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SPACE-TIME STRUCTURE OF RADIATIVE CORRECTIONS

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1. INTRODUCTION

The Schwinger source theory (ST) [1,2] deals with symbolic idealizations of realistic phenomena observed experimentally. The space-time description is preferred. The basic notion of the theory is the physical source, an abstraction of any processes that lead to the creation or absorption (detection) of a real particle in the localized to a certain extent space-time region. This region is a carrier of the function describing the source in the coordinate space. The source function is proportional to the amplitude of the quantum state in question creation probability.

The ST ideology is well illustrated by the free-particle consideration. If only two sources are in the laboratory, one emits a particle in some quantum state and the other detects the same state (destroying the latter), one says about the free propagation of a ("physical") particle between two sources (Fig.1). It is important that the boundary points of particle evolution are situated on finite, experimentally controlled distances, since the infinitely remote source cannot be physically realized. The smooth limit from arbitrary large to infinite distances, which is in fact assumed [3] in quantum field theory (QFT), is absent here. One may say that ST is reduced to QFT as the sources are shifted to the infinity and thus are no longer physical sources.

Notice that the primary notion in ST is an interaction act (the source) through which the free particles in particular are determined.

ST has some essential advantages over QFT. The ultraviolet divergences are absent, there are no asymptotic states and related difficulties. It is just diagrams that have a physical meaning.

The appearance of a nonphysical pole [2] of the total photon propagator in ST like in QFT [4] looked strange in such a prosperous picture.

The present paper shows that in ST nonphysical singularities do not arise provided the basic principles are kept carefully. Thus, the momentum-transfer region, where the theory is formally applicable, is expanded. Our consideration is suitable for any types of Green functions, however, we consider only the photon propagator.

Fig. 1

In papers [5], where the ϕ^4_4 model is considered, and in ref. [6], devoted to the Wess-Zumino model, the tendency towards conservation of nonphysical singularities in an exact expression for total propagators is marked. It is remarkable that these singularities are the same as in one-loop leading-logarithms approximation for a polarizable operator [4]. Thus the investigation of this approximation acquires an additional sense.

In section 2 the calculation of the photon propagator in the two-particle exchange approximation (analogous to the QFT one-loop approximation) within the framework of ST is briefly reproduced and the reasons for divergences to be absent are discussed. In section 3 an important property of the extended source localizability is shown. In section 4 a modification of such a source is demonstrated. In section 5 an expression without nonphysical singularities, however, being analytical in the coupling constant $\alpha = 1/137$ around zero is constructed. Finally, in section 6 the comparison with QFT is done and the reasons for nonphysical singularities are established.

2. PHOTON PROPAGATOR IN ONE-LOOP APPROXIMATION

Sources turn the vacuum of a given quantum state into this very state and vice versa. Suppose, we have an emission source $J^\mu_E(x)$ and an absorption source $J^\mu_A(x)$ (Fig.1), the source J_A being localized in time later than J_E. Then the vacuum-persistence probability amplitude $\langle 0_+ | 0_- \rangle^J$ (we mind the probability of non-emission of photons or the emission with subsequent absorption) equals [1]:

$$\langle 0_+ | 0_- \rangle^J = \exp(iW), \qquad (1)$$

$$W = \frac{1}{2}\int dx\,dx' J^\mu(x) D_0(x-x') J_\mu(x'), \qquad (2)$$

$$\partial_\mu J^\mu(x) = 0, \quad J^\mu = J^\mu_A + J^\mu_E ,$$

where $J^\mu(x)$ is real.

$$D_0(x-x') = \int \frac{d^4p}{(2\pi)^4}\,\frac{\exp[ip(x-x')]}{-p^2 - i0}. \qquad (3)$$

Terms in the exponential expansion (1) describe [1] the exchange of a various number of free photons. In particular, the one-photon term includes the contribution:

$$\dots + i\int dx\,dx' J^\mu_A(x)\,[\,i\int d\omega_p\, e^{ip(x-x')}]\,J_{\mu E}(x') + \dots , \qquad (4)$$

where $d\omega_p = \frac{d^3p}{(2\pi)^3}\,\frac{1}{2p^0}$, $p^0 = +\sqrt{\vec{p}}$,

and (3) is partially integrated bearing in mind the inequality $x_0 > x'_0$.

