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BETHE-SALPETER EQUATION FOR TWO-QUARK SYSTEM IN THE FOCK GAUGE

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1. INTRODUCTION

A consistent description of the composite system in QCD requires the usage of the wave function defined in a gauge-invariant way. This follows from the fact that the wave function is directly connected with such measurable quantities as the $\pi \to \mu \nu$ and $\Psi \to 3\gamma$ decay widths. Moreover, the gauge-noninvariant values do not exist in QCD in the general case because of the presence of infrared divergences

A meson, that can be considered as a bound state of quark q and antiquark $\bar{\mathbf{q}}$ interacting with each other via the gluon gauge field $\hat{\mathbf{A}}_{\mu}\left(\mathbf{x}\right)$ can be described by a gauge-invariant wave function*

$$\chi_{p}(x_{1}, x_{2}) = Sp < 0 | T{q(x_{1})}\overline{q}(x_{2}) \exp[ig \int_{x_{1}}^{x_{2}} dx^{\mu} \hat{A}_{\mu}(x)]}|P\rangle$$
, (1)

where the spur is calculated by the colour indices, and $|P\rangle$ is the state vector of a colourless meson with the 4-momentum P. In the general case the presence of the exponential factor in (1), which makes the standard Bethe-Salpeter wave function to be a gauge-invariant object, hinders the derivation of a dynamical equation for the wave function $\chi_P(x_1,x_2)$.

The choice of the light-cone gauge $A_+ = A_0 + A_3 = 0$ allows one to leave in the gauge exponent in (1) only transfer variables that are considered to be small at large momentum transfer Q. Thus, it is possible to get rid of the exponential factor in (1) up to the power corrections Q^{-2} . This method of ruling out the gauge exponent has been used, for example, in papers 4.5.1/

Nowadays it becomes clear that while describing the dynamical quantities an essential role can be played by the interactions at large distances, for which such an approximation is irrelevant. Besides, the description of the properties of composite systems, of the form factors, for example, on the basis of the dynamical equations of quantum field theory without any complementary suppositions like factorization hypothesis, requires the knowledge of the wave function at small and large distances.

In the present paper we shall derive the Bethe-Salpeter equation for the gauge-invariant wave function (1) without using any approximations.

The starting point in our consideration will be the analysis of expression (1) in the Fock gauge*

$$(x - x_0)^{\mu} \hat{A}_{\mu}^{F}(x; x_0) = 0,$$
 (2)

where \mathbf{x}_0 is some fixed point in the 4-dimensional space-time. In contrast with widely used gauges such as the Lorentz gauge $\partial_{\mu}\hat{\mathbf{A}}^{\mu}(\mathbf{x})=0$, the Coulomb $\partial_{i}\hat{\mathbf{A}}^{i}(\mathbf{x})=0$, (i=1,2,3) and axial gauge $\mathbf{n}_{\mu}\hat{\mathbf{A}}^{\mu}(\mathbf{x})=0$, condition (2) has no translation invariance. This fact will be reflected in an explicit form of the Green function of the gauge field $\hat{\mathbf{A}}_{\mu}(\mathbf{x})$, which will contain in the momentum space (as we shall see below (see (25)) the ordinary term proportional to $\delta^{(4)}(\mathbf{p}+\mathbf{k})$ and additional terms of a different structure.

An important feature of the Fock gauge (2) consists in the presence of a simple formula that connects the potential $\hat{A}_{\mu}^{F}(x;x_{0})$ with the field strength $\hat{G}_{\nu\mu}(y)^{/6/}$ (see Appendix)

$$\hat{A}_{\mu}^{F}(x;x_{0}) = \int_{0}^{1} d\alpha (x - x_{0})^{\nu} d\hat{G}_{\nu\mu}[\alpha (x - x_{0}) + x_{0}].$$
 (3)

Because of the gauge invariance of the field strength $\hat{G}_{\nu\mu}(y)$, it follows from (3) that the behaviour of $\hat{A}_{\mu}^{F}(x,x_{0})$ at translations is given by the formula $(U_{a}^{-1}\hat{G}_{\nu\mu}(y)U_{a}=\hat{G}_{\nu\mu}(y-a))$

$$U_{a}^{-1} \hat{A}_{\mu}(x;x_{0}) U_{a} = \hat{A}_{\mu}^{F}(x;x_{0}) = \hat{A}_{\mu}^{F}(x-a;x_{0}-a).$$
 (4)

Let us mention also that the theory does not contain ghosts in gauge (2).

