объединенный
институт
ядерных
исследований
дубна

E2-83-28

R.P.Zaikov

ON CONFORMAL INVARIANCE IN GAUGE THEORIES: QUANTUM ELECTRODYNAMICS

Submitted to "ТМФ"

1983

1. INTRODUCTION

As is well known (see refs. [1,2], where a complete list of original papers is given), the conformal invariance allows the complete two- and three-point functions to be determined up to a few constants only on invariance considerations. However, in the important case of gauge fields, i.e., the four-vector fields with a scale dimension to be equal to one (in units of the inverse length), the corresponding conformal covariant two-point functions have only a longitudinal part, i.e., these fields are pure gauge. To avoid this difficulty and to construct the nontrivial conformal invariant theory in the case of gauge fields, Backer and Johnson [2,3] proposed the hypothesis that special conformal transformations follow from some restricted class of gauge transformations. In papers [4,5] using this hypothesis the nontrivial model of nontrivial conformal-invariant quantum electrodynamic was constructed.

To clarify the origin of above-mentioned difficulties of unification of conformal and gauge symmetries, it is essential to consider the corresponding representations of the conformal group. In general the symmetric traceless tensor field of rank n and scale dimension d is transformed by the representation $\chi = \{d, n\}$ [1,2]. When $n=1$ and $d \neq 1, 3$, the representations $\chi = \{d, 1\}$ are irreducible. However, this is not the case, when $d = 1, 3$, the so-called exceptional points, for which the representations χ are reducible but not decomposable. For the representation $\chi = \{1, 1\}$ there is one invariant subspace formed by longitudinal vector-fields $\partial_\mu \phi$, i.e., pure gauges, and for the representation $\tilde{\chi} = \{3, 1\}$ the corresponding invariant subspace is constructed from conserved currents $\partial^\mu j_\mu = 0$. This can explain a pure longitudinal nature of two-point functions for the electromagnetic potentials and a pure transversal nature of the corresponding functions for conserved currents transforming according to the representation $\tilde{\chi} = \{3, 1\}$. Nondecomposability of the representations $\chi = \{1, 1\}$, by which the electromagnetic potentials are transformed, is used in papers [6], where also an Euclidean nontrivial model of quantum electrodynamic has been proposed.

In the present paper we also use the nondecomposability of the representations $\chi = \{1, 1\}$ and $\tilde{\chi} = \{3, 1\}$ of the conformal group SO(4,2) according to which the electromagnetic potentials and currents are transformed. It is supposed that the potentials and currents are transformed according to the nonbasic representations, i.e., such representations of conformal group which are nondecomposable for any scale dimension (see refs. [1,2]). A short description of the nondecomposable representations is given in the second section. These representations are characterized by that the generators of the special conformal transformations act on the fields $A_\mu(x)$ at point $x = 0$ in a nontrivial way, i.e., $[A_\mu(x), K_\lambda]_{x=0} = 0$. As a consequence we see that the invariant two-point functions have a nonvanishing transversal part. The explicit form of these two-point functions is found in the third section. In the forth section the invariant action is given for the model under consideration from which the equations of motion are derived. In sect. 5 the problem of quantization is discussed.

In part two of the present paper the above results will be generalized to the nonabelean case.

2. NONDECOMPOSABLE REPRESENTATIONS OF THE CONFORMAL GROUP

Here we will cite some results from ref. [1], which will be used later. The irreducible representations (IR) of the conformal group SO(4, 2) (in the Euclidean case SO(5, 1)) in general are labelled by three numbers $\chi = \{d, \nu_1, \nu_2\}$, where d is the scale dimension and ν_1 and ν_2 are the numbers labelling the IR of the Lorentz subgroup (SO(4)). Here we consider only the scalar $\nu_1 = \nu_2 = 0$ and four-vector $\nu_1 = \nu_2 = 1$ representations, which are labelled by $\chi = \{d, 0\}$, $\chi = \{d, 1\}$, respectively. As has been pointed out for the exceptional points $d = 1, 3$ the four-vector representations are not decomposable.