The photon field $\mathcal{A}^\mu(x)$ is defined in the Lorentz gauge by

$$\mathcal{A}^\mu(x) = \int dx' D_0(x-x') J^\mu(x') \qquad (5)$$

and satisfies the equation

$$-\partial^2 \mathcal{A}^\mu(x) = J^\mu(x). \qquad (6)$$

An analogous treatment is suitable for spinor sources $\eta(p)$, their field $\psi(x)$ and the propagator $G_0(x-x')$ too.

The photon source J^μ_E can produce the e^+e^- pair (such a source is called the extended source) due to the photon-fermion interaction in the following way. The source emits a virtual ($k > 4m^2$) photon which cannot propagate at macroscopic distances, but is able to turn into the real pair capable to reach the remote absorption source J^μ_E, where the reverse process can take place (Fig.2). Such a mechanism leads to the alterations in the propagator D_0.

Indeed, the term appears (see page 29 in [2]) in the vacuum amplitude (1)

$$-\int_{(2m)^2}^{+\infty} dM^2 \int d\omega_k\, \mathcal{A}^\mu_A(-k)\, I(M^2)\, \mathcal{A}_{\mu E}(k), \quad M^2 = k^2 . \qquad (7)$$

Here

$$I(M^2) = \frac{\alpha}{3\pi} M \left(1 + \frac{2m^2}{M^2}\right)\left(1 - \frac{4m^2}{M^2}\right)^{1/2} . \qquad (8)$$

Fields $\mathcal{A}_{A,E}$ are connected, respectively, with sources $J_{A,E}$ through the type (5) correlation. For example,

$$\mathcal{A}^\mu_E(k) = \frac{1}{-k^2} J^\mu_E(k) = -\frac{1}{M^2} J^\mu_E(k). \qquad (9)$$

The substitution of expression (9) into (7) gives

$$i\int dM^2 \frac{I(M^2)}{M^4}\, i\int d\omega_k\, J^\mu_A(-k) J_{\mu E}(k). \qquad (10)$$

But

$$i\int d\omega_k\, J^{\mu}_{A}(-k)\, J_{\mu E}(k) =$$

$$= \int dx\, dx'\, J^{\mu}_{A}(x)\,[\,i\int d\omega_k\, e^{ik(x-x')}\,]\, J_{\mu E}(x').$$

In brackets we recognize the quantity analogous to (4). We would remind that here $x_0 > x_0'$, $k^2 = M^2$.

Joining this term that is bilinear in sources, with the corresponding term (4) from (1), we get in the momentum representation

$$\tilde{D}(p^2) = \frac{1}{-p^2 - i0} + \int_{(2m)^2}^{+\infty} dM^2\, \frac{I(M^2)}{M^4}\, \frac{1}{-p^2 + M^2 - i0}, \qquad (11)$$

what is identical with the one-loop approximation in QFT where the renormalization is performed at the zero point /3/.

It should be stressed that there are no ultraviolet divergences in the expression (11). This gratifying circumstance is a straightforward result from the multiplier M^{-4} in expressions (10) and (11) that has appeared in the course of transition (9) from fields to sources. It means that the production of the corresponding functions turns out to be well defined. That situation became possible because of the consideration of sources situated at finite distances. The refusal of the description based on such sources immediately leads to the appearance (see page 10 in ref. /2/) of ultraviolet divergences.

It should be stressed that ST is not reduced to the packet formulation in QFT, as the packet obeys the homogeneous field equation (cf. (6)) and thus could not be created by any physical source.

3. LOCALIZABILITY OF EXTENDED SOURCE

ST is based on two principles /1/. These are the space-time uniformity (this principle is expressed through the Euclidean postulate) and the causality principle. The latter in ST means the requirement that a limited (localized) in space-time region of the emission and correspondingly absorption may be pointed out for every real particle or a system of real particles. The absorption region is localized, of course, in future with respect to the emission one.

For instance, if the source of a real particle is realized through a collision of the other particles, the emission region is the region of intersection of the initial-particle beams.

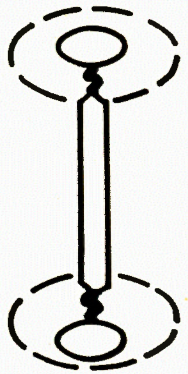

Fig. 2

An e^+e^- pair source formed by an extended photon source is localized near the photon one, as a virtual ($k^2 > 4\,m^2$) photon cannot leave the source region. In Fig.2 the localized sources are encircled by dashes.