2. THE WAVE FUNCTION

For simplicity let us consider that quarks have equal masses and define the following variables: a coordinate of the center of mass of the system of two particles $X = \frac{x_1 + x_2}{2}$ and a relative coordinate $x = x_2 - x_1$. Let us choose for the contour of integration in (1) an interval of a straight line that connects the points x_1 and x_2 . In this case the integral in (1) can be represented in the form

^{*}The discussion of the role of the operator $\exp\{ \inf_{x} \int_{1}^{x} dx^{\mu} \hat{A}_{\mu}(x) \}$ can be found, for example, $\inf^{3/3}$.

^{*}Gauge (2) has been considered also in refs. /7-11/ where it is named "fixed point gauge" or Schwinger gauge /7/.

$$\int_{x_{1}}^{x_{2}} d\xi^{\mu} \hat{A}_{\mu}^{F}(\xi, x_{0}) = \hat{I}(x, X; x_{0}) =
= \int_{-\frac{1}{2}}^{\frac{1}{2}} d\beta x^{\mu} \hat{A}_{\mu}^{F}(\beta x + X; x_{0}).$$
(5)

The behaviour of expression (5) at translations is evidently given by the formula

$$U_a^{-1} \hat{I}(x, X; x_0) U_a = \hat{I}(x, X - a; x_0 - a),$$
 (6)

i.e., the translations do not influence the relative coordinate. Let us take now for a fixed point x_0 in (2) a coordinate of the center of mass of the system $x_0 = X$, i.e., we shall choose a gauge condition

$$(x - X)^{\mu} \hat{A}_{\mu}^{F} (x) = 0$$
 (7)

for the potential $\hat{A}_{\mu}^{F}(x)$ (in what follows we shall omit the fixed point $x_0 = X$ from a set of variables of the potential $\hat{A}_{\mu}^{F}(x; x_0 = X) = \hat{A}_{\mu}^{F}(x)$. While dividing in (1) the relative motion and the motion of the system as a whole, we use the translation operator U_X . Formula (6) allows one to represent the wave function (1) in the form

$$\chi_{P}(x_{1}, x_{2}) = e^{-iPX} \chi_{P}(x), \qquad (8)$$

where the wave function of the relative motion is

$$X_{P}(x) = Sp < 0 \mid T \mid q(-\frac{x}{2}) | q(\frac{x}{2}) | exp[igl(x,0;0)] \mid P > .$$

But from (5) with the use of inversion formula (3) for a gauge (7), we find

$$\hat{\mathbf{I}}(\mathbf{x},0;0) = \int_{-\frac{1}{2}}^{\frac{1}{2}} d\beta \cdot \beta \cdot \int_{0}^{1} d\alpha \cdot \alpha \cdot \mathbf{x}^{\nu} \hat{\mathbf{G}}_{\nu\mu}(\alpha\beta\mathbf{x}) \mathbf{x}^{\mu} = 0.$$
 (9)

It is easy to see that in a more general case, we have also

$$\hat{1}(x, X; X) = 0.$$
 (10)

Thus, for the wave function $\chi_{p}(x)$ in the gauge (7), we have finally

$$\chi_{P}(x) = Sp < 0 \mid T \mid q(-\frac{x}{2}) \overline{q}(\frac{x}{2}) \mid P > .$$
 (11)

3. EQUATION FOR THE WAVE FUNCTION

It is obvious that in the Fock gauge due to formulae (8) and (11) the wave function (!) in the case of absence of an interaction satisfies the equation

$$\left(i\frac{\hat{\partial}}{\partial x_1} - m\right)\left(i\frac{\hat{\partial}}{\partial x_2} - m\right)\chi_p\left(x_1, x_2\right) = 0. \tag{12}$$

In the framework of the Fock gauge the Bethe-Salpeter equation in the ladder approximation is

$$\left[i\frac{\partial}{\partial x_1} - m\right]\left[i\frac{\partial}{\partial x_2} - m\right]\chi_P(x_1, x_2) =$$
(13)

$$= -ig^{2} \gamma^{(1)\mu} D_{\mu\nu}^{F} (x_{1}, x_{2}) \gamma^{(2)\nu} \chi_{P} (x_{1}, x_{2}),$$

where $D_{\mu\nu}^{F}(\mathbf{x}_{1},\mathbf{x}_{2})$ is the Green function of the vector field in the gauge (7).