The same is true for the scalar representations with scale dimensions $d = 0$ and 4. The representation space is denoted by C_χ. Corresponding invariant subspaces are:

$$
\begin{aligned}
&C_{\{0,0\}} \supset F_{\{0,0\}} = \{ f \in C_{\{0,0\}} ;\ f = \mathrm{const} \}, \\
&C_{\{1,1\}} \supset F_{\{1,1\}} = \{ f_\mu \in C_{\{1,1\}} ;\ f_\mu = \partial_\mu g,\ g \in C_{\{0,0\}} \}, \\
&C_{\{3,1\}} \supset D_{\{3,1\}} = \{ f_\mu \in C_{\{3,1\}} ;\ \partial^\mu f_\mu = 0 \}, \\
&C_{\{4,0\}} \supset F_{\{4,0\}} = \{ f \in C_{\{4,0\}} ;\ f = \partial^\mu g_\mu,\ g_\mu \in C_{\{3,1\}} \}.
\end{aligned}
\tag{2.1}
$$

The peculiarity of these representations can be established by considering the conformal group Casimir operators. The second Casimir operator is given by [1]

$$\hat{C}_2 = \frac{1}{2} J^{AB} J_{AB} = \frac{1}{2} \Sigma^{\mu\nu} \Sigma_{\mu\nu} - \Delta^2 + 4i\Delta \ , \tag{2.2}$$

where Δ and $\Sigma_{\mu\nu}$ are the dilatational and Lorentz generators acting at point $x = 0$. Then for the basic symmetric traceless tensor fields

$$[\hat{C}_2, \Phi_{\mu_1,\dots,\mu_n}(x)] = [n(n+2) + d(d-4)]\Phi_{\mu_1,\dots,\mu_n}(x) \tag{2.3}$$

from which it follows that for any of the considered four representations the r.h.s. of (2.3) vanishes. As a consequence of this degeneration of the spectrum of the Casimir operators, for some values of scale dimension and tensor rank there is the above-mentioned nondecomposability of the corresponding representations. As is known (see ref. [1]), the conformal group has a class of representations, the so-called nonbasic representations. These representations are nondecomposable for any scale dimension and are characterized by the action of special conformal generators at point $x = 0$, which is nontrivial, i.e.,

$$[\Phi(x), K_\mu]_{x=0} = k_\mu \Phi(0) \neq 0 \ . \tag{2.4}$$

Here k_μ is a nilpotent operator $(b^\mu k_\mu)^\ell = 0$, $\ell = 2,3,\dots$ and b_μ is an arbitrary four-vector. In the case of nonbasic representations the second Casimir operator is given by

$$\hat{C}_2 = \frac{1}{2} J^{AB} J_{AB} = \frac{1}{2} \Sigma^{\mu\nu} \Sigma_{\mu\nu} - \Delta^2 + 4i\Delta + k^\mu P_\mu \ , \tag{2.5}$$

where $P_\mu = i\partial_\mu$ is the translational generator.

Consider the following five-component potential fields

$$A(x) = \begin{pmatrix} R(x) \\ A_\mu(x) \end{pmatrix} , \tag{2.6}$$

where $R(x)$ is a scalar field with scale dimensional d and A_μ is the four-vector field with scale dimension $d+1$. When $d = 0$, A_μ is the electromagnetic vector potential. Suppose that the field $A(x)$ has the following transformation properties with respect to special conformal transformations

$$\left[\begin{pmatrix} R(x) \\ A_\rho(x) \end{pmatrix}, K_\mu \right] =$$

$$= i \begin{pmatrix} 2x_\mu(d + x^\nu\partial_\nu) - x^2\partial_\mu & 0 \\ 2\lambda g_{\mu\rho} & [2x_\mu(d+1+x^\nu\partial_\nu) - x^2\partial_\mu]\delta^\sigma_\rho + 2ix^\nu(\Sigma_{\mu\nu})^\sigma_\rho \end{pmatrix} \begin{pmatrix} R \\ A_\sigma \end{pmatrix} , \tag{2.7}$$