Let us consider now the space-time structure of the contribution to the propagator (Fig.2). Two sources interact by exchanging a real e^+e^- system at arbitrary large distances while virtual photons are localized in a microscopic vicinity of the sources.

The localizability problem is discussed in ref. /1/ (pages 267, 362, 364) and in ref. /2/ (page 389) in detail. Here it will be exemplified for free spinless particles. The field ϕ created by an arbitrary source distribution K is (Δ is the causal propagator)

$$\phi(x) = \int dx'\, \Delta(x - x')\, K(x')$$

$$= \int \frac{dp}{(2\pi)^4}\, \frac{e^{ip(x-x')}}{-p^2 + m^2 - i0}\, K(p).$$

Being interested in the virtual-particle contribution we put the source equal to zero on the mass hyperboloid, that means the elimination of this hyperboloid from the integration region. From the Sokhotski relation

$$\frac{1}{-p^2 + M^2 - i0} = P\,\frac{1}{-p^2 + M^2} + i\pi\delta(-p^2 + M^2), \qquad (12)$$

it follows that the elimination of the hyperboloid is equivalent to the change of the rule for going around the poles in the propagator:

$$\frac{1}{-p^2 + M^2 - i0} \to P\,\frac{1}{-p^2 + M^2}. \qquad (13)$$

This rule is also confirmed by the following reasoning. The probability of the virtual time-like particle exchange between the source and remote detector is obviously equal to zero, because only the real particle has the propagation characteristics. Hence it appears /1/ (cf. (1), (2)), that

$$|\langle 0_+ | 0_- \rangle^K|^2 = \exp(-2\,\mathrm{Im}W) = 1,$$

that is,

$$\mathrm{Im} \int dx\, dx' K(x) \Delta(x - x') K(x') = 0 .$$

The last expression combined with (2) leads to the rule (13). Notice that the presence of real and virtual particles in ST on equal status is a consequence of the Euclidean postulate [2].

Let us return to the extended sources. Let the source has emitted a time-like virtual ($k^2 = M^2 > 0$) photon (No 1) that rapidly has decayed into a system (No 1) of real particles (electrons, positrons, and (or) photons). Suppose (see Fig. 3) that having not yet reached the remote detector, the system No 1 has recombined into the virtual photon No 2 rapidly decaying into the real system No 2. The decay of the photon No 2 is the source of the real system No 2. However, this source is not localized elsewhere as the kinematics of the experiment does not give the possibility of determining where(when)the creation and decay of the photon No 2 has happened. An additional detector is necessary to recognize this. An important conclusion is that the creation and recombination (and in particular reiteration of this process) of systems of real particles is possible only in the microscopic vicinity of a localized source or a detector, i.e., the extended source of a real system must be localizable.

We proceed to construct an expression for such a source. The rule (13), with M as a spectral mass now, will play a special role here.

4. MODIFICATION OF THE EXTENDED SOURCE

While calculating the correction (Fig.2) to the propagator, we used expression (9) for the virtual photon field which enters into the expression for the fermion-pair source:

$$\mathcal{J}^{\mu}(k) = D_0^P(k^2)\, J^{\mu}(k) . \qquad (14)$$

The propagator obtained from the causal one by the rule (13) is denoted by P.

The inclusion of the processes discussed at the end of section 3 (Fig.3) leads to the replacement of (14) by

$$\mathcal{J}^{\mu}(k) = \partial^P(k^2)\, J^{\mu}(k) , \qquad (15)$$

where $\partial^P(k^2)$ is the virtual photon propagator with arbitrary number of iterations of the process shown in Fig.3.