Passing to a Fourier transform with respect to a relative corrdinate $x = x_2 - x_1$, we obtain from (13)

$$e^{-iPX} \left[\left(\frac{1}{2} \hat{P} - \hat{q} \right) - m \right] \left[\left(\frac{1}{2} \hat{P} + \hat{q} \right) - m \right] \chi_{p} (q) =$$

$$= -ig^{2} \int dx \gamma^{(1)\mu} D_{\mu\nu}^{F} \left(X - \frac{x}{2}, X + \frac{x}{2} \right) \gamma^{(2)\nu} e^{-iqx} e^{-iPX} \chi_{p} (x) .$$
(14)

In the case of translationally invariant gauges the Green function $D_{\mu\nu}$ (x,y) depends on the difference of arguments x and y, i.e., in equation (14) $D_{\mu\nu}(X-\frac{x}{2},X+\frac{x}{2})$ depends on a relative coordinate x, and the separation of the motion of the system as a whole from (14) can be done in an evident way. In the translationally noninvariant Fock gauge (2) $D_{\mu\nu}$ (x,y) does not depend on $x-y_{\rm F}$ only. Nevertheless, it will be shown in the next section that $D_{\mu\nu}(X-\frac{x}{2},X+\frac{x}{2})$ does not depend on X in the gauge

(7). Taking into account this fact the Bethe-Salpeter equation we are looking for will be

$$[(\frac{1}{2}\hat{P} - \hat{q}) - m][(\frac{1}{2}\hat{P} + \hat{q}) - m]\chi_{P}(q) =$$

$$= -ig^{2} \int \frac{d^{4}q'}{(2\pi)^{4}} \gamma^{(1)\mu} \tilde{D}_{\mu\nu}(q - q') \gamma^{(2)\nu} \chi_{P}(q'), \qquad (15)$$

where the function $\tilde{D}_{\mu\nu}(q)$ is connected with the Green function $D^F_{\mu\nu}$ in the gauge (7) as follows:

$$\tilde{D}_{\mu\nu}(q) = \int d^4 x e^{iqx} D_{\mu\nu}^{F} \left(-\frac{x}{2}, \frac{x}{2}\right) = \\
= 2^4 \int \frac{d^4 k}{(2\pi)^4} D_{\mu\nu}^{F} (2q + k, k). \tag{16}$$

From (16) one can easily see that in the case of translationally invariant conditions $\tilde{D}_{\mu\nu}(\mathbf{q})$ is an ordinary propagator of a vector particle.

4. GREEN FUNCTION

We shall consider here some different ways of deriving the Green function of a vector boson in the Fock gauge. The first of them is a standard method for the gauge theories 12. The others use the specific features of the Fock gauge, namely the presence of the inversion formula (3).

Let us put for simplicity a fixed point at the origin of the coordinate system (i.e., we pass to c.m.s.) and consider an Abelian case. The potential in this case obeys the gauge condition

$$x^{\mu} A_{\mu}(x) = 0. \tag{17}$$

The generating functional of the free Green function is written in the form of the functional integral

$$Z_{0}[J] = N^{-1} \int DA \prod_{x} \delta(x^{\mu} A_{\mu}(x)) \times \\ \times \exp\{i \int dx[-\frac{1}{4}(\partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu})^{2} + J_{\mu}(x) A^{\mu}(x)]\}.$$
 (18)

Let us use a representation

$$\prod_{\mathbf{x}} \delta[\mathbf{x}^{\mu} \mathbf{A}_{\mu}(\mathbf{x})] = \int D \Lambda \exp[-i \int d\mathbf{x} \cdot \dot{\mathbf{x}}^{\mu} \mathbf{A}_{\mu}(\mathbf{x}) \Lambda(\mathbf{x})]. \tag{19}$$