where $(\Sigma_{\mu\nu})^\sigma_\rho = i(\delta^\sigma_\mu g_{\nu\rho} - \delta^\sigma_\nu g_{\mu\rho})$ are generators of the Lorentz transformations in the vector representation. Consequently, k_μ for the considered here representation is given by

$$k_\mu = 2i\lambda \begin{pmatrix} 0 & 0 \\ g_{\mu\rho} & 0 \end{pmatrix} . \tag{2.8}$$

Substitutung (2.8) in (2.5) (for $d = 0$) and taking into account (2.3) we have

$$[A(x), C_2] = -2\lambda \begin{pmatrix} 0 \\ \partial_\mu R \end{pmatrix} ,$$

i.e., the action of the Casimir operator (2.5) on the field (2.6) for $d = 0$, is given by the projection on the invariant subspace $F_{\{1,1\}}$, i.e., on the subspace of longitudinal functions.

3. COVARIANT TWO-POINT FUNCTIONS

Consider the two-point function of the five-component potential field A (2.6) in the case of arbitrary scale dimension. For our purposes it is convenient to find this function in the Euclidean momentum space:

$$G(x-y) = \int \frac{d^4p}{(2\pi)^4} e^{ip(x-y)} \tilde{G}(p) = \tag{3.1}$$

$$= \langle A(x) A^T(y) \rangle_0 = \begin{pmatrix} \langle R(x)R(y)\rangle_0 & \langle R(x) A_\nu(y)\rangle_0 \\ \langle A_\mu(x)R(y)\rangle_0 & \langle A_\mu(x) A_\nu(y)\rangle_0 \end{pmatrix} ,$$

where $A^T(x)$ is found from (2.6) by transposition. Note that from the Euclidean two-point function (3.1) a time ordered Green function can be found in the Minkowski space by the substitution

$$p_4 \to ip_0 \, , \ p^2 \to p_0^2 - p^2 - i\epsilon \, , \quad \delta_{\mu\nu} \to g_{\mu\nu} \tag{3.2}$$

and the corresponding Wightman function with the following limit (see ref. [2])

$$W(x_0, x) = \lim_{\epsilon \searrow 0} G_d(x, -\epsilon - ix_0). \tag{3.3}$$

The covariant function (3.1) can be found up to some constants only from the symmetry considerations. Indeed, the covariance with respect to special conformal transformations gives the following equations:

$$<[A(x), K_\mu] A^T(y)>_0 + <A(x)[A(y), K_\mu]^T>_0 = 0, \tag{3.4}$$

where the invariance of the vacuum state is supposed, and the action of the special conformal generators K_μ is given by (2.7). By taking into account the covariance with respect to the Lorentz and dilatational transformations, the solution of eqs. (3.4) is found in the following form

$$\tilde{G}_d = \begin{pmatrix} \frac{id}{\lambda} c_1 & c_1 p_\nu \\ -c_1 p_\mu & \left(\frac{dc_2}{2(1-d)} - \frac{i\lambda c_1}{2(1-d)}\right) \frac{g_{\mu\nu}}{p^2} + c_2 p_\mu p_\nu \end{pmatrix} (p^2)^{d-2}. \tag{3.5}$$