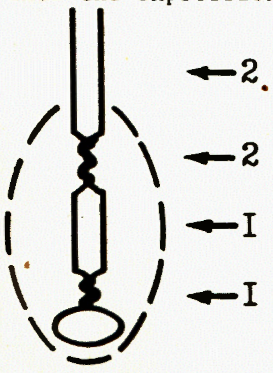

Fig.3

The equation for ∂^P may be constructed in the following way [7]. Remind expression (11), which includes processes shown in Figures 1 and 2:

$$\tilde{D}(t) = D_0(t) + \int_{(2m)^2}^{+\infty} \frac{dt'}{t' - t - i0} D_0^P(t') I(t') D_0^P(t') . \qquad (16)$$

The function I(t) (8) describes the real e^+e^- pair and D_0^P - the virtual photons. Consider the equation

$$\partial(t) = D_0(t) + \int \frac{dt'}{t' - t - i0} D_0^P(t') I(t') \partial^P(t') . \qquad (17)$$

It corresponds to the procedure of consecutive decays and recombinations near the source: the previous decay must be localized by the rule (13) in order that the subsequent one has a physical meaning. Equation (17) meets the case of localization of all loops but the last one before the final decay.Thus only one propagator $D_0^P(t')$ is substituted by $\partial^P(t')$.

The solution of Eq. (17) looks most simply [7] in the phase representation (the latter is in accord with the Kallen-Lehmann representation):

$$\partial(t) = D_0(t) \exp\left[-\frac{t}{\pi} \int_0^{+\infty} \frac{dt'}{t' - t - i0} \frac{\phi(t')}{t'}\right] , \qquad (18)$$

$$\phi(t) = \arctan\left[-\pi D_0^P(t) I(t)\right] .$$

Mention that $\partial^P(t) = \mathrm{Re}\,\partial(t)$. Bearing in mind Eqs. (15), (7) we have constructed the expression for the source of the real system which after arbitrary number of decays-recombinations in the microscopic vicinity of the source J_μ has flied out at the macroscopic distance (time).

5. TOTAL PHOTON PROPAGATOR

We have already seen that the sources exchange by the real e^+e^- pair, as in Fig.2, and any changes due to the iterations of decay-recombinations are concentrated in both the special-type propagators ∂^P describing the virtual photons. Let us come back to expression (16) corresponding to the substitution (14). Now we shall replace both the propagators $D_0^P(t')$ by $\partial^P(t')$, i.e., go to the substitution (15):

$$D(t) = D_0(t) + \int \frac{dt'}{t' - t - i0} \partial^P(t') I(t') \partial^P(t'). \qquad (19)$$

The $\partial^P(t')$ here is given by (18). We have got the representation for the total photon propagator.

All the reasoning and expressions (18), (19) remain obviously true if the words "e^+e^- pair" are changed by the words "arbitrary real-particle system described by the function $I(t)$". The spectral integration then runs from zero to infinity.

The total propagator (19) expressed through the polarizable operator, has the Kallen-Lehmann form. It is easy to make sure that the spectral weight is positive definite. It guarantees the absence of nonphysical singularities.

In paper [8] it is shown that the expression (19) is in accord with the renormgroup equations. The first term of the asymptotical expansion of (19) in the vicinity of point $t = -\infty$ is found in [9]:

$$\frac{D(t)}{D_0(t)} \sim \frac{1}{2}\left(\frac{-t}{4m^2}\right)^{\kappa}, \qquad (20)$$

$$\kappa = \frac{2}{\pi} \arctan\left[\lim_{T\to\infty} \pi T^{-1} I(T)\right] < 1 .$$

The limit exists at least in a finite-loop approximation for $I(t)$. For one-loop (see (8)):

$$t^{-1} I \to a/3\pi; \quad \kappa \approx 2a/3\pi \approx 1.5 \times 10^{-3} .$$

For two loops [2]: $t^{-1} I \to a/3\pi(1 + 3a/4\pi)$, what leaves κ practically unchanged.

The region of nearly reaching the asymptotics (20) will be pointed out below.

6. COMPARISON WITH QFT

It is interesting to compare our expressions (19), (18) for the total photon propagator with the QFT [3] one:

$$D(t) = \frac{1}{-\pi(t) - t - i0}, \qquad (21)$$

where the polarizable operator π renormalized at zero point is expressed through the function $I(t)$ from (19)

$$\pi(t) = t^2 \int \frac{dt'}{t' - t - i0} I(t') \frac{1}{t'^2} .$$

Consider in particular the one-loop approximation for π. Then the expression (21) possesses an unphysical pole [4] at $t_{pole} \approx -4m^2 \exp(3\pi/a)$. This limits the applicability region of expression (21) while the representation (19) is formally applicable at arbitrary t.

Compare two expressions at not very large $|t|$. While both are analytic in a at $a = 0$, we compare term by term the expansions of both the representations in powers of a.