Then $Z_0[J]$ can be rewritten as

$$\begin{split} Z_0[J] &= N^{-1} \int D \Lambda \cdot D A \exp \{ i \int dx [-\frac{1}{2} A^{\mu}(x), \\ &\cdot (-g_{\mu\nu}(-\partial^2 + i0) + \partial_{\mu} \partial_{\nu}) A^{\nu}(x) + J_{\mu} A^{\mu} + x_{\mu} A^{\mu} \Lambda(x)] \}. \end{split} \tag{20}$$

Let us perform the translation

$$A_{\mu}(x) \rightarrow A_{\mu}(x) - \mathring{A}_{\mu}(x); \Lambda(x) \rightarrow \Lambda(x) - \mathring{\Lambda}(x),$$
 (21)

where $\overset{\circ}{\mathbf{A}}_{\mu}$ and $\mathbf{\Lambda}$ satisfy the equations

$$[(-\partial^2 + i0)g_{\mu\nu} + \partial_{\mu}\partial_{\nu}]\mathring{A}^{\nu} + J_{\mu} - x_{\mu}\mathring{\Lambda} = 0,$$

$$x_{\mu}\mathring{A}^{\mu} = 0 \tag{22}$$

in the coordinate space and the equations

$$[(p^{2} + i0)g_{\mu\nu} - p_{\mu}p_{\nu}]\mathring{A}^{\nu} + J_{\mu} + i\frac{\partial}{\partial p^{\mu}}\Lambda = 0,$$

$$\frac{\partial \mathring{A}_{\mu}}{\partial p_{\mu}} = 0$$
(23)

in the momentum space. We shall then have

$$Z_{0}[J] = \exp\{-\frac{i}{2} \int dx J^{\mu}(x) \mathring{A}_{\mu}(x)\} =$$

$$= \exp\{-\frac{i}{2} \int dx dy J^{\mu}(x) D_{\mu\nu}^{F}(x,y) J^{\nu}(y)\}. \tag{24}$$

Here $D_{\mu\nu}^{F}(x,y)$ is the Green function we are looking for. Thus, to find the function $D_{\mu\nu}^{F}(x,y)$, one has to solve equations (22) and (23). After solving them (see Appendix) we find in the momentum space

$$D_{\mu\nu}^{F}(p,k) = -\frac{g_{\mu\nu}\delta^{(4)}(p+k)}{p^{2}+i0} +$$

$$+ \frac{1}{p^{2} + i0} \left\{ -\frac{\partial}{\partial p_{\mu}} p_{\nu} \int_{1}^{\infty} d\beta \, \delta^{(4)} (\beta p + k) + \right.$$
 (25)

+
$$p_{\mu} \frac{\partial}{\partial p^{\nu}} \int_{0}^{1} d\alpha \delta^{(4)}(\alpha p + k) + p_{\mu} \frac{\partial^{2}}{\partial p^{2}} p_{\nu} \int_{0}^{1} d\alpha \int_{1}^{\infty} d\beta \delta^{(4)}(\alpha \beta p + k)$$
.

The first term in the right-hand side of (25) is proportional to $\delta^{(4)}$ (p+k), like in the case of translationally invariant gauges.

In the coordinate space we find

$$(2\pi)^2 D_{\mu\nu}^F (x_1, x_2) = g_{\mu\nu} \frac{1}{(x_1 - x_2)^2 - 10}$$

$$-\partial_{1\mu} x_{1\nu} \int_{0}^{1} d\alpha \frac{1}{(\alpha x_{1} - x_{2})^{2} - i0} - \partial_{2\nu} x_{2\mu} \int_{0}^{1} d\beta \frac{1}{(x_{1} - \beta x_{2})^{2} - i0} +$$
 (26)

$$+ \partial_{1\mu} \partial_{2\nu} \int_{0}^{1} da \int_{0}^{1} d\beta \frac{x_{1\mu} x_{2\nu}}{(ax_{1} - \beta x_{2})^{2} - i0}$$

From (26) it is clear that only the first term in (26) depends on the difference of x_1 and x_2 , while the others as well as the $D^F_{\mu\nu}(x_1,x_2)$ function as a whole have a more complicated structure.