Here $c_{1,2}$ are constants, one of which can be determined from the normalization condition. We point out that for any value of the scale dimension $d+1$ of the four-vector potential, but the value $d=1$ for which (3.5) is singular, the covariant function has the transversal part. Consequently, for $d=0$ (3.5) contains the conformal covariant two-point Green function for nontrivial electromagnetic field. In the limiting case $d \to 0$ from (3.5) we have

$$\tilde{G}(p) = \lim_{d \to 0} \tilde{G}_d(p) = \begin{pmatrix} -\frac{1}{2\lambda^2}\delta^{(4)}(p) & \frac{2i}{\lambda}\frac{p_\nu}{(p^2)^2} \\ -\frac{2i}{\lambda}\frac{p_\mu}{(p^2)^2} & -\frac{g_{\mu\nu}}{p^2} + c\frac{p_\mu p_\nu}{(p^2)^2} \end{pmatrix}, \tag{3.6}$$

where the limit is taken in the sense of generalized functions and $c_1 = -2i/\lambda$ is substituted to find the Green function of the electromagnetic field in the standard normalization. From (3.6) it follows, that the conformal-covariant two-point Green function is given in arbitrary gauge - the conformal invariance gives no restriction of the gauge fixing parameter c. As has been mentioned above, the corresponding covariant time-ordered Green function in the Minkowski space can be found from (3.6)

by the substitutions (3.2), and the Wightman function by the limit (3.3). The Fourier kernel of the Wightman function has the following form

$$\tilde{W}(p) = \Theta(p_0) \begin{pmatrix} -\frac{1}{2\lambda^2}\delta^{(4)}(p) & \frac{2i}{\lambda} p_\nu \delta'(p^2) \\ -\frac{2i}{\lambda} p_\mu \delta'(p^2) & -g_{\mu\nu}\delta(p^2) + c p_\mu p_\nu \delta'(p^2) \end{pmatrix}. \tag{3.7}$$

Here $\Theta(p_0)$ is the ordinary theta-function, ensuring the spectrality condition.

We point out that for any value of the parameter c the Green function is nondegenerate and its inverse which is found from the condition

$$\tilde{G}^{-1}(p)\tilde{G}(p) = \tilde{G}(p)\tilde{G}^{-1}(p) = I \tag{3.8}$$

has the following form

$$\tilde{G}^{-1}(p) = \begin{pmatrix} \frac{\lambda^2}{4}(1-c)(p^2)^2 & \frac{i\lambda}{2} p_\nu p^2 \\ -\frac{i\lambda}{2} p_\mu p^2 & -g_{\mu\nu} p^2 + p_\mu p_\nu \end{pmatrix}. \tag{3.9}$$

<u>Remark</u>. Strictly speaking (3.8) is not satisfied as an operator equation on the subspace of functions V_0 satisfying the following equations

$$G\mathcal{F} = 0, \quad G^{-1}\mathcal{F}' = 0,$$

where $\mathcal{F}, \mathcal{F}' \in V_0$.

It is easy to check that the Wightmann function (3.7) satisfies the homogeneous equation

$$\tilde{G}^{-1}(p)\tilde{W}(p) = 0. \tag{3.10}$$

Also it is easy to check that G^{-1} is the covariant Green function for the five-component current

$$J(x) = \begin{pmatrix} D(x) \\ J_\mu(x) \end{pmatrix}, \tag{3.11}$$

which is transformed under special conformal transformations by the law

$$\left[\begin{pmatrix} D(x) \\ j_\tau(x) \end{pmatrix}, \tilde{K}_\mu\right] =$$

$$= i\begin{pmatrix} 2x_\mu(3 + x^\nu\partial_\nu) - x^2\partial_\mu & -2\lambda g_{\mu\tau} \\ 0 & [2x_\mu(3 + x^\nu\partial_\nu) - x^2\partial_\mu]\delta_\tau^\rho + 2ix^\nu(\Sigma_{\mu\nu})_\tau^\rho \end{pmatrix} \begin{pmatrix} D(x) \\ j_\rho(x) \end{pmatrix}. \qquad (3.12)$$

