] = -\frac{1}{i\hbar}\partial_1 (N_1 - N_2) , \tag{A.3}$$

$$N_i(x) = \Psi_i^+(x)\Psi_i(x) \qquad i = 1,2$$

From (A.2) and (A.3) we easily find

$$\partial^\mu j_{5\mu} = \frac{1}{i\hbar}\left[H_I, N_2 - N_1\right] = \frac{e^2}{\pi\hbar}\sqrt{\frac{1}{\pi\hbar}}\,\hat{\Phi}$$

where

$$H_I = \frac{-e^2}{4}\iint dx\,dy\, j_0(x)|x-y| j_0(y)$$

is the interaction part of the **Hamiltonian.**

Appendix B

Using the explicit solution of the Schrödinger equation (21) with spectrum (31), we obtain the spectral representation of Green's function of the free gauge field:

$$G(N(t_2)|N(t_1)) = \frac{1}{2\pi}\sum_K e^{-\frac{i\hbar(2\pi K+\theta)^2(t_2-t_1)}{2M}} e^{i(2\pi K+\theta)(N_2-N_1)} \qquad (B.1)$$

where

$$M = \frac{1}{V}\left(\frac{2\pi\hbar}{e}\right)^2 , \quad N_i \equiv N(t_i)$$

It can be transformed into a sum over homotopy classes[16] in the configuration space:

$$G(N(t_2)|N(t_1)) = \sqrt{\frac{M}{2\pi i\hbar(t_2-t_1)}}\sum_n e^{in\theta} e^{i\frac{M}{2\hbar}\cdot\frac{(N_2-N_1+n)^2}{(t_2-t_1)}} =$$

$$= \sum_n e^{in\theta}\int \delta N \exp\{iS_n[t_1,t_2;N]/\hbar\}$$

Here

$$S_n[t_1,t_2;N] = \int_{t_1\,(n)}^{t_2} dt\, L(t) = \frac{M}{2}\,\frac{(N_2-N_1+n)^2}{t_2-t_1}$$

is the effective classical action for an n-th homotopy class.

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Квантовые эффекты в модели Швингера

Исследуется роль квантовых движений бозонного и фермионного вакуумов в модели Швингера. Дается новая интерпретация Θ-вакуума в этой модели.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Quantum Effects in the Schwinger Model

The role of quantum motions of bosonic and fermionic vacua in the Schwinger model is studied. A new interpretation of the Θ-vacuum in the model is given.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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