Let us present here another way of deriving the Green function, which is based on the inversion formula (3). For the gauge (17), we have

$$\mathbf{A}_{\mu}(\mathbf{x}) = \int_{0}^{1} d\alpha \left[\mathbf{y}^{\rho} \mathbf{G}_{\rho \mu} (\mathbf{y}) \right]_{\mathbf{y} = \alpha \mathbf{x}}$$
 (27)

wherefrom we find

$$\frac{1}{i}D_{\mu\nu}^{F}(x_{1},x_{2}) = \langle 0 | T\{A_{\mu}(x_{1})A_{\nu}(x_{2})\} | 0 \rangle =$$
(28)

$$= \int_{0}^{1} da_{1} \int_{0}^{1} da_{2} [y_{1}^{\rho} y_{2}^{\sigma} < 0 | T \{ G_{\rho \mu}(y_{1}) G_{\sigma \nu}(y_{2}) \}].$$

The field strength $G_{\mu\nu}$ (x) is a gauge invariant value. Thus, the term $<0\,|\,T\,|\,G_{\rho\,\mu}\,(y_1)\,G_{\sigma\,\nu}\,(y_2)\,|\,0>$ can be easily calculated in any suitable gauge like the diagonal gauge.

Let us consider now the third method of derivation of the Green function $D_{\mu\nu}^F$ (x,y) and prove the statement of the previous section that $D_{\mu\nu}^F(X-\frac{x}{2}; X+\frac{x}{2})$ is independent of X in the gauge (7). The potential A_{μ} (x) in the Fock gauge (7) is related with the potential A_{μ} (x) in the diagonal gauge by

$$A_{\mu}^{F}(x) = A_{\mu}(x) - \partial_{\mu}\Lambda(x). \tag{29}$$

Let us multiply both parts of (29) by $(x - X)^{\mu}$. As a result we get the following equation for the $\Lambda(x)$ -function:

$$(\mathbf{x} - \mathbf{X})^{\mu} \partial_{\mu} \Lambda(\mathbf{x}) = (\mathbf{x} - \mathbf{X})^{\mu} \mathbf{A}_{\mu}(\mathbf{x}), \tag{30}$$

whose solution has the form (see Appendix)

$$\Lambda(x) = \int_{0}^{1} d\alpha (x - X)^{\mu} A_{\mu} [\alpha x + (1 - \alpha) X] + \Lambda(0).$$
 (31)

Thus, the T-product of the operators A_{μ}^{F} in the Fock gauge (7) can be easily expressed through the known T-product of the operators A_{μ} in the diagonal gauge

$$\overrightarrow{A}_{\mu}^{F}(x)\overrightarrow{A}_{\nu}^{F}(y) = \overrightarrow{A}_{\mu}(x)\overrightarrow{A}_{\nu}(y) -$$

$$-\frac{\partial}{\partial y^{\nu}} \tilde{A}_{\mu}(x) \tilde{\Lambda}(y) - \frac{\partial}{\partial x^{\mu}} \tilde{\Lambda}(x) \tilde{A}_{\nu}(y) + \qquad (32)$$

$$+ \frac{\partial^2}{\partial x^{\mu} \partial y^{\nu}} \stackrel{\frown}{\Lambda(x) \Lambda(y)},$$

where the operator $\Lambda(\mathbf{x})$ is connected with the operator $\mathbf{A}_{\mu}(\mathbf{x})$ according to (31).

The first term in the r.h.s. of (32) is the Green function in the diagonal gauge. Let us consider in detail the second term in (32). Simple calculations give

$$\begin{split} &\frac{\partial}{\partial y^{\nu}} \, \mathbf{A}_{\mu} \, (\mathbf{x}) \, \boldsymbol{\Lambda}(\mathbf{y}) \, = \\ &= \frac{\partial}{\partial y^{\nu}} \int\limits_{0}^{1} \mathrm{d}\alpha \, \frac{1}{4 \, \pi^{2}} (\mathbf{y} - \mathbf{X})_{\mu} \, \frac{1}{\left[\, \mathbf{x} - \alpha \, \mathbf{y} \, - (1 - \alpha) \, \mathbf{X} \, \right]^{2} - \mathrm{i} \, 0} \, = \end{split}$$

$$= \frac{1}{4\pi^2} g_{\mu\nu} \int_0^1 d\alpha \frac{1}{\left[x - \alpha y - (1 - \alpha)X\right]^2 - i0} + \frac{1}{4\pi^2} \int_0^1 d\alpha \left(y - X\right)_{\mu} \frac{2\alpha \left[x - \alpha y - (1 - \alpha)X\right]_{\nu}}{\left[\left[x - \alpha y - (1 - \alpha)X\right]^2 - i0\right]^2}.$$