Here $D(x)$ is a scalar field with scale dimension four, and $j_\mu(x)$ is a vector current with scale dimension three. It can be established that the inverse Green function (3.9) is the interwining operator of representations T and $\tilde{T}$ which transform the fields (2.6) and (3.11), i.e.,

$$G^{-1}T = \tilde{T}G^{-1}. \qquad (3.13)$$

In an infinitesimal form (3.13) has the following form

$$G^{-1}X_{AB} = \tilde{X}_{AB}G^{-1}, \qquad (A, B = 0,1,2,3,5,6), \qquad (3.13')$$

where X_{AB} are generators of the conformal group. For generators of the Lorentz subgroup and dilutations the equality (3.13') can be checked directly. In the case of special conformal transformations, by substituting K_μ from (2.7) and $\tilde{K}_\mu$ from (3.12) after some algebraic operations we find that (3.13') is also satisfied.

At the end of this section it is necessary to point out that the scalar field $R(x)$ has extraordinary properties which follow from (3.6) and in the x-space read

$$\langle R(x), R(0)\rangle_0 \sim \text{const}, \quad \langle R(x), A_\mu(0)\rangle_0 \sim \frac{x_\mu}{x^2}.$$

Consequently, for the field

$$\tilde{R}(x) = R(x) - \langle R(x)\rangle_0$$

we have

$$\langle \tilde{R}(x)\tilde{R}(0)\rangle_0 = 0, \quad \langle \tilde{R}(x)A_\mu(0)\rangle \sim \frac{x_\mu}{x^2}. \qquad (3.14)$$

To prove the existence of the field $\tilde{R}(x)$ with the properties (3.14), it is necessary to propose the nilpotent properties for

$\tilde{R}(x)$. One representation with such properties is given by

$$\tilde{R}(x) = \begin{pmatrix} 0 & r(x) \\ 0 & 0 \end{pmatrix}$$

and the degenerate vacuum state $|0\rangle = \begin{pmatrix} |0\rangle \\ |0\rangle \end{pmatrix}$ also must be supposed.

4. COVARIANT EQUATIONS OF MOTION

Consider the following invariant bilinear form

$$I = \int d^4x \,\{ \frac{1}{2}A^T(x)G^{-1}(\partial)A(x) + A^T(x)J(x)\} =$$

$$= \int d^4x\{\frac{1}{2}A^\mu(x)(g_{\mu\nu}\Box - \partial_\mu\partial_\nu)A^\nu(x) + \frac{\lambda}{4}(R(x)\Box\partial^\mu A_\mu(x) -$$

$$- A^\mu(x)\Box\partial_\mu R(x)) + \frac{\lambda^2(1-c)}{8}R(x)\Box^2 R(x) +$$

$$+ A^\mu(x)j_\mu(x) + R(x)D(x)\}, \qquad (4.1)$$

where $G^{-1}(\partial)$ is obtained from (3.9) with the substitution $p \to i\partial$, and $J(x)$ is the five-component electromagnetic current of the matter fields, transformed by the law (3.12). For the matter fields the standard basic representations are supposed (see refs. /1,2/.

The invariance of (4.1) with respect to the Lorentz and dilatational transformations is evident. To show the invariance of (4.1) also with respect to special conformal transformations consider first the interaction term of the Lagrangian

$$\mathcal{L}_{int} = A^T(x)J(x) = A^\mu(x)j_\mu(x) + R(x)D(x). \qquad (4.2)$$

The variation of Lint under the infinitesimal special conformal transformations has the form

$$\partial\mathcal{L}_{int} = 8(x\delta a)\mathcal{L}_{int} + 2\lambda\delta a^\mu R(x)j_\mu(x) -$$

$$- 2\lambda\delta a^\mu R(x)j_\mu(x) = 8(x\delta a)\mathcal{L}_{int},$$

where a is the parameter of special conformal transformations. Consequently, $\mathcal{L}_{int}$ is transformed as in the case of the basic fields. As the Jacobian of special conformal transformations is given by

$$\det\left|\frac{\partial(x_\mu + \delta x_\mu)}{\partial x^\nu}\right| = -8(x\delta a),$$

the interaction part of (4.1) is invariant with respect to special conformal transformations.