A simple but bulky computation demonstrates the identity of terms of both the series in powers a^0, a, a^2. In power a^3 and higher a slight discrepancy smoothly increasing with $|t|$ takes place.

This situation allows us to conclude that the version suggested of quantum electrodynamics will retain the excellent agreement with low-energy experiments.

The analysis shows [9] that the expression (19) is in good agreement with (21) while $|t|$ is increasing from zero. Strong deviations begin when t_{pole} comes nearer. Expression (19) quickly achieves the asymptotics (20) here.

Consider the Dyson equation corresponding to (21):

$$D(x - x') = D_0(x - x') + \int d\xi\, d\xi' D_0(x - \xi)\pi(\xi - \xi')D(\xi' - x'). \qquad (22)$$

Let us establish whether the latter contradicts any ST properties and whether such an equation can be constructed here. Consider the electromagnetic field produced by a localized source J:

$$\mathcal{A}^\mu(x) = \int dx' D(x - x') J^\mu(x').$$

Substituting (22) into this expression we obtain

$$\mathcal{A}^\mu(x) = \int dx' D_0(x - x') J^\mu(x') +$$
$$+ \int dx' d\xi\, D_0(x - x')\pi(x' - \xi)\mathcal{A}^\mu(\xi).$$

In the second term the role of the source is played by the combination

$$\int d\xi \pi(x' - \xi)\mathcal{A}^\mu(\xi). \qquad (23)$$

Even if the field $\mathcal{A}^\mu(\xi)$ is localized in some region, the combination (23) does not possess any localization. This in particular means that the virtual ($k^2 > 0$) photon is able to propagate at arbitrary large distances. The completely nonlocalized source (23) cannot correspond to any really existing process and thus breaks the causality principle in ST (see section 3). Hence the Dyson equation (22) may be constructed in ST only with violating the source localizability property and simultaneously the causality principle in the form cited in section 3. The small value of discrepancies in expressions (19) and (21) at not very large $|t|$ is explained simply by a small value of the coupling constant a.

7. DISCUSSION OF RESULTS

On the example of the construction of the total photon propagator we have seen that ST is free from internal inconsistency, i.e., it is self-consistent. The equations for the electrodynamics total Green functions are necessarily consistent with spectral representations for these functions. This statement which deals both with exact and approximate solutions of such equations is absolutely not transparent in a standard QFT approach in electrodynamics.

Since the proposal about a small value of the coupling constant has been never exploited, the formal applicability region must be more wide as compared with QFT.

The departure from the Feynman diagram technique, connected with clearing up the difference in properties and roles of real and virtual particles, has allowed us to give the manifestly causal description of intermediate processes in the diagrams and to observe the basic principles of the theory at every stage of calculation.

We have come to the requirement that any transitions of real systems into virtual time-like systems and vice versa must occur in a microscopic (i.e., experimentally uncontrolled) vicinity of sources. This property leads to the picture of radiative corrections where remote sources exchange only by the real system of particles. In the space between sources no such transformations occur with those real systems (Figs.2,3).

The problem of construction of the approximate expressions for $D(t)$ in Kallen-Lehmann form has been solved in papers [10,11] with the help of the summation rule in the spectral integrand. The finite value of D/D_0 at $t \to \infty$ and the non-analyticity of $D(a)$ near $a = 0$ were observed. It is seen that the method investigated in the present paper is quite different from the mentioned one. It does not require additional hypotheses and utilizes only the basic principles of the theory.

It should be mentioned that the total photon propagator representation without nonphysical pole and simultaneously analytical in coupling constant is offered for the first time.

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Пространственно-временная структура радиационных поправок

На основе теории источников Швингера развит метод получения уравнений для полных функций Грина в электродинамике, точные и приближенные решения которых свободны от нефизических сингулярностей, а приближенные решения аналитичны по константе связи. Дана пространственно-временная картина вкладов виртуальных и реальных процессов и полные функции Грина. Проведено сравнение с уравнениями квантовой теории поля.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Vyshensky S.V. E2-83-264

Space-Time Structure of Radiative Corrections

Method of obtaining equations for the total Green functions of quantum electrodynamics, exact and approximate solutions of which are free of nonphysical singularities and approximate ones are analytic in the coupling constant at zero, is developed on the base of the Schwinger source theory. The corresponding space picture of virtual- and real-process contributions to the total Green function is given. A comparison with quantum field theory equations is presented.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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