While substituting (32) and (33) into equation (14), one should change the variables

$$x \rightarrow X - \frac{x}{2}$$
, (34)

$$y \rightarrow X + \frac{x}{2}$$
.

The expressions that enter (3) should change according to the rule

$$y - X \rightarrow \frac{x}{2}$$
,

$$x - ay - (1 - a) X \rightarrow -(1 + a) \frac{x}{2}$$
 (35)

Thus, we see that expression (33) becomes independent of the variable X and after calculating the integrals over α , it takes the form:

$$\frac{1}{4\pi^2} \left[\frac{2g_{\mu\nu}}{x^2 - i0} + \frac{x_{\mu} x_{\nu}}{(x^2 - i0)^2} \right]. \tag{36}$$

Analogously the independence of the third and fourth terms in (32) can be shown in changing x and y in (32) accreding to (34).

Thus, the vector field propagator $D_{\mu\nu}^F(X-\frac{x}{2};X+\frac{x}{2})$ in equation (14) can be calculated with any value of X; with the use of (26) (gauge (7) at X=0) this leads to the following form of an interaction kernel in equations (14) and (15):

$$D_{\mu\nu}^{F}(X - \frac{x}{2}; X + \frac{x}{2}) = \widetilde{D}_{\mu\nu}^{F}(x) = \frac{g_{\mu\nu} - \frac{x_{\mu}^{X}\nu}{x^{2} - i0}}{x^{2} - i0}, \qquad (37)$$

$$\widetilde{D}_{\mu\nu} (q) = \int d^4 x e^{iqx} \widetilde{D}_{\mu\nu}^F (x). \qquad (38)$$

From (37) it is clear that the Fock gauge in the coordinate representation has the same form as the Lorentz gauge in the momentum space, and the form of the $\tilde{D}^F_{\mu\nu}$ (x) function (37) is analogous to that of the Green function in the diagonal gauge in the momentum representation.

In our next publications we shall study in more detail equation (15) with an interaction kernel (37) and (38).

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APPENDIX

Here we shall consider the solutions of equations (22) and (23). By differentiating the first equation, we obtain an equation for a function

$$\frac{\partial}{\partial x_{\mu}} [x_{\mu} \mathring{\Lambda}(x)] = \frac{\partial J_{\mu}(x)}{\partial x_{\mu}}, \qquad (A.1)$$

or

$$[4 + x_{\mu} \frac{\partial}{\partial x_{\mu}}] \mathring{\Lambda}(x) = \frac{\partial J_{\mu}(x)}{\partial x_{\mu}}, \qquad (A.2)$$

Equations of the (A.2) type appear often in using the Fock gauge. For example, they appear at deriving an inversion formula (3). Let us study the equations of such a type in more detail. The general form of this equation is as follows:

$$\left[\kappa + x_{\mu} \frac{\partial}{\partial x_{\mu}}\right] f(x) = \phi(x). \tag{A.3}$$

From (A.3) it follows

$$a^{\kappa-1} \left[\kappa + y \frac{\partial}{\partial y_{\mu}}\right] f(y)_{y=\alpha x} =$$
(A.4)

$$= a^{\kappa-1} \left[\kappa + a \frac{d}{da} \right] f(ax) = \frac{d}{da} \left[a^{\kappa} f(ax) \right] = a^{\kappa-1} \phi(ax).$$