As a consequence of the fact that $G^{-1}(\partial)$ is an intertwining operator of the potential (2.7) and current (3.12) representations the kinetic part of (4.1) is also invariant.

Suppose that (4.1) is an invariant action of the nontrivial conformal invariant model of electrodynamics under consideration, from which we get the following equations of motion

$$G^{-1}(\partial)A(x) + J(x) = 0, \tag{4.3}$$

In terms of the components A_μ and $R(x)$ eqs. (4.3) read

$$(g_{\mu\nu}\Box - \partial_\mu\partial_\nu)A^\nu(x) - \frac{\lambda}{2}\Box\partial_\mu R(x) + j_\mu(x) = 0, \tag{4.4}$$

$$\frac{\lambda^2(1-c)}{4}\Box^2 R(x) + \frac{\lambda}{2}\Box\,\partial^\mu A_\mu(x) + D(x) = 0. \tag{4.5}$$

Equations (4.4) are the Maxwell equations in the presence of sources (given by the matter current). If it is required that this current is conserved $(\partial^\mu j_\mu = 0)$, from (4.4) we get that the scalar field $R(x)$ satisfies the fourth-order free field equation

$$\Box^2 R(x) = 0. \tag{4.6}$$

With the latter equation, eq. (4.5) becomes

$$\frac{\lambda}{2}\Box\partial^\mu A_\mu(x) + D(x) = 0. \tag{4.7}$$

The conformal covariance of eqs. (4.3) or (4.4) and (4.5), that can be checked directly, is a consequence of the invariance of the action (4.1). We point out that the parameter c depending on the choice of gauge in the propagator of electromagnetic field (3.6) enters only into eq. (4.5) for the scalar field $R(x)$.

To investigate the behaviour of the action (4.1) under the gauge transformations

$$A_\mu(x) \to A_\mu(x) + \partial_\mu\phi(x), \quad R(x) \to R(x) \tag{4.8}$$

we write down the free field part of the action in an equivalent (up to the full divergent terms) form

$$I' = \int d^4x \left\{-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{\lambda}{2}B(x)\partial^\mu A_\mu(x) + \frac{\lambda^2(1-c)}{8}B^2(x)\right\}, \tag{4.9}$$

where the notation $B(x) = \Box R$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is used. It is evident that (4.4) coincides with the action of a free electromagnetic field with the gauge fixing term [10]. Note that the equations following from (4.9) when $B(x)$ is considered as a Lagrange multiplier

$$\partial^\mu F_{\mu\nu} - \frac{\lambda}{2}\partial_\nu B = 0,$$

$$\partial^\mu A_\mu(x) + \frac{\lambda(1-c)}{2}B(x) = 0,$$

are not conformal covariant.

5. CONFORMAL COVARIANT QUANTIZATION OF THE ELECTROMAGNETIC FIELD

Quantization of the electromagnetic field will follow the procedure proposed in ref. [5], where the formalism of Gupta-Bleuler for the nontrivial conformal invariant models is generalized. The intrinsic difference from the standard Gupta-Bleuler formalism is that the Lorentz condition (as expected) separating the physical space is replaced by eq. (4.7) (as expected). Notice that eq. (4.7) is conformal invariant, that is not the case of the Lorentz condition $\partial^\mu A_\mu = 0$. The condition (4.7) with $D = 0$ is considered in papers [8,9], where it has been observed that the Lorentz condition is inconsistent in the presence of strong interactions. Note also that the subsidiary condition (4.7) is different in form in the free case $(D(x) = 0)$ and in the interaction case $(D(x) \neq 0)$.