From (A.4) we find

$$f(x) = \int_{0}^{1} d\alpha \, \alpha^{\kappa-1} \phi(\alpha x) + \text{const}, \qquad (A.5)$$

where

const =
$$\lim_{\alpha \to 0} \alpha^{\kappa-1} \phi(\alpha x)$$
,

Thus, the solution of equation (A.2) has the form

$$\stackrel{\circ}{\Lambda}(\mathbf{x}) = \int_{0}^{1} d\alpha \cdot \alpha^{3} \frac{\partial J_{\mu}(\mathbf{y})}{\partial y_{\mu}} \Big|_{\mathbf{y} = \alpha \mathbf{x}},$$
(A.6)

or in the momentum space

$$\mathring{\Lambda}(p) = -i \int_{0}^{1} d\alpha \cdot (\alpha)^{-1} \frac{p_{\mu}}{\alpha} J^{\mu}(\underline{p}) =$$

$$= -i \int_{0}^{\infty} d\beta p_{\mu} J^{\mu}(\beta p). \tag{A.7}$$

From (23) it follows that a general representation of the function A_{μ} (p) can be written in the following form:

$${\stackrel{\circ}{A}}_{\mu}(p) = -\frac{1}{p^2 + 10} [J_{\mu}(p) + i \frac{\partial \hat{\Lambda}(p)}{\partial p^{\mu}} - p_{\mu} f(p)]. \tag{A.8}$$

An equation for the function f(p) can be obtained if we choose a gauge condition $\partial_\mu\,A^\mu(p)=0$ for the potential (A.8). As a result, we have

$$[2 + p_{\mu} \frac{\partial}{\partial p_{\mu}}] f(p) = \frac{\partial J^{\mu}(p)}{\partial p^{\mu}} + i \frac{\partial^{2} \Lambda(p)}{\partial p^{2}} , \qquad (A.9)$$

where we have taken into account that

$$p^{\mu}J_{\mu}(p) + ip_{\mu}\frac{\partial \mathring{\Lambda}(p)}{\partial p_{\mu}} = 0. \qquad (A.10)$$

The solution of (A.9) according to (A.5) has the form

$$f(p) = \int_{0}^{1} d\alpha \cdot \alpha \left[\frac{\partial J^{\mu}(q)}{\partial q^{\mu}} + i \frac{\partial^{2} \mathring{\Lambda}(q)}{\partial q^{2}} \right]_{q = \alpha p}$$
(A.11)

By substituting the found values of $\mathring{\Lambda}(p)$ and f(p) into (A.8), we obtain

$$\int \frac{d^4p}{(2\pi)^4} J^{\mu}(p) \mathring{A}_{\mu}(p) =$$

$$= \frac{1}{(2\pi)^4} \int d^4 p d^4 k J^{\mu}(p) \left\{-\frac{1}{p^2 + i0} \left[g_{\mu\nu} \delta^{(4)}(p + k) + \right]\right\}$$

(A.12)

$$+\frac{\partial}{\partial p^{\mu}} p_{\nu} \int_{1}^{\infty} d\beta \delta^{(4)}(p\beta + k) - p_{\mu} \frac{\partial}{\partial p^{\nu}} \int_{0}^{1} d\alpha \delta^{(4)}(\alpha p + k) -$$

$$-p_{\mu} \frac{\partial^{2}}{\partial p^{2}} p_{\nu} \int_{0}^{1} d\alpha \int_{0}^{\infty} d\beta \delta^{(4)} (\alpha \beta p + k) J^{\nu}(-k).$$

The expression in figure brackets has the sense of the Green function of the gauge field in the Fock gauge.

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Капшай В.Н., Скачков Н.Б., Соловцов И.Л. E2-83-26 Уравнение Бете-Салпетера для системы двух кварков в калибровке Фока

Калибровочное условие В.А.Фока $(x-x_0)^{\mu}A_{\mu}(x)=0$ использовано для вывода уравнения Бете-Салпетера для калибровочно-инвариантной волновой функции двухчастичной составной системы. Установлен вид функции Грина векторного поля в фоковской калибровке.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1983

Kapshay V.N., Skachkov N.B., Solovtsov I.L. E2-83-26 Bethe-Salpeter Equation for Two-Quark System in the Fock Gauge

The gauge condition of Fock $(x-x_0)^\mu A_\mu(x)=0$ is used to derive the Bethe-Salpeter equation for a gauge-invariant wave function of the two-particle system. The Green function of the vector field is found in the Fock gauge.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1983