For simplicity consider first the free field case, when eq. (4.7) has the form

$$\Box\partial^\mu A_\mu(x) = 0, \tag{5.1}$$

which is conformal invariant if eq. (4.5) is satisfied. Consider the following one-particle states

$$|\Phi_1\rangle = \int d^4x\,\{\xi^\mu(x)A^+_\mu(x)|0\rangle + \eta(x)R^+(x)|0\rangle\}, \tag{5.2}$$

where $A^+_\mu(x)$ and $R^+(x)$ are positive frequency parts of the corresponding fields, $\xi^\mu(x)$ and $\eta(x)$ are arbitrary functions $\xi_\mu(x),\ \eta(x) \in S(R^4)$, which are transformed by the current representation (3.12). To ensure the transversality of the physical states (5.2) and consequently, their gauge invariance, the following condition is required

$$\partial^{\mu}\xi_{\mu}(x) = 0 , \tag{5.3}$$

i.e., $\xi_{\mu}(x) \in D_{\{3,1\}}$ (see (2.1)). Note that (5.3) is a sufficient condition for the separation of states $|\Phi_1>$ with a positive definite norm. Indeed, from (5.2) we have

$$\begin{aligned} <\Phi_1|\Phi_1> = \int d^4x d^4y \{ & \xi^{\mu}(x)\xi^{\nu}(y)W_{\mu\nu}(x-y) + \\ + & \xi^{\mu}(x)\eta(y)W_{\mu,}(x-y) + \eta(x)\xi^{\nu}(y)W_{,\nu}(x-y) + \\ + & \eta(x)\eta(y)W_{,}(x-y)\} , \end{aligned} \tag{5.4}$$

where $W_{,}(x-y)$ are components of the Wightman function (3.7) in x-space representation. Going to the momentum-space representation in (5.4) and substituting (3.7) we have

$$<\Phi_1|\Phi_1> = \int \frac{d^3p}{2|\underline{p}|}\{\vec{\xi}^{\,2}(\underline{p})(1-\cos^2\Theta) + \frac{\lambda^2}{4}|\eta(0)|^2\} > 0 , \tag{5.4'}$$

where $\cos\Theta = \xi\cdot p/|\xi|\,|p|$, $0 \le \Theta < \pi$, and it is taken into account that $\mathrm{Im}\lambda \ne 0$ and the condition (5.3) holds.

As a consequence of equation (3.10) we get that in the free case the equations of motion are satisfied as expected on the average (for the one-particle states), i.e.,

$$<0|G^{-1}(\partial)A(x)|\Phi_1> = 0 , \tag{5.5}$$

which in terms of components are written down

$$<0|(g_{\mu\nu}\Box - \partial_{\mu}\partial_{\nu})A^{\nu}(x) - \frac{\lambda}{2}\Box\,\partial_{\mu}R(x)|\Phi_1> = 0 , \tag{5.6}$$

$$<0|\partial^{\mu}\Box A_{\mu}(x)|\Phi_1> = 0 . \tag{5.7}$$

Note that the subsidiary condition (5.7) is satisfied not only for the physical states for which (5.3) is satisfied, but also for any one-particle states. To find a connection between the condition (5.3) separating the physical space and eq. (5.7), recall that the transversal components do not form an invariant subspace (see (2.1)), i.e., the corresponding transformed components depend also on longitudinal components. Using the invariance of eq. (5.7) and following paper /5/, we require that

$$\partial^{\mu}\Box\frac{\partial x'^{\rho}}{\partial x^{\mu}}<0|A_{\rho}(x')|\Phi_1> = 0 , \tag{5.8}$$

where $x'^{\rho} = (x^{\rho} + a^{\rho}x^2)/(1 + 2ax + a^2x^2)$ is a special conformal transformation of the coordinates. It is checked that (5.8) is satisfied if $\xi_{\mu} \in D_{\{3,1\}}$, i.e., the transversality condition (5.3) is fulfilled.

Notice that the requirement that $\xi_{\mu}(x)$ and $\eta(x)$ are transformed by the currents representation (3.12) results in the identity

$$<0|A'_{\mu}(x)|\Phi_1> = <0|A_{\mu}(x)|\Phi'_1(x)> , \tag{5.9}$$

where the transformation law for states is given by

$$|\Phi'> = \int d^4x\{\xi'^{\mu}(x)A^{+}_{\mu}(x)|0> + \chi'(x)R^{+}(x)|0>\} . \tag{5.10}$$

At the end we point out that the above given procedure of quantization can be generalized to any n-particle states, including the interacting case. In the interaction case eq. (4.3) should be satisfied on the average, i.e.,

$$<\Phi_j|G^{-1}(\partial)A(x) + J(x)|\Phi_k> = 0 , \tag{5.11}$$

$$(j, k = 0, 1, \dots)$$

from which we get the following subsidiary condition separating the physical states

$$<\Phi_j|\frac{\lambda}{2}\Box\,\partial^{\mu}A_{\mu} + D(x)|\Phi_k> = 0 , \qquad (j, k = 0,1,\dots) .$$

Here $j_{\mu}(x)$ and $D(x)$ are currents of external sources or matter currents.

A consistent scheme of canonical quantization for the considered model and the corresponding nonabelian model will be considered in the following papers on this subject.

ACKNOWLEDGEMENTS

We are pleased to thank Ch.Ya.Christov, I.T.Todorov, S.P.Kuleshov, and D.Robaschik for the stimulating interest and helpful discussions. Interesting discussions with M.Ya.Palchik and G.M.Sotkov are also acknowledged.

REFERENCES

1. Dobrev V.K., Mack G., Petkova V.B., Petkova S.G., Todorov I.T. Harmonic Analysis in the n-Dimensional Lorentz Group and Its Application to Conformal Quantum Field Theory. Lecture Notes in Physics, Springer, 1977, p.63.
2. Todorov I.T., Mintchev M.C., Petcova V.B. Conformal Invariance in Quantum Field Theory, Scuola Normale Superiore, Piza 1978.
3. Baker M., Johnson K. Physica, 1979, 96A, No. 1, p. 120.
4. Sotkov G.M., Stoyanov D.Tz., J.Physica A, 1980, 13, p. 2087.
5. Стоянов Д.Цв., Сотков Г.М. ОИЯИ, Р2-81-655, Дубна, 1981.
6. Palchik M.Ya. Preprint IAE, No. 180, Novosibirsk, 1982.
7. Bayen F., Flato M. J.Math.Phys., 1976, 17, p. 1112.
8. Haller K., Landowitz F. Phys.Rev., 1968, 117, p. 1749.
9. Tomczak S., Haller K. Nuovo Cim., 1972, B8, p. 1.
10. Nakanishi N. Prog.Theor.Phys., 1966, 35, p. 1111.

Received by Publishing Department
on January 21, 1983.

Зайков Р.П. E2-83-28

О конформной инвариантности в калибровочных теориях. Квантовая электродинамика

В работе предлагается нетривиальная конформно-инвариантная модель квантовой электродинамики. Основным является предположение, что электромагнитный потенциал вместе с дополнительным скалярным полем преобразуется по неосновному и, следовательно, неразложимому представлению конформной группы. Получены нетривиальные функции распространения, инвариантное действие и выведенные из него уравнения движения. Рассматривается ковариантная процедура квантования и показано, что норма одночастичных состояний положительна.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1983

Zaikov R.P. E2-83-28

On Conformal Invariance in Gauge Theories: Quantum Electrodynamics

In the present paper another nontrivial model of the conformal quantum electrodynamics is proposed. The main hypothesis is that the electromagnetic potential together with an additional zero scale dimensional scalar field is transformed by a nonbasic and, consequently, nondecomposable representation of the conformal group. There are found nontrivial conformal covariant two-point functions and an invariant action from which equations of motion are derived. There is considered the covariant procedure of quantization and it is shown that the norm of one-particle physical states is positive definite.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1983